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Discounting climate change

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Abstract In this paper I offer a fairly complete account of the idea of social discount rates as applied to public policy analysis. I show that those rates are neither ethical primitives nor observables as market rates of return on investment, but that they ought instead to be derived from economic forecasts and society's conception of distributive justice concerning the allocation of goods and services across personal identities, time, and events. However, I also show that if future uncertainties are large, the formulation of intergenerational well-being we economists have grown used to could lead to ethical paradoxes even if the uncertainties are thin-tailed. Various modelling avenues that offer a way out of the dilemma are discussed. None is entirely satisfactory.

Keywords Utilitarianism · Prioritarianism · Intergenerational well-being · Social discount rates · Uncertainty · Inequality aversion · Risk aversion · Rate of time preference · Hyperbolic discounting · Rate of return on investment · Precautionary principle · Elasticity of marginal felicity · Risk-free discount rates · Thin-tailed distributions

JEL Classification C61 · D53 · D9 · G12

Imagine someone who has been reading articles and watching documentaries on climate change. She is persuaded that rising concentrations of carbon dioxide in the

The ideas I apply here were presented in my Plenary Lecture to the World Congress of Environmental and Resource Economists, held in Monterey, California, June 2002, and were explored in Dasgupta (2001: Ch. 11). For discussions and correspondence over the years, I am very grateful to Kenneth Arrow, Geir Asheim, and Karl-Göran Mäler. While revising the paper I have benefited greatly from the comments of William Cline, William Nordhaus, W. Kip Viscusi, and Martin Weitzman.

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atmosphere is a major contributor to the process. She knows that even though the global warming associated with climate change is slow in comparison to the speed of contemporary economic growth, the carbon concentrations expected to be reached at the end of this century under business as usual haven't been harboured by Earth's atmosphere in the past several million years. This scares her. However, she realises that although the investment required to curb the process—controlling carbon emissions, enlarging sequestration possibilities, and investing in alternative energy technologies—are large, the benefits will be enjoyed only many decades from now. Which is why she is not only anxious about climate change, she is also at a loss to know how to think about the matter.

As our protagonist is a citizen of a functioning democracy, she wants to instruct her political leaders to start discussions with governments of other countries on what, as she sees it, is a global commons problem. That is why she now seeks a grammar that can join her understanding of the way the world works (the ways in which people would choose under various circumstances, the pathways Nature chooses, the consequences of those choices, and so on) to the basis on which alternative global investment policies ought to be evaluated. As carbon emissions involve massive externalities, she realises that in her role as a citizen she shouldn't rely exclusively on her private interests, but should instead adopt something like a social point of view, one that would appear reasonable not only to her, but to others as well. This makes her want to take others into account when deliberating over the costs and benefits of alternative investment policies. But she realises that when it comes to climate change, most of those others will be people who are yet to be born. So, she wants to know what contemporary economics has to say about her dilemma.

Our protagonist asks an economist friend to give her a reading list, complaining to him that correspondents even in the most prominent newspapers never write about the questions that are vexing her. The friend assures her that economics does have the conceptual tool she seeks, and that it has already been put to use by contemporary economists for studying the economics of climate change. He gives her three books to read: Cline (1992), Nordhaus (1994), and Stern et al. (2006)—henceforth Stern (2006).

Some days later our protagonist calls her friend to complain. She says she has now read the books, but remains confused. Cline and Stern, she says, urge immediate, strong global action to combat climate change—Stern, she notes, recommends what amounts to an annual expenditure of 2% of the GDP of rich countries. But Nordhaus, she observes, claims that despite the threats climate change poses to the global economy, it would be more equitable and efficient to invest in reproducible capital and human capital now so as to build up the productive base of economies—including, especially, poor countries—and to put into effect controls on carbon in an increasing, but gradual manner, starting several decades from now. What confounds her, our protagonist remarks, is that Cline and Stern, on the one hand, and Nordhaus, on the other, reach very different conclusions even though they are all agreed that global GDP per capita can be expected to continue to grow over the next 100 years and more even under business as usual—at something like 1–2% a year. What, she asks, is going on?

This article offers an account of what she wants to know.

1 Facts and values

Reading the many reports on Stern (2006), published in newspapers and magazines at its launch (31 October 2006)—interestingly, reading the book itself—would give one the impression that the case built by the authors for strong, immediate action rests wholly on insights drawn from the new and more refined global circulation models of climate scientists. In fact the conclusions reached by Stern and his co-authors are implications of their choice of a pair of fundamental ethical parameters—namely, the time discount rate and the elasticity of marginal felicity (defined below)—they aren't driven so much by the new climatic facts the authors have stressed. It so happens, Cline (1992) postulated values for that same pair of parameters that, at least in the context of the economic model of climate change he studied, were very close to the ones assumed in Stern's book (see below). In a symposium on his book, Cline (1993:4) summarised his findings in words that reflect a point of view strikingly similar to that in Stern (2006): "My central scenario shows that...if risk aversion is incorporated by adding high-damage and low-damage cases and attributing greater weight to the former, benefits comfortably cover costs (with a benefit–cost ratio of about 1.3 to 1). Aggressive abatement is worthwhile even though the future is much richer, because the potential massive damages warrant the costs." (There is but a perfunctory reference to Cline (1992) in Stern (2006); and no mention at all of the similarities between its conclusions and those reached in the earlier publication.)

In contrast, the figure chosen for one of the two ethical parameters, namely, that for the *time discount rate* (see below), in Nordhaus (1994) is so much higher than the ones chosen by Cline (1992) and Stern (2006), that it leads him to advocate the upward-sloping "climate policy ramp" of ever tightening reductions in carbon emissions our protagonist noticed in his work. The higher figure chosen by Nordhaus obliges him to use a considerably higher rate to discount the future costs and benefits associated with public economic policies.

All this must be well known to those who have followed recent discussions on the economics of climate change. What hasn't been studied in the commentaries, however, are the reasons underlying the differences between Nordhaus, on the one hand, and Cline and Stern, on the other, over the choice of the time discount rate. I believe those reasons have to do with differences in the way welfare economics is read by the authors. So, although I begin by belabouring what could appear to readers as rather self-evident points, I do so because it will prove useful for drawing out two alternative ways of reading welfare economics.

Policy evaluation involves comparisons of different people's well-being. We will call the person doing the evaluation the *social evaluator*. The social evaluator could be a citizen (thinking about things before casting his vote on political candidates), she could be an ethicist hired to offer guidance to the government, he could be a government decision maker, and so on. In what follows, I frequently adopt modern convention by replacing the "social evaluator" by "society" and say, for example, that "society entertains the view...," when I mean "the social evaluator entertains the view..."

Assume, as Cline, Nordhaus, and Stern do, that each person's felicity (i.e. instantaneous utility) depends solely on his or her consumption level. By the "fundamental ethical parameters", I mean two things: (i) the tradeoffs that ought to be made between the felicities of the present and future generations, given that future generations will be here only in the future; and (ii) the tradeoffs that can justifiably be made between the consumptions people enjoy, regardless of the date at which they appear on the scene. Technically, (i) is reflected in the *time discount rate* (we denote it here by δ); and (ii) is reflected in the elasticity of the social weight that ought to be awarded to a small increase in an individual's consumption level (we denote it here by η). We confirm later that δ reflects the way the future is seen through today's telescope; while η is a measure of society's aversion to interpersonal inequality and risk in consumption. In the formulation we adopt here, η is the elasticity of marginal felicity.

δ and η, as we have defined them, are fundamental because they help to determine the rates at which society ought to discount changes in future consumption. The other factor that helps to determine those rates is society's forecast of future consumptions. Discount rates on consumption changes combine "values" with "facts."

