Problem Set 2, Game Theory 2, 2013-14

1. There are two bidders, whose valuations are independently drawn from a uniform distribution on $[0, 2]$, in a first price sealed bid auction.

a) Find an increasing, symmetric equilibrium bidding strategy.

b) Compute the expected revenue of the seller.

c) Now suppose that the seller has imposed a reserve price r on this auction. That is, if all bids are less than r then the object remains unslod. Find an increasing, symmetric equilibrium bidding strategy in this case.

d) Supposing that the seller gets no utility from retaining the object, compute the expected revenue from reserve price auction.

e) Compare (b) with (d). Find the revenue maximising value of r.

2. In a sealed bid all pay auction, the highest bidder receives the good but every buyer pays the seller the amount of her bid regardless of whether she wins. Suppose there are two bidders whose valuations are independently drawn from a uniform distribution on $[0, 1]$.

a) Find an equilibrium bidding strategy (without using the revenue equivalence principle).

b) Compute the expected revenue from this auction.

c) Rewrite all-pay auction as a mechanism and check that it implements the efficient outcome in Bayes-Nash equilibrium (that is truth telling is Bayes-Nash equilibrium and the outcome is efficient).

3. Consider a first price auction, where the highest bidder pays his bid but receives the object with probability 0.5 (the seller keeps the object with probability 0.5).

a) Find an equilibrium bidding strategy.

b) Find the seller's expected revenue.

c) Can you use the 'revenue equivalence principle' to compare (b) with the expected revenue from first price auction? Why or why not?

4. Consider a queueing problem. There is a server, which can serve one individual at a time. Thus, individuals have to wait in a queue. However waiting

in a queue is costly for each individual. The server's objective is to order the individuals in a queue efficiently so as to minimise the aggregate waiting cost. For instance, if there are n agents and agent i comes at the $k-th$ position of a queue, then his cost is $\theta_i(k-1)-t_i$, where θ_i is the per unit waiting cost of agent i and t_i is the transfer. Suppose that there are three agents and $\theta_i \in [0,1]$ for all $i = 1,2,3$.
a) Find the efficient queue.
b) Suppose that the waiting costs are private information. Model this situation as a mechanism design problem.
c) Find the pivotal mechanism. Is it budget balanced?
d) Can you suggest a mechanism, which is **strategy-proof (truth telling is weakly dominant strategy equilibrium)** and budget balanced?

5. A monopolist seller produces a good with constant marginal cost $c \geq 0$. The monopolist sells to a consumer whose utility from consuming x units of the product is $\theta\sqrt{x}-t$, where t is the payment to the monopolist.
a) Find the socially efficient outcome.
b) Suppose that θ is only known to the buyer. The monopolist knows that θ is drawn randomly from a uniform distribution on $[0,1]$. Model it as a mechanism design problem.
c) Find the monopolist's revenue from a mechanism which is **Bayesian incentive compatible (truth telling is Bayes-Nash equilibrium)**, efficient and individually rational.

6. Suppose there is a seller with privately known cost $c \in [0,1]$ of producing a single unit of indivisible good. Utility of the seller is $t_S - c$ if he sells the good and get a price t_S. There is a buyer who wants to buy one unit of that good with privately known utility $v \in [0,1]$. Utility of the buyer is $v - t_B$ if he buys the good and pay a price t_B. No trade utility is 0 for both the agents. Assume that c and v are independently distributed.
a) Model this problem as a mechanism design problem.
b) Find the pivotal mechanism.
c) Is it individually rational for all types of buyers and sellers?

d) Find a Clarke Groves mechanism which is individually rational for all types of buyers and sellers. Is it budget balanced?
e) Using the characterization result of all Bayesian incentive compatible mechanisms in the auction domain, show that there is no efficient, budget balanced and individually rational mechanism which is Bayesian incentive compatible.

7. Suppose that a public good costs c. There are two agents, who can be either high type or low type, depending on their valuation of the public good. Valuation of high type is v_H and valuation of low type is v_L. Types of agent 1 and 2 are independently distributed random variables. Each agent knows her type but does not know about the other. Suppose that, $v_H + v_L > c > 2v_L$.
a) Write this as a mechanism design problem.
b) Identify the efficient outcome.
c) Consider the following cost division: if both the agents are of same type then the cost is equally divided, otherwise high type pays the full cost. Is the above cost allocation rule (along with efficient outcome) strategy proof?
d) Is there a cost division rule, which (along with the efficient outcome) is strategy proof, budget balanced and individually rational.

8. There is a cake of size c, which has to be divided between n claimants. Satiation point of agent i is denoted by b_i. Utility of an agent increases with consumption till her satiation point is reached and then it decreases monotonically (no free disposal). Suppose that b_i is private information. Show that the following sharing method is strategy proof.
If $\sum_i b_i \leq c$, each agent gets her satiation point. Otherwise if $\sum_i b_i > c$ then find λ such that $\sum_{i=1}^n \min\{\lambda, b_i\} = c$. Share of agent i is $x_i = \min\{\lambda, b_i\}$.

9. Let $N = 2$ and $X = \{a, b, c\}$. Suppose $\Theta_1 = \Theta_2 = \{\theta, \theta'\}$.
preference at θ and θ' are as follows, $aI(\theta)bP(\theta)c$ and $aP(\theta)bP(\theta)c$
Consider the following social choice function, $f(\theta, \theta) = b$ and $f(.,.) = a$ otherwise.
a) Show that f is strategy proof.
b) Show that this game has another dominant strategy equilibrium, which

fails to implement f.

10. While proving Gibbard Satterthwaite Theorem, we showed that strategy proof social choice function must also satisfy monotonicity.

(a) Show that the dictatorial rule satisfies monotonicity and unanimity.

(b) Find a social choice rule which satisfies monotonicity but fails unanimity.

(c) Find a social choice rule which satisfies unanimity but fails monotonicity.

(d) Show that the G-S theorem fails when there are two alternatives.