UNIVERSITY OF DELHI

M.A. ECONOMICS SUMMER SEMESTER
COURSE 002. INTRODUCTORY MATHEMATICAL ECONOMICS
Final Examination
30th November 2012

Maximum marks: 70 **Time:** $2\frac{1}{2}$ hours

Instructions: There are 3 parts to the exam. The questions in Part A & B are compulsory. In Part C, attempt any two questions. Use separate booklet for each part.

PART A

1. A primitive farmer seeks to maximize the present value of her utility from the production of apples. Let x_t be the number of apples at time t, and z_t is the rate at which she eats the apples, then change in the stock of apples at time t is expressed: $\dot{x} = a - bx_t - z_t$.

   Her objective is to maximize the present value of utility, which she gets from two sources, the sale of apples at the price p, which occurs at the end of the season, T, and her personal consumption of apples during the season, $u(z_t) = ln(z_t)$. The constraints $x_t > 0$ and $z_t > 0$ can be assumed to hold (i.e., you do not need to explicitly write these constraints).

   (a) Formally state the decision maker's optimization problem.

   (b) Specify the Hamiltonian (current or present value) and the first-order necessary conditions for an optimum.

   (c) Give an economic interpretation of the each of the first-order conditions and the transversality condition.

   (d) Solve for the optimal path for consumption of the apples. Be sure to express your solution in terms of the known parameters of the model, a, b, p, and T. $(3, 4, 4, 5)$

PART B

2. (a) Let S be and invertible $n \times n$ matrix and let A be an $n \times m$ matrix. Show that the dimension of the kernel of A is equal to that of the kernel of SA and that the dimension of the image of A is equal to that of the image of SA.

   (b) Prove that if $P \in L(V)$ is such that $P^2 = P$ and every vector in null P is orthogonal to every vector in range P, then P is an orthogonal projection..

   (c) For every $T \in L(V, W)$ prove the followings -

   (i) dim null $T^* =$ dim null $T+$ dim$W -$ dim$V$, and

   (ii) dim range $T^* =$ dim range T. $(4, 4, 4)$

3. Consider the subspace $V = \{\overrightarrow{x} \in \Re^3 | 2x_1 + 6x_2 - 4x_3 = \overrightarrow{0}\}$.

   (a) Find a basis for V. What is its dimension of this basis?

   (b) Extend the basis from part (a) to a basis for $\Re^3$.

   (c) Write the matrix of the linear transformation $T : \Re^3 \to \Re$ such that $T\overrightarrow{x} = \overrightarrow{x}.[.]$, where [.] is the third vector in part (b). $(2, 2, 2)$

PART C

4. Solve the following problem.

   Minimize $e^{(x_1-x_2)}$ s.t.

$$\begin{aligned} &e^{x_1} + e^{x_2} \leq 20 \\ &0 \leq x_1. \end{aligned} \qquad (18)$$

5. Define a function $f : \Re^n \to \Re$ to be *coercive* if for every sequence (x_k) in $\Re^n$ s.t. $||x_k|| \to \infty$, we have $f(x_k) \to \infty$. Now suppose f is coercive and continuous. Show that then, for all $\alpha \in \Re$, the lower contour set $LCS(a) \equiv \{x | f(x) \leq \alpha\}$ is compact. (18)

6. Consider the following system of differential equations.

$$Dx_1 = (A - Ax_2 - Bx_1)x_1$$
$$Dx_2 = (Cx_1 - C)x_2$$

   where $A, B > 0$, $A > B$, $C > 1$, and $AC > B$. Dx_1 and Dx_2 are time derivatives.

   (a) Interpret the equations as a predator-prey model. How does it differ from the Lotka-Volterra model. Be very brief. (4)

   (b) Find all equilibria in which $x_1 > 0$. Using the linearization theorem, comment on the stability of these equilibria. (14)