Delhi School of Economics
Department of Economics

Entrance Examination for Ph. D/M.Phil. Economics
June 27, 2009
Series 01

Time: 3 hours

Maximum marks: 100

General Instructions: Please read carefully.

- Do not break the seal on this booklet until instructed to do so by the invigilator. Anyone breaking the seal prematurely will be evicted from the examination hall and his/her candidature will be cancelled.
- Immediately after you receive this booklet, fill in your Name and Roll Number in the designated space at the bottom of this page.
- Check that you have a bubble-sheet and an answer booklet accompanying this examination booklet.
- Following the instructions on the bubble-sheet, fill in the required information in Boxes 1, 2, 4, 5 and 6. The invigilator will sign in Box 3. In Box 4, enter your roll number as a 4-digit number, e.g. 0123. In Box 6, enter 01 as your series number.
- The examination has two parts. Questions in Part I are to be answered on the bubble sheet only, and will be checked by a machine. Therefore, it is very important that you follow the instructions on the bubble-sheet.
- Fill in your Name and Roll Number on the detachable slip on the answer booklet. Use this booklet to answer questions in Part II. You can also use the spare pages of the answer booklet for rough work for both Parts.
- Keep your admission ticket easily accessible for verification by the invigilators.
- When you finish, hand in this booklet, the answer booklet and the bubble-sheet to the invigilator.
- Do not disturb or talk to your neighbours at any time. Anyone engaging in illegal examination practices will be immediately evicted and his/her candidature will be cancelled.
- Only after the invigilator announces the start of the examination, break the seal on this booklet and follow the instructions on Page 2.
The Examination has two parts, Attempt both parts.

PART I (Maximum Marks 50)

Instructions:

1. First check that this booklet has pages numbered 1 through 9. Bring any missing pages to the attention of the invigilator.

2. This part of the examination consists of 25 multiple-choice questions. Each question is followed by four possible answers, one of which is correct. Indicate the correct answer on the bubble sheet, NOT on this booklet.

3. Each correct choice will earn you 2 marks. However, you will lose 2/3 mark for each incorrect choice. If you shade none of the bubbles, or more than one bubble, you will get 0 for that question.

4. Use the blank spaces in this booklet or pages in your answer booklet for rough work. This rough work will not be read or checked.

5. You may begin now. Good luck.

1. The nine digits 1, 2, ..., 9 are arranged in random order to form a nine digit number, which uses each digit exactly once. Find the probability that 1, 2 and 3 appear as neighbours in the increasing order.
   a) 1/12
   b) 1/72
   c) 1/84
   d) (2/3)^3

2. In a survey of 102 Timarpur residents in 2009, the average income was found to be Rs. 4635 per month. Previous studies show the population variance of income in this locality to be Rs. 12342 per month. It is asserted that the average monthly income is Rs. 4650 in this locality. Which conclusion below can be asserted from this information?
   a) The assertion is rejected at the 10% level.
   b) The assertion is rejected at the 5% level.
   c) The assertion is not rejected at the 10% level.
   d) None of the above.

3. A company has 100 employees, 40 men and 60 women. There are 6 male executives. How many female executives should there be for gender and rank to be independent?
   a) 9
   b) 6
   c) 10
   d) 8
4. Consider two events A and B with P(A) = 0.4 and P(B) = 0.7. The maximum and minimum values of P(A ∩ B) respectively are:
   a) (0.4; 0.1)
   b) (0.7; 0.4)
   c) (0.7; 0.1)
   d) (0.4; 0)

5. Jai and Vijay are taking a exam in statistics. The exam has only three grades A, B, and C. The probability that Jai gets a B is 0.3, the probability that Vijay gets a B is 0.4, the probability that neither gets an A, but at least one gets a B is 0.1. What is the probability that neither gets a C but at least one gets a B?
   a) 0.1
   b) 0.6
   c) 0.8
   d) Insufficient data to answer the question

6. You've been told that a family has two children and one of these is a daughter. What is the probability that the other child is also a daughter?
   a) 1/2
   b) 1/3
   c) 1/4
   d) 3/4

Questions 7 and 8. Suppose least squares is used to fit a line relating y and x, namely
   \[ y = \beta_0 + \beta_1 x + \epsilon_i \]
   Assume that in our data not all the x's are identical, so that at least some of the x’s are different from their sample mean \( \bar{x} \). Now consider the following possible assumptions about our data.

