Econometric Analysis of Discrete Choice

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Binary Decisions

- Regression analysis concerns effect of X on Y (e.g., effect of study hours on exam results)
- Y continuously distributed in population
- But many important decisions are discrete
- Send daughter to school, ride train/car/bicycle, retire/work
- Want to know effect of covariates (education/income/caste/gender/price) on the probability of decision
- Important for policy analysis (e.g., tuition subsidy), how to target intervention

Example: Enrol teenage daughter in school in India

- School S, no-school N
- Monthly price of school P, monthly hhd expenditure W (No income data in NSS)
- **Economic** Model : Choose school if $U(S, W-P) - U(N, W) \geq 0$
- **Econometric** Model: Choose school if $U(S, W-P, \eta) - U(N, W, \eta) \geq 0$

Model of Choice

- Linear approximation

$$\begin{aligned} & U(S, W-P, \eta) - U(N, W, \eta) \\ \simeq \quad & \beta_0 + \beta_1 P + \beta_2 W + \eta \end{aligned}$$

- Latent desire to enrol $\longrightarrow$ Observed decision

$$\begin{aligned} Y^* &= \beta_0 + \beta_1 P + \beta_2 W + \eta, \\ Y &= \begin{cases} 1, \text{ if } y^* \geq 0 \\ 0, \text{ if } y^* < 0 \end{cases} \end{aligned}$$

Data and Statistical Assumptions

- Each observation in NSS is one teenager, indexed by i, observe P_i, W_i, Y_i
- $Y_i = 1\{\beta_0 + \beta_1 P_i + \beta_2 W_i + \eta_i \geq 0\}$, η_i indep of P_i, W_i
- Observed outcome nonlinear in error term η
- $\eta_i \sim N(0,1)$: Probit Model

Conditional Choice Probability

- Conditional Choice Probability

$$\begin{aligned} q(p,w) &\equiv \Pr(School_i = 1 | P_i = p, W_i = w) \\ &= \Pr(\beta_0 + \beta_1 P_i + \beta_2 W_i + \eta_i \geq 0 | P_i = p, W_i = w) \\ &= \Pr(\eta_i \geq -(\beta_0 + \beta_1 p + \beta_2 w) | P_i = p, W_i = w) \\ &= \Pr(\eta_i \geq -(\beta_0 + \beta_1 p + \beta_2 w)), \text{ by independence} \\ &= 1 - \Phi(-(\beta_0 + \beta_1 p + \beta_2 w)) = \Phi(\beta_0 + \beta_1 p + \beta_2 w) \end{aligned}$$

- Marginal Effect

$$\frac{\partial}{\partial p} \Pr(Y_i = 1 | P_i = p, W_i = w) = \beta_1 \times \phi(\beta_0 + \beta_1 p + \beta_2 w) \neq \beta_1$$

- Predicted value given price=500, income=10000

$$\begin{aligned} & \Phi(\beta_0 + 500 \times \beta_1 p + 10000 \times \beta_2) \\ \neq \quad & \beta_0 + 500 \times \beta_1 p + 10000 \times \beta_2 \end{aligned}$$

Parameter Estimation

- Once we know β's, we can predict the probability of enrolment at different price and income
- General method of estimation: Maximum likelihood
- Intuitively equivalent to minimum distance, analogous to least squares for linear model

$$\min_{\beta_0,\beta_1,\beta_2} \sum_{i=1}^{n} \left(School_i - \Phi\left(\beta_0 + \beta_1 P_i + \beta_2 W_i\right)\right)^2$$

- Get estimates $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$, can predict enrolment at price p and income w by $\Phi\left(\hat{\beta}_0 + \hat{\beta}_1 p + \hat{\beta}_2 w\right)$
- Std error, confidence intervals etc.