The ethical viewpoint I explore here is self-consciously anthropocentric. Nature has an intrinsic value, but I ignore it because the three books on the economics of climate change I am responding to ignore it. I don't even accommodate the fact that people care about certain types of natural capital as stocks (e.g., places of scenic beauty or sacred sites), because the books I discuss here don't consider it.[1]

2 Consumption discount rates: basics

For simplicity of exposition, let us suppose that the vector of consumption goods in the economy can be aggregated into a single commodity, called *consumption* (C).[2] Again, for simplicity of exposition, imagine that a generation's felicity can be aggregated from individual felicities in such a way that it depends solely on the generation's average consumption level. Next imagine that society entertains no uncertainty and has made a forecast of future consumptions. Society now conducts a thought experiment on its forecast by asking how much additional consumption it would demand on behalf of tomorrow's people in payment for a reduction in today's consumption by one unit. We say that the "social rate of discount" between today's and tomorrow's consumptions is that additional consumption demanded, less unity. So, if ρ is that rate, society would demand $(1+\rho)$ units of additional consumption tomorrow as a price for giving up one unit of consumption today; meaning that society regards an additional unit of consumption tomorrow to be worth $1/(1+\rho)$ units of additional consumption today. In order to stress that society is deliberating over a *consumption* swap between today and tomorrow, we say that ρ is the *consumption discount rate*. As would be expected, consumption discount rates play a central role in social cost–benefit analysis (Marglin 1963; Arrow and Kurz 1970; Dasgupta et al. 1972; Lind 1982; Arrow et al. 1996; Portney and Weyant 1999).

[1] Heal (1998, 2007) and Hoel and Sterner (2007) study inter-generational welfare economics when individual felicities depend on stocks of environmental capital.

[2] For the analysis involving multiple consumption goods, see Sterner and Persson (2008).

The definition of consumption discount rates given above is very general: it isn't based on any particular conception of intergenerational justice, nor on any specific formulation of the idea of intergenerational well-being. In order to put the definition to work, we need to specify the latter. In Section 3, I do so. However, any mention of "discount rates," and one thinks immediately of positive numbers. But should society discount future consumption costs and benefits at a positive rate?

There are two reasons why it may be reasonable to do so. First, an additional unit of consumption tomorrow would be of less value than an additional unit of consumption today if society is impatient to enjoy that additional unit now. Therefore, impatience is a reason for discounting future costs and benefits at a positive rate. Second, considerations of justice and equality demand that consumption should be evenly spread across the generations. So, if future generations are likely to be richer than us, there is a case for valuing an extra unit of their consumption less than an extra unit of our consumption, other things being equal. Rising consumption provides a second justification for discounting future consumption costs and benefits at a positive rate.

A number of questions arise: How should society choose consumption discount rates? How are they related to notions of intergenerational justice and equity? Should they be constant over time or could they depend on date? Do they reflect the "opportunity cost" of capital; if so, how should society determine what that cost is? Can they be inferred from "market observables," such as risk-free interest rates on government bonds? Must consumption discount rates be positive or are there circumstances when they would be negative? And how should we price future consumption when that future is uncertain?

In this paper I discuss tentative answers to those questions. I do that in stages. Section 3 considers a deterministic world. In Sections 4 and 5 I introduce "small" and "large" uncertainties, respectively, in future technology. Unfortunately, even the simplest analytical model of the economics of global climate change (Dasgupta et al. 1999) is a lot more complicated than is necessary for our discussion here. So, although climate change motivates this paper—I refer to it repeatedly—the model I use as my workhorse doesn't contain the phenomenon. Just so that we know how to translate statements in the economic model studied here into corresponding statements in economic models of climate change, we note that, to be concerned about future generations in models of climate change means investing heavily so as to tame that change or to withstand the deleterious effects of that change; whereas, to be concerned about future generations in our model translates into high investment rates. Either way, the "present" foregoes consumption in favour of the "future".

3 Intergenerational well-being: the deterministic case

As climate change involves the long run, we assume that population size is a constant, N. Individuals are indexed by i ($i=1,2,\ldots,N$). Time is denoted by t and is taken to be discrete: $t=0,1,2,\ldots$. The present is $t=0$. When we come to perform numerical exercises below, we will often take the unit of time to be a year.

Assume, as Cline, Nordhaus, and Stern do, that each generation's felicity is the sum of the felicities of its members. I follow the three authors in supposing that an individual's felicity depends solely on his current consumption level.[3] If C_{it} and $U_i(C_{it})$, respectively, are i's consumption level and felicity at t, then *social felicity* at t is[4]

$$V_t =_i \sum U_i(C_{it}). \tag{1}$$

Cline, Nordhaus, and Stern focus on the *inter*generational distribution of consumption. So, we also bypass *intra*-generational issues by supposing that a generation's felicity depends only on its average consumption level, C. One way to conceptualise the assumption is to imagine a world with identical individuals. Write C_t for consumption at t and $U(C_t)$ for felicity at t. We take it that marginal felicity is positive ($U'(C) > 0$), but declines with increasing consumption ($U''(C) < 0$).[5] The curvature of $U(C)$, as measured by the elasticity of $U'(C)$ plays a crucial role in intergenerational welfare economics. In keeping with a vast literature, I assume that the horizon is infinite (but see Section 6). $\{C_t\}$ denotes the infinite sequence $(C_0, C_1,\ldots, C_t,\ldots)$ and $\{U(C_t)\}$, the corresponding felicity sequence $(U(C_0), U(C_1),\ldots, U(C_t),\ldots)$.

If the *time discount rate* is δ (≥ 0), *intergenerational well-being* at $t = 0$, which we write as W_0, is understood to be the present-value of the $U(C_t)$s. Thus,

$$W_0 = U(C_0) + U(C_1)/(1+\delta) + \ldots + U(C_t)/(1+\delta)^t + \ldots =_{t=0} \sum{}^{\infty} \left[U(C_t)/(1+\delta)^t\right]. \tag{2}$$

In expression (2) U is unique up to positive affine transformations.[6]

The time discount rate in expression (2) is constant. In contrast, Arrow (1999) has appealed to a form of "agent-relative" consequentialist ethics to recommend a variable time discount rate. Using a functional form made famous by Phelps and Pollak (1968), Arrow proposes that each generation should award equal weight to the felicities of all subsequent generations, but a higher weight to its own felicity relative to that awarded to subsequent generations. Arrow's formulation is a special case of *hyperbolic* time discounting.

As I understand it, though, agent-relative ethics (e.g., Scheffler 1992) responds to the demands an individual is justified in making when he deliberates over alternative courses of action in the *private* sphere. In this paper I study decisions in the *public*

[3] This rules out the influence on an individual's felicity of habitual consumption or the average consumption of the person's peer group. The implications of habitual consumption on social rates of discount have been studied by Ryder and Heal (1973); the influence of peer group by Layard (1980, 2005) and Arrow and Dasgupta (2007), among others.

[4] Expression (1) has the structure of "utilitarianism", though not necessarily its classical interpretation (see below). Some ethicists have proposed an ethical theory they call "prioritarianism", which says that an increase in the well-being of a rich person (i.e., someone who enjoys a high consumption level) should be assigned less social value than the same increase in the well-being of a poor person (someone whose consumption level is low). I have not understood why such an *ad hoc* ethical principle should be awarded a name. I would have thought the utilitarian who is averse to inequality in consumption has it right: he assigns a lower social value to an increase in the *consumption* level of a rich person than to the same increase in the consumption level of a poor person.

[5] We write $U'(C) = dU(C)/dC$ and $U''(C) = dU'(C)/dC$.