(1) \( E(\epsilon_i) = 0 \).
(2) \( \text{Cov}(x_i, \epsilon_j) = 0 \).
(3) Homoskedasticity: \( \text{Var}(\epsilon_i) = \sigma^2 \), a constant.
(4) No autocorrelation: \( \text{Cov}(\epsilon_i, \epsilon_j) = 0 \) for \( i \neq j \).

Indicate the one best answer to each question below.

7. When are the least-squares estimators unbiased?
   a) Only if our data satisfy assumptions (1) and (2).
   b) Only if our data satisfy assumptions (1), (2), (3), and (4).
   c) Only if our data satisfy assumptions (1), (2), and (3).
   d) Only if our data satisfy (1), (2), and (4).

8. When are the least-squares estimators "best" (lowest variance) of any unbiased estimators?
   a) Only if our data satisfy assumptions (1) and (2).
   b) Only if our data satisfy assumptions (1), (2), (3), and (4).
   c) Only if our data satisfy assumptions (1), (2), and (4).
   d) Cannot be determined, without additional assumptions.
9. Romeo and Juliet have 96 chocolates to divide between them. Romeo has the utility function \( U = R^a J^b \) and Juliet has the utility function \( U = R^c J^d \) where \( R \) is Romeo's chocolate consumption and \( J \) is Juliet's chocolate consumption. Which of the following is true:

a) Romeo would want to give Juliet some chocolates if he had more than 62.
b) Juliet would want to give Romeo some chocolates if she had more than 60.
c) Romeo and Juliet would never disagree about how to divide the chocolates.
d) Juliet would want to give Romeo some chocolates if she had more than 64 chocolates.

10. A consumer spends an income of Rs. 100 on only two goods, A and B. Assume non-satiation, i.e., more of any good is preferred to less. Suppose the price of B is fixed at Rs. 20. When the price of A is Rs. 10, the consumer buys 3 units of B. When the price of A is Rs. 20, she buys 5 units of A. From this we can conclude:

I. A is an inferior good.
II. A is a Giffen good.
III. B is a complement of A.

a) I only.
b) I and II.
c) I and III.
d) I, II and III.

Questions 11 and 12. "Suraksha" is the sole producer and supplier of security systems in India and the sole employer of locksmiths in the labour market. The demand for security systems is \( D(p) = 100 - p \), where \( p \) is the price. The production of security systems only requires locksmiths and the production function is given by \( f(L) = 4L \), where \( L \) is the number of locksmiths employed. The supply curve for locksmiths is given by \( L(w) = \max \left( 0, \frac{w}{2} - 20 \right) \), where \( w \) is the wage rate.

11. How many locksmiths will "Suraksha" employ?
   a) 5
   b) 10
   c) 15
   d) 20

12. If the government sets the minimum wage is 70, how many locksmiths will "Suraksha" employ?
   a) 5
   b) 10
   c) 15
   d) 20
Questions 13 and 14 Suppose that a typical graduate student at the Delhi School of Economics lives in a two good world, books \((x)\) and movies \((y)\), with utility function \(u(x, y) = x^{1/2} y^{1/2}\). Prices of books and movies are 50 and 10 respectively. Suppose the University is considering the following schemes.

Scheme 1: 750 is paid as fellowship and additional 250 as book grant. Naturally, the grant can only be spent on books.

Scheme 2: 1000 as scholarship and gets one movie free on each book they purchase.

Believing that books and movies are perfectly divisible, compute the optimal consumption bundle under each scheme.