Endogenous Income

- W and η correlated, eg, parents' education is omitted variable
- Then probit regression does not give us what we want to know

$$q(p, w): \text{ what is the probability of school if given wealth } w$$

- If linear model then can use IV and two-stage least squares
- In nonlinear models need to use **Control Functions** – active research area
- Smith-Blundell (Ecta86), Blundell-Powell (ReStud04), Imbens-Newey (Ecta, 09)

Control function

- Find IV for income, eg., avg neighborhood income, Z
- Z indep of individual's own taste, Z correlated with W

1. Regress W_i on Z_i, P_i, residual $\hat{V}_i$
2. Add residual as additional regressor in Probit

$$\Pr(S_i = 1) = \Phi\left(\beta_0 + \beta_1 P_i + \beta_2 W_i + \underbrace{\beta_3 \hat{V}_i}_{\text{Correction term}}\right)$$

3. Finally average across $\hat{V}_i$

$$\hat{q}(500, 10000) = \frac{1}{n}\sum_{i=1}^{n} \Phi\left(\beta_0 + 500 \times \beta_1 + 10000 \times \beta_2 + \beta_3 \hat{V}_i\right)$$

Going deeper

- Simple Question

  What is the expected enrolment if govt offers a tuition subsidy of 100 rupees?

$$\frac{1}{n}\sum_{i=1}^{n}\Phi\left(\hat{\beta}_0+\hat{\beta}_1(P_i-100)+\hat{\beta}_2 W_i\right)$$

- Deeper Welfare Question

  What is the "Cash-Equivalent" (Equivalent Variation) of this subsidy, what is the deadweight loss?

My Research

- Because η varies across people, same subsidy has different cash-equivalents for different people
- **Average** EV of a Rs 100/month subsidy
- Challenge: EV based on utilities, not observable directly
- How to infer welfare-relevant features of utilities from choice data?

EV of a subsidy

- Recall $Y = 1\{U(S, W-P, \eta) - U(N, W, \eta) \geq 0\}$
- Price fall from P to $P-100$ due to subsidy
- For people with income w and facing price p_0, EV is defined by

$$\begin{aligned} & \max\{U(S, w+EV-p_0, \eta), U(N, w+EV, \eta)\} \\ = \quad & \max\{U(S, w-(p_0-100), \eta), U(N, w, \eta)\} \end{aligned}$$

- For fixed w, p_0, the EV is a function of η
- Distbn of $\eta \longrightarrow$ distbn of EV $\longrightarrow$ AverageEV

Bhattacharya (Ecta 2015)

$$
\begin{aligned}
E\left(EV\right) &= \int_{p_1}^{p_0} q\left(p, w + p - \left(p_0 - 100\right)\right) dp \\
&= \int_{p_0 - 100}^{p_0} \Phi\left(\hat{\beta}_0 + \hat{\beta}_1 p + \hat{\beta}_2 \left(w + p - \left(p_0 - 100\right)\right)\right) dp
\end{aligned}
$$

Change in Marshallian consumer surplus: Area under dd curve

$$
\begin{aligned}
&\int_{p_0 - 100}^{p_0} \Phi\left(\hat{\beta}_0 + \hat{\beta}_1 p + \hat{\beta}_2 w\right) dp \\
\neq\ & E\left(EV\right), \text{ unless } \textbf{zero income-effect}.
\end{aligned}
$$

Deadweight Loss

Efficiency cost of subsidy

$$\begin{aligned} E\left(DWL\right) &= 100 \times q\left(p_0 - 100, w\right) - \int_{p_0-100}^{p_0} q\left(p, w + p - p_1\right) dp \\ &= 100 \times \Phi\left(\hat{\beta}_0 + \hat{\beta}_1\left(p_0 - 100\right) + \hat{\beta}_2 w\right) \\ &\quad - \int_{p_0-100}^{p_0} \Phi\left(\hat{\beta}_0 + \hat{\beta}_1 p + \hat{\beta}_2\left(w + p - \left(p_0 - 100\right)\right)\right) dp \end{aligned}$$

These results bring together "empirical program evaluation" and "classical welfare theory" of public finance.