[6] If we wished to study the intra-generational distribution of consumption as well, the simplest move would be to disaggregate each generation by imagining that there are N people at each date ($i = 1,2,\ldots,N$), as in expression (1), and assuming that people have the same felicity function, U. Intergenerational well-being at $t = 0$ would then be $W_0 =_{t=0} \sum^{\infty} \left[V_t/(1+\delta)^t\right] =_{t=0} \sum^{\infty} \left[_i \sum \{U(C_{it})/(1+\delta)^t\}\right]$.

sphere (after all, climate change involves a "commons" problem). Expression (2) offers a form of ethical guidance that discourages public officials from being self-indulgent, or from contaminating his judgment with his personal interests. In any event, I want to stay close to Cline, Nordhaus, and Stern, all of whom have used expression (2) as the basis of their studies.

Although it is ubiquitous in intergenerational welfare economics, expression (2) suffers from a serious conceptual weakness: it doesn't admit any concept of the "self" that lives through time. The ethical calculus at the basis of the formula treats differences between an individual's felicities in two periods of time in the same way as it treats differences between the felicities of two individuals in those same two periods of time. The lifetime well-being of a person is constructed in the same way as intergenerational well-being is constructed; which is to say that, even though a person lives for many periods, she is regarded as a distinct self in each period. It can be argued, however, that for someone to ask oneself, "how much should I save for my children?" involves ethics that are different from those pertinent when that same person asks, "how should I spread out *my* consumption over time?" Expression (2) encapsulates a framework for addressing the former question and is the one used in each of the three books I am discussing here. So I make use of it.[7]

How should the social evaluator choose U? It has become customary in the welfare economics of climate change to infer the felicity function from the choices people make as they go about their lives (Cline 1992; Nordhaus 1994, 2007; Nordhaus and Boyer 2000; Stern 2006; Weitzman 2007a, Section 3.4). But there are several philosophical viewpoints that give rise to expression (2) in which U is not necessarily felicity in the sense that has become familiar in the literature on "revealed preference." For example, Harsanyi (1955) constructed a theory that was independently developed by Rawls (1972) into a far-reaching theory of justice based on choice behind a "veil of ignorance" as to the position the chooser would occupy in society. In the context of intergenerational justice, the chooser's ignorance would be about the generation he is to join. Unlike Rawls, Harsanyi argued that a rational chooser would interpret his ignorance to be an "equal" chance of belonging to any generation. Dasgupta and Heal (1979: Ch. 9) used an argument due to Yaari (1965) to show that intergenerational well-being in Harsanyi's theory would be expression (2) if society faces extinction at a constant hazard rate, $\delta>0$ (see Section 4). Moreover, if the chooser were risk averse behind the veil of ignorance, U in the Harsanyi-Rawls theory would not be felicity, but an increasing, concave function of felicity.

In contrast, Koopmans (1960, 1972) studied the idea of intergenerational well-being by imposing a set of ethical requirements on orderings over felicity sequences. It was found, remarkably, that if an ordering is continuous and monotonic (that is, if

[7] In work under preparation, I have tried to construct a framework that builds an intergenerational welfare economics admitting the idea of selfhood. The model I have constructed permits someone to discount his own future felicities in any way he likes (that's the demand of his "self"), but requires of him to give a weight to the lifetime well-being of each of his children that equals the weight he gives to his own lifetime well-being. The model would seem to reconcile the widespread finding from consumption behaviour that people do discount their future felicities at a non-negligible positive rate (see below) and the philosophical injunction that many people would seem to adhere to, namely, that they should not discriminate against their children's futures (see below).

one felicity sequence is never smaller than another and is larger at one or more points in time, then it is judged to be socially superior), it must necessarily reflect impatience. A further requirement imposed by Koopmans, which he named "stationarity", is a near-cousin of the demand that value judgments be universalizable, which in the present context means that the ordering over a set of felicity sequences should be the same no matter which generation constructs it. Koopmans showed that if a further requirement, "independence", is added, intergenerational well-being takes the form of expression (2).[8] Impatience means that $\delta>0$. Although Koopmans didn't study the issue of equity across the generations, equity considerations would demand that U in expression (2) be an increasing, concave function of felicity. (Rawls (1972) would call Koopmans' formulation "intuitionist".)

In further contrast, Ramsey (1928) interpreted expression (2)—with $\delta=0$—in classical utilitarian terms. But he didn't presume that U is to be calibrated from market choices. (Rawls (1972) would call Ramsey's formulation "teleological".) As I am not restricting myself to classical utilitarianism, let alone utilitarianism founded on revealed preference, we will be able to explore a far wider range of ethical considerations than have been admitted in the recent economics literature. Which is why we should note that by felicity (U) I shall mean a generation's *well-being*.

3.1 Consumption discount rates in the imperfect economy

Begin by imagining that our social evaluator is given a consumption forecast $\{C_t\}$ at $t=0$, which he converts into a forecast of well-beings $\{U(C_t)\}$. Assuming that the series in expression (2) converges, this yields a figure for intergenerational well-being W_0. δ is the *time* discount rate in expression (2). We now provide a formula for the *consumption* discount rate, ρ_t, defined earlier.

Let ΔC_t and ΔC_{t+1} denote "small" variations in C_t and C_{t+1}, respectively, and assume that the pair of variations leaves the numerical value of W_0 unchanged. Denote by $g(C_t)$ the percentage rate of change in the consumption that has been forecast between t and $t+1$.[9] Let η be the *elasticity of marginal felicity*, which is a measure of the curvature of $U(C)$.[10] Although there is no obvious reason why η should be independent of C, I follow Cline, Nordhaus, and Stern and assume that η is a constant. The class of Us for which η is constant is given by the form

$$\begin{aligned} &U(C) = C^{(1-\eta)}/(1-\eta), && \text{for } \eta > 0 \text{ and } \eta \neq 1, \\ \text{and}\quad &U(C) = \ln C, && \text{corresponding to } \eta = 1. \end{aligned} \tag{3}$$

The larger is η, the greater is the curvature of $U(C)$. Notice that $U(C)$ is bounded above but unbounded below if $\eta>1$, whereas $U(C)$ is bounded below but unbounded above if $\eta<1$.[11]

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[8] Roughly, the "independence" assumption amounts to the requirement that the marginal rate of societal indifference between felicities in any two periods is independent of the felicities in all other periods.

[9] So, $C_{t+1}/C_t = 1 + g(C_t)$. (F1)

[10] Formally, $\eta = -CU''(C)/U'(C) > 0$.

[11] Arrow (1965) observed that the simplest U that is bounded at both ends is one for which η is an increasing function of C and is less than 1 at low values of C and greater than 1 at high values of C.

On using expression (2), we obtain[12]

$$1 + \rho_t = (1 + \delta)(1 + g(C_t))^\eta. \tag{4}$$

Equation 4 gives a precise expression to the intuitive reason that was offered earlier as to why the social evaluator would be ethically correct to discount changes in future generations' consumption levels when comparing them with changes in the consumption level of the present generation.

The formula for ρ_t reduces to a familiar approximation when δ and $g(C_t)$ are both small. So, suppose they are small. Then Eq. 4 becomes[13]

$$\rho_t \approx \delta + \eta g(C_t). \tag{4a}$$

If the interval between dates was to be made smaller and smaller, expression (4a) would be a better and better approximation. It is simple to prove that if time is continuous, expression (4a) is an exact equality (see, e.g., Arrow and Kurz 1970).