13. Optimal consumption bundle under scheme 1 is:
   a) (4 books, 80 movies)
   b) (3 books, 75 movies)
   c) (6.5 books, 57.5 movies)
   d) (10 books, 50 movies)

14. Optimal consumption bundle under scheme 2 is:
   a) (4 books, 80 movies)
   b) (4 books, 84 movies)
   c) (5 books, 75 movies)
   d) (5 books, 80 movies)

15. Let \(X\) stand for the consumption set and let \(R\), \(I\), \(P\) respectively stand for the weak preference relation, indifference relation and strict preference relation of a consumer. The weak preference relation \(R\) is said to satisfy Quasitransitivity if and only if for all \(x, y, z\) belonging to \(X\), \(xPy \land yPz \rightarrow xPz\). Which of the following preference relations over \(X = \{x, y, z\}\) satisfies Quasitransitivity?
   a) \(xPy \land yPz \land zPx\)
   b) \(xPy \land yPz \land zPz\)
   c) \(xPy \land yPz \land zPz\)
   d) \(yPz \land yPz \land xPz\)

16. Consider an exchange economy with two consumers (A&B) and two goods (\(x\&y\)). Assume that total amount of \(x\) available is 4 and total amount of \(y\) available is 2 which is to be optimally distributed between A & B. A’s utility function is \(U_A = x_A^2 + 4x_A y_A + 4y_A^2\) and B’s utility function is \(U_B = x_B + y_B\). The contract curve for this exchange economy will be:
   a) the entire boundary of the edgeworth box
   b) allocations satisfying \((x_A = 0, 0 \leq y_A \leq 2)\) and \((0 \leq x_B \leq 4, y_B = 0)\)
   c) allocations satisfying \((0 \leq x_A \leq 4, y_A = 0)\) and \((x_B = 0, 0 \leq y_B \leq 2)\)
   d) all points inside the edgeworth box.
25. Consider the following three definitions for a country's current account surplus. Which of them is correct?

(i) equal to its trade balance plus net income from abroad
(ii) equal to its trade balance plus foreign direct investment
(iii) equal to the change in its claims against the rest of the world

a) (i) and (ii)
b) (ii) and (iii)
c) (i) and (iii)
d) none
QUESTION 6. If there is a one shot increase in the parameter $\lambda$,
(a) equilibrium output unambiguously increases
(b) equilibrium output unambiguously decreases
(c) equilibrium output remains unchanged
(d) equilibrium increases or decreases depending on whether $\lambda \geq 1/2$

QUESTION 7. If there is a one shot increase in the parameter $c_A$,
(a) equilibrium output unambiguously increases
(b) equilibrium output unambiguously decreases
(c) equilibrium output remains unchanged
(d) equilibrium increases or decreases depending on whether $\lambda \geq 1/2$

QUESTION 8. If there is a one shot increase in the parameter $c_B$,
(a) equilibrium output unambiguously increases
(b) equilibrium output unambiguously decreases
(c) equilibrium output remains unchanged
(d) equilibrium increases or decreases depending on whether $\lambda \geq 1/2$

QUESTION 9. Suppose $X_1$ and $X_2$ are real-valued random variables with $f$ as their common probability density function. Suppose $(x_1, x_2)$ is a sample generated by these random variables. The expectation of the number of observations in the sample that fall within a specified interval $[a, b]$ is:
(a) $\left( \int_a^b f(x) \, dx \right)^2$
(b) $\int_a^b x^2 f(x) \, dx$
(c) $2 \int_a^b f(x) \, dx$
(d) $\int_a^b x f(x) \, dx$

QUESTION 10. Suppose $X_1, \ldots, X_n$ are observed completion times of an experiment with values in [0, 1]. Each of these random variables is uniformly distributed on [0, 1]. If $Y$ is the maximum observed completion time, then the mean of $Y$ is:
(a) $[n/(n+1)]^2$
(b) $n/2(n+1)$
(c) $n/(n+1)$
(d) $2n/(n+1)$
QUESTION 11. Suppose the random variable $X$ takes values in the set $\{-1, 0, 1\}$ and the probability of each value is equal. Let $Y = X^2$. Which of the following statements is true?

