

Game Theory 2, Final Examination 2012

Maximum score:70, Time: 2.5 hr

Part A: Maximum score in Part A is 30

1. Consider an economy with two goods and two agents. Agent 1 has an endowment of 1 unit of good Y and agent 2 has an endowment of 1 unit of good X . Utility functions are as follows, $u_1(x_1, y_1) = x_1 y_1$ and $u_2(x_2, y_2) = x_2 + y_2$.
a) Show that, core allocations are: $x_1 = y_1 = t$, $x_2 = y_2 = (1 - t)$, where $t < 0.5$. [4]

Consider a two fold replica of the above, that is, an economy with two agents of type 1 and two agents of type 2.

b) Compute the characteristic function, that is, $v(S)$ for all $S \subseteq N$, for this replica economy. [8]

c) Find a core allocation of the replica economy. [8]

2.

a) State Arrow's impossibility theorem. [2]

b) Define the axioms used in Arrow's impossibility theorem. [6]

c) Show that it is a 'tight' result, that is, if you relax one axiom, there are rules other than the dictatorial rule which satisfy the remaining axioms. [8]

d) Show that Arrow's impossibility theorem does not hold when there are just two alternative policies. [4]

Part B: Maximum score in Part B is 40.

3. Consider the voting domain.

a) Show that strategy proof implies monotonicity. [10]

Consider a voting rule which picks the alternative x irrespective of preference profile, that is $f(P_1, \dots, P_n) = x$ for all P_1, \dots, P_n .

b) Show that the above rule is strategy proof. [2]

c) Does this violate Gibbard-Satterthwaite theorem? [3]

Consider the following voting problem. There are three candidates $\{L, C, R\}$ and n voters. Voters are of the following types: left wing ($L \succ C \succ R$), right wing ($R \succ C \succ L$) and centrists (either $C \succ L \succ R$ or $C \succ R \succ L$).

- d)* Show that there are non-dictatorial and strategy proof voting rules. [8]
- e)* Does this violate Gibbard-Satterthwaite theorem? [2]

4. Suppose that two bidders, whose valuations are independently drawn from a uniform distribution on $[0, 1]$, take part in a first price sealed bid auction with reserve price r .

- a)* Find an equilibrium bidding strategy. [10]
- b)* Compute the expected revenue from this auction. [8]
- c)* Rewrite this auction as a mechanism [4]
- d)* Is it an efficient mechanism? [3]