Notice the way δ, η, and the forecast, $g(C_t)$, together determine ρ_t. Observe in particular that ρ_t increases with δ and $g(C_t)$, respectively, and increases with η *if and only if* $g(C_t) > 0$. I have highlighted the qualifier "if and only if" for a good reason. In studying long run economic development, it has become a habit among economists to confine attention to forecasts in which consumption increases indefinitely. Equation 4 or, equivalently, Eq. 4a, says that when $g(C_t) > 0$, δ and η play similar roles in the determination of ρ_t: a higher value of either parameter would reflect a greater aversion toward consumption inequality. Which may explain why it hasn't been uncommon to suppose that higher values of δ reflect a greater ethical concern for consumption equality. But if $g(C_t) < 0$, δ and η assume diametrically opposite features: in contrast to η, higher values of δ raise ρ_t, implying an ethical preference for even greater inequality in consumption across the generations.

In expression (2), $\{C_t\}$ is assumed to be a forecast, nothing more. At this point we are not assuming that $\{C_t\}$ is an optimum consumption programme for society (but see Section 3.2). The forecast is based on society's reading of technological possibilities, household preferences, current and future government policies, and so forth. To make a forecast requires an understanding of the political economy of society.

Equations 4 and 4a give quantitative expression to the pair of reasons offered earlier for discounting future consumption gains and losses—namely, "impatience" and "intergenerational equity." As noted earlier, the larger is δ, the larger is ρ_t, other things being equal. So we turn to the influence of η on ρ_t. η is an index of the

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[12] Proof: Because the pair of variations ΔC_{t+1} and ΔC_t leave the numerical value of expression (2) unaltered,

$$U'(C_t)\Delta C_t/(1+\delta)^t + U'(C_{t+1})\Delta C_{t+1}\Big/(1+\delta)^{t+1} = 0. \tag{F2}$$

By definition,

$$\rho_t = -\Delta C_{t+1}/\Delta C_t - 1, \tag{F3}$$

where ΔC_{t+1} and ΔC_t satisfy equation (F2). Now use equations (3), (F1)–(F3) to obtain equation (4) in the text.

[13] Proof: Take the logarithm of both sides of equation (4) and, using the fact that if x is small in absolute value, $\ln(1+x) \approx x$, the approximate equation 4a follows.

curvature of U. Equations 4 and 4a say that if $g(C_t) \neq 0$, the larger is η, the larger is the absolute value of ρ_t, other things being equal. This proves

Proposition 1 *η is the index of the aversion society ought to display toward consumption inequality among people—be they people in the same period or in different periods.*

It will prove useful to table the most-preferred values of δ and η in Cline (1992), Nordhaus (1994), and Stern (2006).

Cline: $\delta=0$; $\eta=1.5$

Nordhaus: $\delta=3\%$ a year; $\eta=1$

Stern: $\delta=0.1\%$ a year; $\eta=1$

In the context of Eq. 4a, notice how close the authors are in their choice of η. Notice also how close Cline and Stern are in their specifications of δ. In Section 3.4 we ask why, among the three, Nordhaus is such an outlier in his choice of δ. Here we note that to say that $\eta=1$ is to insist that any proportionate increase in someone's consumption level ought to be of equal social worth to that same proportionate increase in the consumption of anyone else who is a contemporary, no matter how rich or poor that contemporary happens to be. It is also to insist that, if in addition $\delta=0$, any given proportionate increase in consumption today ought to be of equal social worth to that same proportionate increase in consumption at any future date, no matter how rich or poor people will be at that future date. Taken at face value, though, it isn't immediate whether such tradeoffs are ethically reasonable. In Section 3.2 we run more informative tests. They confirm that the pair ($\delta\approx0$, $\eta=1$) can recommend bizarre policies in classroom models of consumption and saving.

For computational purposes, it helps to assume that expression (4a) is a good approximation. I summarise the points it makes:

(a) ρ_t is not a primary ethical object, it has to be derived from an overall conception of intergenerational well-being and the consumption forecast: consumption discount rates cannot be plucked from air. (b) Just as growing consumption provides a reason why discount rates in use in social cost–benefit analysis should be positive, declining consumption would be a reason why they could be negative. Example: Suppose $\delta=0$, $\eta=2$, and $g(C_t) = -1\%$ per year. Then $\rho_t=-2\%$ per year. Such reasoning assumes importance when we come to discuss that people in the tropics, who are in any case very poor, will very likely suffer greatly from climate change under business as usual (Section 3.5). The reasoning takes on an interesting application when we come to consider uncertainty in future consumption (Sections 4 and 5).[14] (c) If inter-temporal external diseconomies are substantial, as is the case with climate change under business as usual, both ρ_t and private rates of return on investment could be positive for a period of time, even while the social rate of return on investment is negative.[15]

[14] I have friends in the US who find illustrations involving negative economic growth to be unrealistic. In fact a number of countries in sub-Saharan Africa suffered from negative growth during the period 1970–2000. What discount rates should government project evaluators there have chosen in 1970 if they had an approximately correct forecast of the shape of things to come?

[15] See Dasgupta et al. (1999). This parallels the well-known fact that if the external disbenefits arising from anyone's use of a commodity are large enough, the commodity's shadow price will be negative even when its market price is positive.

(d) Only in a fully optimizing economy (Section 3.2) is it appropriate to discount future consumption costs and benefits at the rate that reflects the direct opportunity cost of capital. In imperfect economies ρ_t should be used to discount consumption costs and benefits, but the capital deployed in projects ought to be re-valued so as to take account of the differences between ρ_t and the various rates of return on investment (Section 3.3). Note though that the re-valued cost of capital would be less than the price of consumption if the social rate of return on investment in that form of capital is less than ρ_t. (e) Unless consumption is forecast to remain constant, social discount rates depend on the *numeraire*: $\rho_t=\delta$ if and only if $g(C_t) = 0$. (f) If $g(C_t)$ varies with time, so does ρ_t. For example, suppose it is forecast that long-run consumption growth is not sustainable but will decline at a constant rate of 1% a year—from the current figure of 2% a year to zero. Suppose $\delta=0$ and $\eta=2$. In that case ρ_t will decline over time at 1% a year, from a current-high 4% a year, to zero. Note that the "hyperbolic" discounting that comes with a declining value of $g(C_t)$ does *not* lead to time inconsistency over project evaluation. As intergenerational well-being is reflected in expression (2), social preferences are inter-temporally consistent. In other words, that ρ_t will decline over time at 1% a year doesn't mean that future generations will be enjoying the gift from the present generation of a "preferential discount rate."

The point estimate of consumption growth under business as usual in Stern (2006) is $g(C_t) = 1.3\%$ a year. Using this in Eq. 4a, we find that:

$\rho_t = 2.05\%$ a year for Cline
$\rho_t = 4.30\%$ a year for Nordhaus
$\rho_t = 1.40\%$ a year for Stern

4.3% a year may not seem very different from 1.4% a year, but is in fact a lot higher when it is put to work on the economics of the long run. Just how much higher can be seen from the fact that the present-value of a given loss in consumption, owing, say, to climate change 100 years from now, if discounted at 4.3% a year is *17* times smaller than the present-value of that same consumption loss if the discount rate used is 1.4% a year. The moral is banal: If the time horizon is long, even small differences in consumption discount rates can mean large differences in the message cost–benefit analysis gives us. The reason Cline (1992) and Stern (2006) have recommended that the world spend substantial sums today to tame climate change, while Nordhaus (1994) has recommended a far more gradualist investment policy can be traced to the difference in their choice of δ.[16] In contrast to these authors, I suggest below that, while it is reasonable to set $\delta\approx0$, values for η larger than 1.5 should be considered, in the range [2,3], but perhaps even beyond that range.

How great is inequality aversion when the figure for η is in the range [2,3]? One way to answer would be to study consumption changes among contemporaries that are judged by expression (1) to be ethically equivalent.[17] Consider two people, 1 and

[16] Nordhaus (2007) confirms this by using Stern's specifications for δ and η in the climate-change model he has developed over the past two decades. It should be noted that Nordhaus' $\rho_t = 4.30\%$ a year is consistent with the US government's discount rate policy. On the latter, see Viscusi (2007).

