Midterm, Game Theory 2 , 2011-12

1. Consider transferable utility (TU) games.
a) Define the 'symmetry' axiom. [2]
b) Show that the Shapley value satisfies 'symmetry'. [2]
c) Give example of an allocation rule which satisfies 'additivity' but violates both 'symmetry' and 'dummy'. [3]
d) Let σ be a permutation on the set of agents N. For any subset S of N, we define $\sigma(S)$ as follows, $\sigma(S) = \{\sigma(i) \mid i \in S\}$. Given a TU game (N, v), we can create a new game (N, w) such that $w(S) = v(\sigma(S))$ for all $S \subseteq N$. An allocation ψ satisfies 'anonymity' if $\psi_k(w) = \psi_{\sigma(k)}(v)$ for all $k \in N$.
Show that if an allocation ψ satisfies 'anonymity' then ψ also satisfies 'symmetry'. [3]

2. Check whether the following social welfare rankings satisfy 'invariance under affine transformation', and 'equity', where $|N| = 2$.
Ranking1: x is as good as $y \Leftrightarrow \omega x_1 + (1-\omega)x_2 \geq \omega y_1 + (1-\omega)y_2$
Ranking2: x is as good as $y \Leftrightarrow \omega[\max(x_1, x_2)] + (1-\omega)[\min(x_1, x_2)] \geq \omega[\max(y_1, y_2)] + (1-\omega)[\min(y_1, y_2)]$.
Here x_i denotes the utility of agent i under policy x and $0 < \omega < 1$. [10]

3. Seller S owns an object, which buyer B values at v. Utility of B and S are as follows; $u_S = p$; $u_B = v - p^2$ if B gets the object and 0 otherwise. Here, p denotes the price paid by B.
a) Model this situation as an axiomatic bargaining problem. [2]
b) Find the Nash allocation price. [3]
c) How should a 'good' bargaining allocation respond to an increase in v - propose an axiom. [3]
d) Does Nash allocation satisfy your axiom? [2]