Delhi School of Economics

M.A. ECONOMICS SUMMER SEMESTER
COURSE 002. INTRODUCTORY MATHEMATICAL ECONOMICS
Midterm 1
24th September 2012

Instructions. *Time: 70 minutes. Maximum Marks 15. Closed book closed notes exam. Attempt all the three questions. Marks for each question given in the parenthesis. Some internal options are available.*

1. (a) Let $A = \begin{bmatrix} 4 & -3 \\ 1 & 0 \end{bmatrix}$. Prove that $A^n = \frac{3^n-1}{2}A + \frac{3-3^n}{2}I_2$. Do not use iterative process. [3]

   (b) Prove the following statement:
   A function $f : X \to \Re$ is quasiconcave *iff* its *better set* is a convex set for all $x \in X$. [2]

2. Let $S \in \Re^2$ be the set of vectors $[x, y]$. Then check whether S be a vector subspace of $\Re^2$ under the following conditions separately one at a time - [4]

   (a) S satisfying $x = 2y$.

   (b) S satisfying $x = 2y + 1$.

   (c) S satisfying $xy = 0$.

   (d) S satisfying $x \geq 0$ and $y \geq 0$.

3. (a) Suppose that $X_1, X_2, ..., X_m$ form a basis for a subspace S. Can you check if
   $X_1, X_1 + X_2, ..., X_1 + X_2 + ... + X_m$ also form a basis for S or not. [3]

   (b) Prove that no subset of $V \in \Re^n$ with fewer than n vectors can $span\{V\}$. [3]

   Or,

   (b). Use the Gram-Schmidt process to orthogonalise the vector space $S \in V^3$. Where $S = (v_1, v_2) = \left(\begin{bmatrix} 1 \\ 1+i \\ 1 \end{bmatrix}, \begin{bmatrix} -i \\ 1 \\ 1+i \end{bmatrix} \right)$. Can you find an orthogonal basis for V^3? Note that orthogonal basis will have 3 vectors in it.