[17] I am grateful to William Cline for correspondence on this way of studying how η should be chosen.

2, with identical U-functions, whose annual consumptions (at purchasing power parity) are \$360 and \$36,000, respectively. The former is below even the World Bank's "dollar-a-day" person, while the latter is well above the annual income of the average resident of the European Union, which is approximately \$29,000 (see World Bank 2007). An easy calculation shows that if $\eta=2$, our social evaluator would regard a 50% decrease in person 2's consumption to be ethically equivalent to a 1% decrease in person 1's consumption.

Does that look reasonable? A 50% decrease in 2's consumption will give him \$18,000, which is still a huge figure (the per capita GDP in the World Bank's "upper middle-income" country is \$11,000), whereas a 1% drop for individual 1 will bring his consumption down further to \$356.40. Some people, but perhaps not many, would find this trade-off to be reasonable. But what if $\eta=3$? A similar calculation shows that in that case a 1% decrease in person 1's consumption would be ethically equivalent to a 93% drop in 2's consumption. A 93% drop in consumption leaves individual 2 with \$2,520 (which is the per capita GDP in the World Bank's "low-income" country), as against the \$356.40 going to individual 1. If this trade-off feels unreasonable, we should ask whether the thought experiment we are conducting is itself reasonable. After all, our attitude toward income transfers is influenced not only by our concern for equality of outcome, but also by the recognition that incentives matter. Incentives in turn are shaped by the presence of moral hazard and adverse selection. As our thought experiment is oblivious of incentives, it is of little value in testing our intuition. So I turn to a thought experiment that is able to bypass some of those worries by focusing on what a generation should leave behind for its descendents.

3.2 The fully optimum economy

Consider the problem of optimum saving. A consumption sequence $\{C_t\}$ is a *full optimum* if it maximizes expression (2) in the set of all technologically feasible $\{C_t\}$s. We want to uncover how the optimum $\{C_t\}$ varies with η. For example, if a particular choice of η requires great sacrifices from earlier generations in order that later generations will be able to enjoy very high consumption, the η in question would not capture the idea of intergenerational equity in consumption.

It has proved unfruitful to test ethical intuitions in the "integrated assessment model" of climate change that Nordhaus has studied and in the global climate models described in Stern (2006), because it isn't possible to track what is influencing what in huge computer runs. Simple classroom production models are far better at informing us how η affects the relative ethical merits of alternative consumption sequences. Moreover, as many people regard value judgments to be "universalizable," the range of ηs that is chosen for consideration should not only lead to reasonable outcomes in the world we think we know, but also in worlds that are possible. The simplest production structure by far is the pure capital model, in which output is a fixed proportion of wealth. By wealth I mean not only reproducible capital, but also human capital (skills, knowledge, and health) and those types of natural capital whose stocks generate a flow of production services (e.g., ecosystem services). The rate of return on investment is taken to be a positive constant, r. To eschew diminishing returns to the factors of production may seem odd in a paper that addresses the economics of climate change, but in "wealth" I include every possible capital asset.[18] And as a check against unbridled optimism, I assume that there is no exogenous technological progress. The latter assumption, however, requires justification.[19] If our model economy were to enjoy exogenous productivity growth, consumption could be made to increase faster than any constant exponential rate. There is no evidence such patterns of growth have ever been experienced over any extended period of time. In any case, we shouldn't expect exogenous productivity growth in our model: as no capital asset is left out from the production function, accounting for economic growth doesn't leave behind a "residual." If labour productivity rises in our model economy, it is because of investment in various forms of capital.

A more common way to model production is to assume that reproducible capital and labour are imperfect substitutes; and that labour is a fixed factor, enjoying exogenous productivity growth. The problem is that it isn't possible to solve analytically for optimum consumption when the latter isn't very close to its long-run steady state. Mirrlees (1967) studied the sensitivity of optimum consumption to δ and η outside steady state, but he had to take recourse to numerical methods. Moreover, Mirrlees' general findings are not at variance with those I report below. That r is constant in the model I pursue allows me to offer a complete account of optimum consumption. In Sections 4 and 5 we will find that the model offers me an easy route for studying the effect of future uncertainty on today's investment decision.

Following Cline, Nordhaus, and Stern, I suppose that $\eta \geq 1$. Consumption is assumed to take place at the beginning of each period. Writing K_t for wealth at t, the economy's accumulation process can therefore be expressed as

$$K_{t+1} = (K_t - C_t)(1 + r), \qquad K_0(> 0) \text{ is given.} \tag{5}$$

In a fully optimum economy, the $\{C_t\}$ that society chooses maximizes expression (2), subject to the accumulation Eq. 5. But infinite sums, as is the case with expression (2), needn't converge. So, we must identify conditions under which an optimum $\{C_t\}$ exists. The parameters that specify our model economy are r, δ, and η. Let us begin by pretending that an optimum $\{C_t\}$ exists and determine the condition it must satisfy. A simple argument shows that an optimum $\{C_t\}$ must satisfy[20]

$$\rho_t = r, \qquad \textit{for all } t \geq 0. \tag{6}$$

Equation 6 says that r is the consumption discount rate in a fully optimum economy and only in a fully optimum economy. *We conclude that it is only in a fully*

[18] Quite obviously, I am making outrageous assumptions regarding aggregation of capital. In this I am no different from contemporary growth economists.

[19] It requires justification because Professor Brad De Long took me to task over it in the critique he posted on his blog on November 30, 2006, under the title, "Partha Dasgupta Makes a Mistake". His piece was a response to the review of Stern (2006) that was published in Dasgupta (2007).

[20] Proof: If ρ_t is less than r, society would be advised to save a bit more at t. But to save a bit more at t is to consume a bit less at t, and this tilts consumption more toward the remaining future, which in turn raises ρ_t. Alternatively, if ρ_t exceeds r, society would be well advised to save a bit less at t. But to save a bit less at t is to consume a bit more at t, and that tilts consumption more toward t, which in turn lowers ρ_t. It follows that along the optimum C_t, $\rho_t = r$.

optimum economy that the direct opportunity cost of capital should be used for discounting future benefits and costs.

What does an optimum $\{C_t\}$ look like? Using Eqs. 4 and 6, we note that C_t grows at the compound rate, g, where

$$C_{t+1}/C_t - 1 = g = [(1+r)/(1+\delta)]^{1/\eta} - 1. \tag{7}$$

From Eq. 5 it follows that K_t grows at the same rate. If r and δ are small, then g is small, and Eq. 7 becomes the approximation [21]

$$g \approx (r - \delta)/\eta. \tag{7a}$$

Equation 7 tells us that, along the optimum, consumption grows if $r>\delta$, but declines if $r<\delta$. The interesting case is where $r>\delta$.[22] In that case the optimum economy experiences positive growth. In what follows, we assume that $r>\delta$.