Extensions (Bhattacharya, working paper)

- Price of multiple options (school/work/stay-at-home) change simultaneously, merger analysis
- Eliminate Option (e.g. Child Labor legislation)
- Although illustrated with probit, results do not require assumptions on heterogeneity distribution

$$\begin{aligned} E(EV) &= \int_{p_1}^{p_0} q(p, w + p - (p_0 - 100))\, dp \\ q(p, w) &= \Pr(School | P = p, W = w) \end{aligned}$$

- **Nonparametric** Identification

Illustration

- Indian girls aged 15-18, 25000 households from NSS 2004
- Choice between stay-at-home/work/school
- Rs 1= \$0.02

Data Summary

	Mean	Std. Dev.	Min	Max
home	0.19	0.39	0	1
working	0.20	0.40	0	1
inschool	0.61	0.49	0	1
school-price	168.44	383.41	0	41666
local wage	123.59	170.75	0	1800
income	4063.04	6030.37	1	583192
female	0.44	0.49	0	1
age	16.13	1.70	13	18
hhsize	5.84	2.53	1	36
adult_literacy	0.34	0.23	0	1
muslim	0.15	0.35	0	1
scst	0.30	0.46	0	1

Welfare Effects

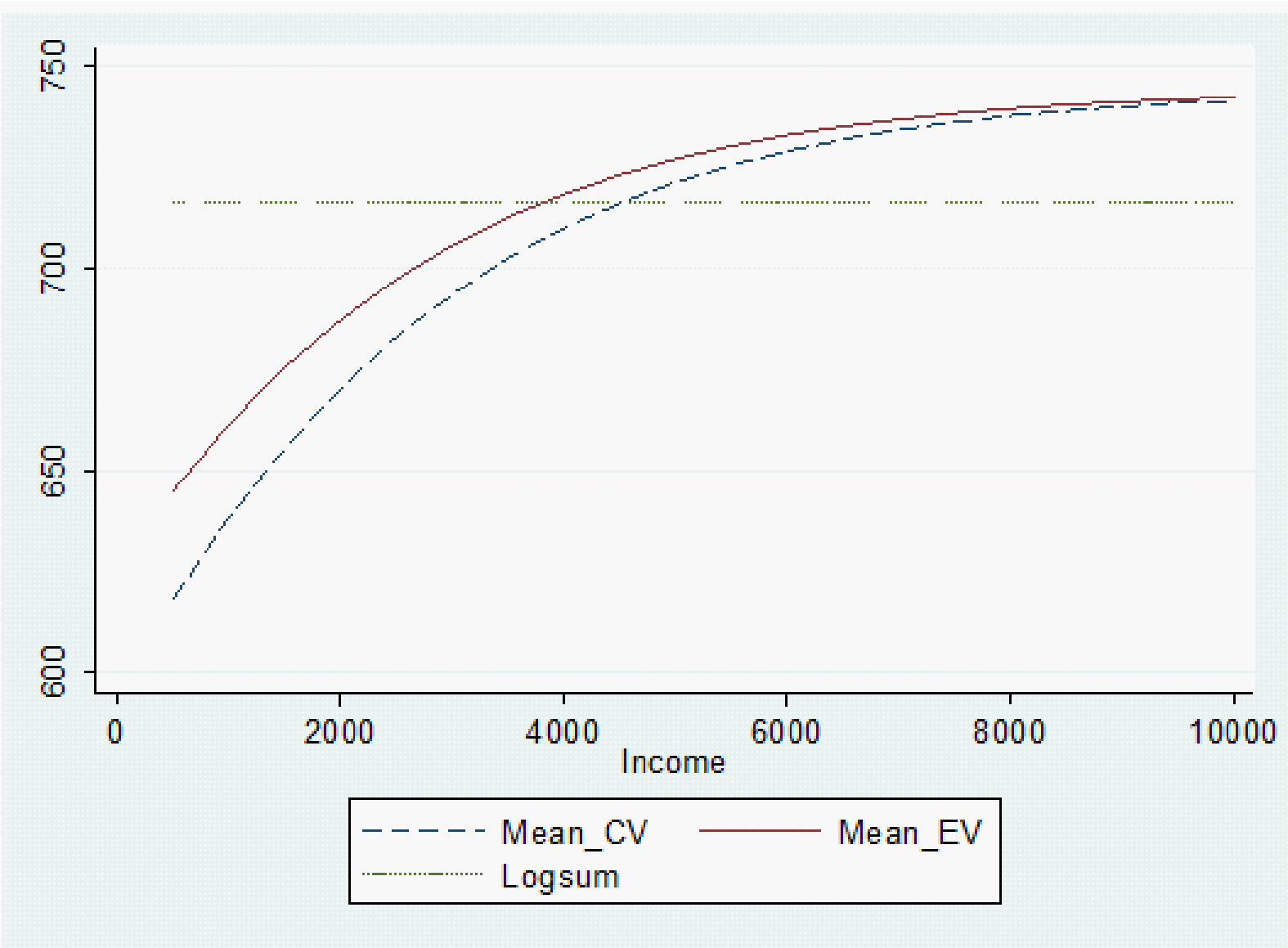
- Policy: Tuition subsidy plus wage rise $\mathbf{p}_0 = (0, -25, 790)$, $\mathbf{p}_1 = (0, -256, 45)$,
- Welfare calculation for Hindu, 15 yr old, family literacy 60%, hhsize 5, by gender and income (MPCE)