(a) $X$ and $Y$ are correlated but independent
(b) $X$ and $Y$ are uncorrelated but dependent
(c) $X$ and $Y$ are dependent and have the same mean
(d) $X$ and $Y$ are correlated and have different means

QUESTION 12. Suppose player 1 has five coins and player 2 has four coins. Both players toss all their coins and observe the number that come up heads. Assuming all the coins are fair, what is the probability that player 1 obtains more heads than player 2?

(a) 1/2
(b) 4/9
(c) 5/9
(d) 4/5

QUESTION 13. Suppose $\theta$ is a random variable with uniform distribution on the interval $[-\pi/2, \pi/2]$. The value of the distribution function of the random variable $X = \sin \theta$ at $x \in [-1, 1]$ is

(a) $\sin^{-1}(x)$
(b) $\sin^{-1}(x) + \pi/2$
(c) $\sin^{-1}(x)/\pi + 1/2$
(d) $\sin^{-1}(x)/\pi + \pi/2$

The next three questions are based on the following data. The number of loaves of bread sold by a bakery in a day is a random variable $X$. The distribution of $X$ has a probability density function $f$ given by

$$f(x) = \begin{cases} 
x, & \text{if } x \in [0, 5) \\
k(10 - x), & \text{if } x \in [5, 10) \\
0, & \text{if } x \in [10, \infty) 
\end{cases}$$

QUESTION 14. As $f$ is a probability density function, the value of $k$ must be

(a) 0
(b) -2/25
(c) 1/25
(d) 2/75

QUESTION 15. Let $A$ be the event that $X \geq 5$ and let $B$ be the event that $X \in [3, 8]$. The probability of $A$ conditional on $B$ is
PART II (Maximum Marks 50)

Note: Attempt any TWO of the following three questions, in the answer book provided to you. Each question is worth 25 marks. The allocation of marks for each part is given alongside.

1. (a) Consider an exchange economy with two consumers A and B and two goods x and y. Initial endowment of A and B are (0,1) and (1,0) respectively. The utility functions of A and B are as follows:

\[ U^A = a \ln x^A + (1-a) \ln y^A \]
\[ U^B = \min(x^B, y^B) \]

Calculate the market clearing price and equilibrium allocation.

(b) A firm's cost function is \( C(w_1, w_2, Q) = w_1^a w_2^b Q \) where \( w_1, w_2 \) are input prices and \( Q \) is output. What restrictions would you put on \( a \) and \( b \)? Give reasons for your answer.

(c) Derive the expenditure function if the utility function is \( U = ax_1 + bx_2 \).

2. (a) Set up the Ramsey-Cass-Koopmans model of optimal growth.

(b) Derive the balanced growth path for this model.

(c) In such a model what is the impact of a fall in the discount rate?

3. A study on industrial employment in the US estimated the following model by OLS using annual data for 22 years:

\[
\begin{align*}
\text{EMP} &= -7.652 + 0.313 \text{INC} - 0.251 \text{WAGE} + 0.622 \text{G} \\
R^2 &= 0.982 \\
D W &= 1.647
\end{align*}
\]

where

\( \text{EMP} = \text{total employment} \); \( \text{INC} = \text{total income} \); \( \text{WAGE} = \text{average hourly wage rate} \)

\( \text{G} = \text{total government expenditure} \)

All variables are in logarithmic terms and the t-statistics are given in parentheses.

(a) Do the signs of the equation conform to your expectations? Explain.

(b) Test the individual significance of the independent variables at the 5% level. Explain your results.

(c) Test for the joint significance of the independent variables at the 5% level.

(d) Calculate the Adjusted \( R^2 \) and interpret it.

(e) Test for serial correlation at the 5% level. What do you conclude?

(f) Explain the consequences of serial correlation in the residuals.

(g) It is suggested that the interest rate should be included in the model. Assuming that this is a valid suggestion, what are the consequences of excluding the interest rate from the above equation?