A macroeconomic variable for which we all have an intuitive feel is the "saving rate." In our model the concept has two meanings. Because consumption takes place at the beginning of each period, the *saving rate* at t is saving as a proportion of wealth at t, that is, $(K_t-C_t)/K_t$. This is the first meaning. And because both C_t and K_t grow at the same rate, the optimum saving rate is a constant. So, our search for the optimum saving rate reduces to a search for that constant rate of saving that maximizes expression (2). Writing the optimum saving rate as s^*, routine calculations show that,

$$s^* = (1+r)^{-(\eta-1)/\eta}(1+\delta)^{-1/\eta}. \tag{8}$$

Now recall Eq. 5. It says that *net* saving is zero if $s = (1+r)^{-1}$, implying that C_t is constant if the saving rate equals $(1+r)^{-1}$. At the other extreme is a saving rate of unity, which is associated with the worst possible consumption sequence, because $C_t=0$ for all t. We therefore want to identify conditions under which s^* in expression (8) is meaningful (i.e., $s^*<1$). We have assumed that $r>\delta$. This means $s^* > (1+r)^{-1}$. We now assume that either (i) $\eta=1$ and $\delta>0$, or (ii) $\eta>1$ and $\delta\geq0$. In either case, $s^*<1$, implying that an optimum consumption programme exists.[23] So we have,

Proposition 2 *The optimum saving rate is a decreasing function of η and δ. If, holding δ and r constant, larger and larger values of η are admitted, s^* declines to $(1+r)^{-1}$.*

The first part of Proposition 2 explains the sense in which η and δ, are fundamental ethical parameters. The second part describes a limiting case. Solow

[21] This is the same as approximation (4a), with $r = \rho_t$.

[22] In Section 3.4 I suggest that in a deterministic world δ should be set equal to zero.

[23] The rigorous argument would have us check that the saving rate in equation (8) satisfies the transversality condition, namely, that the present discounted value of wealth (in well-being units) tends to zero as t tends to infinity. Readers can check that it does.

(1974) observed that in one interpretation of Rawls (1972), $\eta=\infty$. What Proposition 2 says is that, to assume $\eta=\infty$ is to display infinite inequality aversion.

Citing consumer behaviour (Section 3.4), Nordhaus (1994) and Stern (2006) are in agreement that $\eta=1$, which, on using Eq. 8 implies that $s^*=1/(1+\delta)$. But in that case s^* is independent of r, a fact that should alone set off alarm bells that $\eta=1$ reflects bad ethics. To see how bad the ethics is, let us follow Stern by setting $\delta=$ 0.1% a year. Then $s^*=1/1.001$. Is this large or small? To answer, we study the second interpretation of the saving rate in our model.

Because net saving is zero if $s=1/(1+r)$, we should normalise round that figure. Moreover, the maximum possible rate of saving is 1, which implies that the range of non-negative saving rates is $[(1+r)^{-1}, 1]$. Since the saving-wealth ratio is $(K_t-C_t)/K_t$, its normalised value is $[(K_t - C_t)/K_t - (1+r)^{-1}]/[1-(1+r)^{-1}]$. Now, output at $t+1$ is rK_t. It is easy to confirm that the normalised saving-wealth ratio is none other than the more familiar *saving-output ratio*.[24] Let $\tilde{s}^*$ be the optimum saving-output ratio. Let us suppose both r and δ are small. Then routine calculations on expression (8) show that

$$\tilde{s}^* \approx (r-\delta)/\eta r. \tag{8a}$$

If the unit interval of time were made smaller and smaller, (8a) would become a better and better approximation. In the limit, where time is continuous, (8a) is an equality.

Suppose $r=4\%$ a year. Approximation (8a) says that at $\delta=0.1\%$ a year, $\tilde{s}^*$ is 97%. This is an absurdly high rate of saving out of income. Never mind that future generations will be vastly richer: the present generation should not object! $\eta=1$ doesn't reflect much inequality aversion.[25]

If we are to smooth intergenerational consumption, larger values of η have to be admitted. Figures in the range [2,3] suggest themselves. And if we are forced to go empirical on the matter, I can cite Hall (1988), who estimated η to be broadly in the range [2,4] from consumer behaviour in the US. Equation 8a says that if $\eta=2$, the optimum ratio of saving to output is approximately 49%; that if $\eta=3$, it is approximately 32%; and that if $\eta=4$, it is approximately 24%. These are far more palatable figures.

[24] Proof: Re-write equation (5) as

$$K_{t+1} - K_t = rK_t - (1+r)C_t,$$

which says that a consumption level of C_t at the beginning of period t is equivalent to the consumption level $(1+r)C_t$ at the end of that period. So saving out of output at the end of t is $(rK_t-(1+r)C_t)$. Therefore the ratio of saving to output is $(rK_t-(1+r)C_t)/rK_t$, which, as is easily confirmed, equals the normalized saving-wealth ratio.

[25] This result is very old. It dates back to Ramsey (1928). In defense of his choice of $\eta = 1$, Stern (2008) complains that the 97% saving rate I have just obtained is a feature of a very artificial model. Of course it is. But "non"-artificial models, such as those Stern used in his computer runs, don't reveal which parameter is doing what work in generating his findings. How is one to test the robustness of ethical assumptions if not by putting them to work in stark, artificial models?

3.3 Capital revaluation in the imperfect economy

Imagine that, because of imperfections in the capital market, the saving rate doesn't equal s^*, but is a constant, s, and that $(1+r)^{-1} < s < s^*$. (The latter inequality implies that the economy is under-investing for the future, while the former inequality implies that the economy enjoys growth.) Consumption grows at the rate $(s(1+r)-1)$, as do wealth and output. Let ρ be the consumption rate of discount along the optimum. Equation 4 implies that

$$1+\rho = (1+\delta)s^{\eta}(1+r)^{\eta}. \tag{9}$$

Because $s < s^*$, we know from Eqs. 6 and 9 that $\rho < r$.

If costs and benefits associated with investment projects are measured in terms of consumption, ρ is the rate our social evaluator ought to use for evaluating projects. It is commonly argued though, that, because r is the productivity of capital, the correct discount rate to use in social cost benefit analysis is r. To use ρ as the discount rate runs the risk that relatively low-yielding projects will crowd out high-yielding ones, or so the argument continues. And indeed, the practice of using r in public policy contexts is familiar; for example, in the United States (see Viscusi 2007).

The argument's premise is wrong though. In the imperfect economy we are studying, r is not the social rate of return on investment. So, investment needs to be re-valued in social cost–benefit analysis.[26] Let P_k be the shadow price of capital relative to consumption *numeraire*. P_k is the social opportunity cost of capital: when a unit of capital is invested in a project, P_k is the present discounted value of the flow of displaced consumption. Routine calculations yield,[27]

$$P_k = (1-s)(1+\rho)/[(1+\rho) - s(1+r)]. \tag{10}$$

We know that $\rho = r$ if $s = s^*$. But in that case, Eq. 10 says $P_k=1$, which confirms that at a full optimum, consumption and investment are equally valuable at the margin. However, as $s < s^*$ in our imperfect economy, we have $P_k>1$. Moreover, the smaller is s, the bigger is the gap between r and ρ, which in turn means the larger is P_k. So, even though we would use ρ to discount future costs and benefits, a project would have to be high yielding to pass the cost–benefit test. Of course, it may be that the project evaluator chooses investment as *numeraire* (as did Little and Mirrlees 1969). In that case consumption would have to be revalued at $1/P_k$. Choice of *numeraire* has no bearing on project selection.

3.4 Revealed preference and calibration, or, how should society select δ and η?

Because capital is productive, later generations enjoy a natural advantage over earlier generations. The expression for s^* (Eq. 8) says that if $\delta=0$ and $\eta<\infty$, the optimum

[26] See Marglin (1963) and Dasgupta et al. (1972). Among economists writing on climate change, only Cline (1992) has mentioned the need to revalue capital in imperfect economies.

[27] Proof: a marginal additional unit of capital at $t = 0$ yields a small change in consumption, ΔC_t, equal to $(1\text{-}s)(s(1+r))^t$. At the consumption discount rate ρ, the present value of that small change, from 0 to ∞, is the expression for P_k. (Note that, because $s > (1+r)^{-1}$, the present value exists.) Equation (10) is due to Marglin (1963).

policy for each generation is to save so that future generations can be wealthier. That way, or so the ethical reasoning goes, advantage can be taken of the productivity of capital. The lower is η, the larger is the optimum saving rate. Net positive saving ensures that consumption rises indefinitely, implying that generations in the distant future will be far better off than those alive now. If this is in conflict with our immediate intuition regarding distributive justice, we have the choice of considering larger values of either δ, or η, or both.