Price and Income Effects for Schooling: Semi-Elasticities

Covariate	Male	Female
School-fees	-0.02	-0.04
	(-11.41)	(-11.7)
Local wage	-0.006	-0.01
	(-3.08)	(-2.7)
Income	0.11	0.19
	-14.09	-14.6

Mean EV by Income

Female youth; tuition subsidy of Rs 745 and wage rise of Rs 251



Some related research

- Discrete Choice
  - McFadden and Domencik (1975), Small and Rosen (Ecta81), Dagsvik and Karlstrom (ReStud05)
  - Only under strong and unverifiable assumptions, e.g. no income effect
  - Can we do welfare analysis w/o making strong assumptions on preferences?
- Continuous Choice, eg., deadwt loss of taxing gasoline (petrol) consumption
  - Hausman (AER81), Hausman and Newey (Ecta15)

Continuous Choice (Hausman-Newey, Ecta 2016

$$\begin{aligned}\max_q U(q, w - pq, \eta) &\rightarrow q^*(p, w, \eta) \\ &\rightarrow U(q^*(p, w, \eta), w - pq^*(p, w, \eta), \eta) \\ &\equiv V(p, w, \eta), \text{ indirect utility function}\end{aligned}$$

For tax t on initial price p_0, EV solves

$$V(p_0, w - S, \eta) = V(p_0 + t, w, \eta) \rightarrow S(p_0, w, \eta)$$

In typical datasets, observe $\{q_i^*, P_i, W_i\}$, $i = 1, ..., n$

Unobservable from observable

Using Shephard's lemma

$$\begin{aligned} \frac{\partial S(p,w,\eta)}{\partial p} &= -q^*(p,w-S,\eta), \\ S(p_0+t,w,\eta) &= 0. \end{aligned}$$

- But average EV $\bar{S}(p,w) = \int S(p_0,w,\eta)\,dF(\eta)$ cannot be identified without any restriction on preferences: why?

$$\frac{\partial \bar{S}(p,w)}{\partial p} \neq -\underbrace{\bar{q}^*(p,w-\bar{S})}_{Estimable}$$

- Hausman-Newey show that we can get bounds
- Contrast with discrete case where average EV is point identified without any restriction on preferences.

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