Department of Economics

UNIVERSITY OF DELHI

M.A. Economics: Semester IV 2013

Course 104: Game Theory II

Maximum Marks: 70 Time: $2\frac{1}{2}$ Hours

Instruction: Each question carries 20 marks. Attempt as many as you wish and all answers will be graded. However, the maximum that you can score is 70.

1. In a transferable utility cooperative game (N, v), agent i is said to be contributing more than agent j, if $v(S \cup \{i\}) > v(S \cup \{j\})$ for all $S \subset N$ such that $i, j \notin S$. An allocation ψ satisfies 'Raking' if $\psi_i(N, v) > \psi_j(N, v)$ whenever i contributes more than j.

a) Show that the Shapley value satisfies Ranking. [4]

The following transferable utility cooperative game is known as a 'quota game'. Suppose that each agent in N has a 'power index' p_i, where $0 \leq p_i \leq 1$ and $\sum_{i \in N} p_i = 1$. A coalition $S \subset N$ can grab a fixed surplus M, if 'total power' of that coalition is (weakly) greater than a exogenously fixed quota q, otherwise S gets 0. Formally, a quota game (N, W) can be defined as follows,

$$\text{For all } S \subseteq N,\ W(S) = M \text{ if } \textstyle\sum_{i \in S} p_i \geq q \text{ and } W(S) = 0 \text{ otherwise}$$

where $(q, p_1, \ldots, p_n)$ are exogenously given and $\frac{1}{2} < q < 1$.

b) Let Shapley value of the quota game be denoted by $Sh(N, W)$. Show that $p_i > p_j$ implies $Sh_i(N, W) \geq Sh_j(N, W)$. [4]

c) Fix a T, nonempty subset of N. Compute $Sh(N, W)$ where $p_k = \frac{1}{|T|}$ for all $k \in T$ and $p_k = 0$ for all $k \notin T$. [4]

d) Show that if $p_i \leq (1 - q)$ for all $i \in N$, then core of (N, W) is empty. [4]

e) Prove or disprove (by providing a counterexample) the reverse: If core of (N, W) is empty then $p_i \leq (1 - q)$ for all $i \in N$. [4]

2. A rationing problem is denoted by $(T; z_1, \ldots, z_n)$ where T is the total resources and z_i is the claim of agent i. The 'Uniform gain' and 'Uniform loss' methods are defined as follows;

Uniform Gain: $UG_i(T; z_1, \ldots, z_n) = \min(\lambda, z_i)$, where λ is the solution of $\sum_{i \in N} \min\{\lambda, z_i\} = T$

Uniform Loss: $UL_i(T; z_1, \ldots, z_n) = \max(z_i - \mu, 0)$, where μ is the solution of $\sum_{i \in N} \max(z_i - \mu, 0) = T$

a) Show that Uniform Gain is the dual of Uniform Loss. [4]

b) Show that the Uniform Gain satisfies 'Resource Monotonicity', that is, $UG_i(T; z_1, \ldots, z_n) \geq UG_i(T'; z_1, \ldots, z_n)$ for all $i \in N$ whenever $T > T'$. [4]

c) Show that Resource Monotonicity is preserved under dual transformation. That is, if a rule satisfies Resource Monotonicity then its dual should also satisfy Resource Monotonicity. [4]

d) An allocation ϕ is 'Progressive' if $z_i > z_j$ implies $\frac{\phi_i(T; z_1, \ldots, z_n)}{z_i} \leq \frac{\phi_j(T; z_1, \ldots, z_n)}{z_j}$. Using Uniform Gain and Uniform Loss methods, Show that Progressiveness is not preserved under dual transformation. [4]

e) Suppose that $N = 2$ and $z_1 > z_2 > 0$. For each of the following two methods, plot the allocation of agent 1 on $x - axis$ and the allocation of agent 2 on $y - axis$, as T varies from 0 to $(z_1 + z_2)$: (i) Uniform Gain, (ii) Uniform Loss. [4]

3. In a voting, suppose there are odd number of voters. Let X be the set of alternatives and N be the set of voters. Suppose that voters have strict preferences over the alternatives.

A Condorcet winner is an alternative $x \in X$ such that x can defeat every other alternative in pairwise comparisons. Formally, x is a Condorcet winner, if for all $y \in X$, $\{i \in N \mid x \succ_i y\} > \{j \in N \mid y \succ_j x\}$.

a) Show that a Condorcet winner may not always exist. [5]

b) Now, suppose that the set of alternatives X is a finite subset of the interval $[0, 1]$ and voters have single-peaked strict preferences over X. Show that there is a unique Condorcet winner here. [5]

c) Model the above as a mechanism design problem. [5]

d) Show that the Condorcet winner voting rule can be implemented in dominant strategies under the single-peaked domain. [5]

4. Consider an auction domain:

a) Suppose that there are n risk-neutral bidders with independent and identically distributed valuations. Find the symmetric equilibrium bidding strategies in a first-price sealed bid auction. [6]

b) Write the social choice function, which can be obtained through a first price sealed bid auction. [4]

c) Show that the same social choice function can be implemented in Bayes-Nash equilibrium through a direct mechanism. [4]

d) Can you implement the same through an all-pay auction? [6]