One influential school of philosophers has argued that societal impatience is ethically indefensible. They say that to set $\delta>0$ is to favour policies that discriminate against the well-being of future generations merely on the grounds that they are not present today.[28] They also say that values frequently in use among economists, ranging as they do between 2–3% a year, are way too high.

I find their argument hard to rebut. Admittedly, the ethical axioms Koopmans (1972) imposed on infinite consumption sequences implies time discounting, but the axioms don't say how large the discount rate ought to be. Koopmans' axioms are consistent with very, very low values of δ.[29] In contrast, to assume $\delta=2\%$ a year, as is routinely done in the economics literature, is to say that the felicities of the next generation (35 years down the road) ought to be awarded half the weight we award our own felicities. Justifying that is difficult. But once we accept the philosophers' argument, we must turn to the second part of Proposition 2, which tells us that η is an index of aversion to consumption inequality. The problem is that we have very little prior understanding of what η implies as regards intergenerational saving. That's why it is necessary to conduct sensitivity analyses on Eq. 8 by varying η, which is what we have just done. Such exercises are thought experiments, resembling laboratory tests. They give us a sense of how the interplay of facts and values in complicated worlds tells us what we should do. Rawls (1972) called the termination of iterative processes involving such thought experiments, "reflective equilibria".

To illustrate, consider an optimizing society. We know that the growth rate of consumption, $g(C_t)$, satisfies Eq. 7. But that equation says that δ and η play similar roles in determining the character of the optimum $\{C_t\}$: subject to $r>\delta$, the larger is η or δ, the more even is the intergenerational distribution of optimum consumption (which is another way of stating Proposition 2). But the *reasons* δ and η play similar roles should matter; and the reasons differ. As moral philosophers have observed, if we try to achieve greater equality in consumption by increasing δ, we run into a problem of intergenerational inequity. It seems to me we should experiment instead with η, which is the tactic I have adopted here.

Even as I compose this paper, I realise that doing welfare economics is a delicate matter. There is a fine dividing line between ethical thinking and authoritarian

[28] Ramsey (1928: 261) famously wrote that to discount future well-being is "ethically indefensible and arises merely from the weakness of the imagination". That is, of course, not an argument; merely an expression of one's beliefs. Broome (1992) contains a summary of the arguments that support Ramsey's position.

[29] Possible extinction of the human race offers a reason for $\delta > 0$, but that is a different reason for positive time discounting. We discuss that in Section 4. We should also bear in mind that infinite-horizon deterministic models are mathematical artifacts: we know Humanity will not survive forever.

impulses. It is all well and good for the ethicist to assume the high moral ground and issue instructions like a philosopher-king or a Whitehall Mandarin, but social ethics contains an irremediably democratic element. If others aren't persuaded by the conclusions ethicists have reached, the policies they recommend ought to take those others' ethical viewpoints into account. If we are studying the character of optimum policies in a deterministic world, I personally don't know how to justify a δ that is much in excess of zero; but if the protagonist for whom I am writing this paper is not persuaded by me, her view should count equally and we should conduct sensitivity tests on δ as well.[30]

Nordhaus (1994, 2007) holds that δ and η ought to be calibrated to be consistent with: (i) market interest rates (including interest rates offered in government bonds), (ii) observed values of $g(C_t)$, and (iii) rates of private and public saving and investment. This is an interesting, democratic move, in that the idea is to infer δ and η from data generated by people's behaviour as they go about their daily lives—making decisions on how much to consume, how much to spend on their children's education, how much to save for their own future, what public policies to vote for, and so on. However, many ethicists find the move unacceptable. Broome (2008) shows Aristotelian disdain toward anything so crass as a reliance on observed "interest rates" for arriving at figures for δ and η in public decision-making. He calls r in Eq. 5 the "money market" interest rate—the terminological shift from "real" (as in the *real* productivity of capital) to "money" (as in *money* market interest rates) is designed, presumably, to make Nordhaus' move look ethically bogus. Broome dismisses claims that Nordhaus' approach reflects a democratic point of view, and says democracy requires debate and deliberation as well as voting. Viewed from the Common Room it could no doubt appear that citizens in functioning democracies are so thoughtless as not to debate, deliberate, read newspapers, or turn on the news channel; but I rather doubt that Broome's view would resonate with the taxpaying public.

Nevertheless, there is a problem with Nordhaus' stance when the object of study is climate change, which, under "business as usual" involves a massive global commons problem. For all we know, social rates of return on investment in energy intensive activities are negative today. But the market economy wouldn't tell us they are, because private rates of return would perforce be positive (why else would anyone invest?). That alone is a reason why none of the private rates can be regarded to be the consumption discount rate. An alternative would be to imagine that consumers maximize expression (2). Suppose r^* is the private rate of return on investment. The idea now would be to estimate the two ethical parameters, δ and η, by studying consumer behaviour. We could do that by imagining that, rather than Eq. 6, the two parameters satisfy the condition, $\rho_t = r^*$, where ρ_t is defined by Eq. 4. But $\rho_t = r^*$ is only one equation. So we would have to estimate one of the unknowns from other types of data. There is then a problem of consistency in the ways the parameters have been estimated in the different studies. More importantly, it is most doubtful that even thoughtful households maximize expression (2). As we noted

[30] In this context, Arrow (1963) can be interpreted as an attempt to discover an aggregator function of individual *ethical* preferences. It isn't an accident that the title of his classic is "Social Choice and Individual *Values*". I have explored that interpretation in Dasgupta (2005).

earlier, the formula has no room for the "self." So, even if it is accepted that expression (2) should be used to inform public policy, there is a serious possibility that observed behaviour offers a wrong basis for calibrating δ and η.

But in relying exclusively on revealed preference, Nordhaus has been consistent. Cline and Stern would appear not to have bothered at all about consistency. They chose η on the basis of estimates obtained from consumer behaviour, but ignored consumer behaviour entirely when it came to the choice of δ and sought the advice of moral philosophers instead. This is neither good economics nor good philosophy.

Expressions (2) and (3) reduce the ethics underlying intergenerational welfare economics to two parameters: δ and η. If, as I suggested earlier, the appropriate value for δ in a deterministic world is approximately zero, the whole weight of our ethical concerns regarding the distribution of consumption across the generations is borne by η. That is an awful lot of work for a single number to do adequately. But the assumption that η is independent of C has only tractability to commend it. It seems to me many of the ethical puzzles thrown up by intergenerational welfare economics have been due to that assumption. It may be time that economists experiment with Us for which the elasticity of $U'(C)$ is an increasing function of C.

3.5 Consumption smoothing among whom?

Earlier I suggested that if we are to work with constant ηs, the range [2–4] suggests itself. But it can be argued that even $\eta = 3$ flies against the face of revealed preference on foreign aid in the contemporary world. Schelling (1999) has very reasonably noted that the rich world's moral posturing over the problem of global climate change doesn't square with its reluctance to increase foreign aid to poor countries beyond the very small proportion of income allocated to it today. Recalling expression (1), if average consumption in the contemporary poor and rich worlds are C_p and C_r, respectively, and N_p and N_r are the sizes of their populations, world well-being today would be $(N_pU(C_p) + N_rU(C_r))$. Now, N_p exceeds N_r ($N_p \approx 3N_r$) and C_r far exceeds C_p ($C_r \approx 20C_p$). Schelling didn't argue that climate change shouldn't be taken seriously, but rather that it would be more equitable and efficient to invest in reproducible and human capital now, so as to build up the productive base of economies—including, especially, poor countries—and divert funds to meet the problems of climate change at a later date, when people are a lot richer. Schelling's reasoning leads him to a point of view rather similar to that of Nordhaus.

It seems to me though that there is a reason why people in the rich world could justifiably translate their concerns about equity into doing a lot more for "tomorrow's them" than "today's them." That has to do with incentives, governance, and responsibility. We should be anxious over the plight of future generations caused by climate change because we are collectively responsible for amplifying that change; the rich world especially so. If future generations inherit a hugely damaged Earth, it is we who would be in part responsible. In contrast, it isn't possible to trace the source of absolute poverty in today's poor countries solely to a combination of past colonialism and present inequities in the global trading system. There are many other reasons why the world's poorest countries continue not to progress. Bad governance and an absence of social cohesion that lead to communal battles for resources are but two of those reasons; and our protagonist, whom I introduced at the beginning of this paper, could be forgiven for maintaining that, while she does join public demonstrations against the inequities of the global trading system, there isn't much she can do about bad governance and societal conflict in other places. Interfering in foreign countries' affairs excepting under extreme circumstances violates other principles of international justice, such as respecting the autonomy of nations.

Matters are different *within* countries. The rich in Western democracies have been paying a lot more than a mere 2% of their incomes for redistributive purposes. Our protagonist contributes significantly to protect and promote her fellow citizens' well-being. Stern (1976) calibrated η on the basis of income tax rates in the United Kingdom when applied to the timeless model of optimum income taxation due to Mirrlees (1971) and arrived at a specification of $\eta=2$. That said, climate change is predicted to inflict far more damage to the people in the tropics (the poor world) than to the temperate zone (the rich world). Today's rich world, which has been and continues to be the site of the largest emissions of carbon per person, has a particular obligation toward tomorrow's people in today's poor world. Increasing η from 1 to, say, 3 would accentuate that obligation.[31]

I don't believe what I have offered is anything like an airtight argument. All I have done is to draw attention to ethical principles that create an asymmetry between tomorrow's "them" and today's "them." Concern for future generations isn't a case of misplaced ethics.

4 Intergenerational well-being: future uncertainty

Yaari (1965) showed that if Humanity is subject to a constant exogenous risk of extinction—say at the hazard rate δ per year—each generation could reasonably pretend that there is no chance of extinction, but discount future felicities at the hazard rate. Stern (2006) has justified the choice of $\delta=0.1\%$ a year on that very basis.

4.1 Uncertain constant growth rates and hyperbolic discounting

Humanity faces many risks and uncertainties. One particular risk is over future consumption, conditional on Humanity being around. In an influential study, Weitzman (2001) sought to show that the consumption discount rate society ought to choose would be hyperbolic if people differed in their opinion of what the future holds. He invited some 2,800 economists to respond to a questionnaire in which they were asked (p. 271) to submit a *single* number as their "best estimate of the appropriate real discount rate to be used for evaluating environmental projects over a long time horizon." Weitzman also explained his motive (p. 271): "What I am after here is the relevant interest rate for discounting real-dollar changes in future goods and services—as opposed to the rate of pure time preference on (felicity)." In other words, economists were asked to submit their best estimate of consumption discount rates and were told that they could submit only one number. Weitzman's defence of

[31] Barrett (2003) contains an interesting discussion of those obligations.

that restriction was that policy makers find it difficult to grasp consumption discount rates that are not constant. I don't know what policy makers would say if they were informed of that particular view of their intellectual ability, but 2,160 economists from 49 countries responded to the questionnaire. Each supplied a number, of which all but three were non-negative. Forty-six respondents gave zero as their best estimate, while the rest supplied positive numbers (Weitzman 2001: Table 1). Weitzman found that if one were to ignore the 49 non-positive numbers, the responses had the shape of a gamma distribution. He interpreted the responses as draws from an urn by a policy maker who is uncertain of what fixed rate to use for discounting social profits. Being uncertain, the policy maker selects investment projects on the basis of the expected present discounted value (PDV) of the flow of social profits. Weitzman showed that if the uncertain, but fixed, discount rate is governed by a gamma distribution, the expected PDV of a project is proportional to a sure PDV of that same project, but for which the rate used to discount future social profits is not a constant, but decreases over time (starting as a positive number and declining to zero in the long run). Weitzman concluded that the policy maker should use a positive but declining rate to discount social profits of long-term investment projects. (Sozou (1998) independently proved the same mathematical result, but his motivation was entirely different. Sozou sought to explain hyperbolic discounting among starlings and pigeons.)

As I felt unable to respond to Weitzman's questionnaire, I didn't send in a number. The stipulation puzzled me then and it puzzles me even now. The horizon Weitzman wished to consider is 300 years or more. Why should it be insisted that my estimate of the consumption discount rate over such a long period be a constant? To respond to the questionnaire would require of me to suppose that consumption would change at a constant rate over a very long period of time (Eq. 4). It seemed to me that the questionnaire had restricted economic possibilities in an unacceptable way.[32]

Weitzman (2007a) has revisited his earlier work and has altered the motivation behind his study. Suppose that intergenerational well-being under uncertainty is the expected value of expression (2). Imagine that $g(C_t)$—the growth rate in consumption—is an uncertain constant. Let j denote a sample path and g_j the constant growth rate in consumption along that path. Equation 4a then tells us that, if δ and the g_js are all small, the consumption discount rate along j is $\rho_j = \delta + \eta g_j$.

Assume that there are a finite number of possible g_js. Let π_j be the subjective probability that g_j will prevail. Then π_j is also the subjective probability that ρ_j is the

[32] Dasgupta and Maskin (2005) have offered an explanation for hyperbolic discounting (preference reversal, more generally), among starlings and pigeons, that is based on selection pressure over evolutionary time. The authors assume that the decision maker has to choose between two options: (i) a reward, V (> 0), that will appear at an uncertain date (the expected date being T), and (ii) a reward, V^* (> 0), that too will appear at an uncertain date (the expected date being T^*). Assuming $V^* > V$ and $T^* > T$, the authors show that, under quite general circumstances concerning the distributions of the uncertain arrival times, a risk neutral decision maker would display preference reversal (from V to V^*) if neither reward appeared for a while. In the present paper, I am studying social ethics, not private preferences. The viewpoint I am adopting here is that individual behaviour based on hyperbolic discounting is a constraint the social evaluator must take into account when he evaluates public policies, but that the evaluative criterion for social choice should be intergenerational well-being (expression (2)).

appropriate consumption discount rate. Weitzman (2007a) shows that society can equivalently pretend that there is no risk, but use a time varying consumption discount rate α_t, where

$$\alpha_t = -\ln\left(_j \sum\left(\Pi_j \exp\left(-\rho_j t\right)\right)\right)/t. \tag{11}$$

Equation 11 provides a justification for hyperbolic discounting in a societal context. It implies that the certainty-equivalent consumption discount rate should decline over time from $\alpha_0 = _j \sum(\Pi_j \rho_j)$ to the limit, $\alpha_\infty = \min\{\rho_j\}$.

But as with the earlier questionnaire, there is a problem with this line of reasoning. I know of no reason why we should be required to restrict the state space to constant growth paths. Presumably, future consumption is uncertain because the production process is stochastic. So we should model the stochastic process explicitly. In what follows we study optimum consumption plans when future output is risky. The analysis will yield both stochastic and risk-free consumption discount rates along optimum consumption sequences. It will be found that none is hyperbolic. As in Section 3.3, our analysis can be extended to imperfect economies.

4.2 Consumption discount rates in an uncertain production economy

Levhari and Srinivasan (1969) studied optimum policies in a world where, at each date, r in the pure capital model of Section 3 (Eq. 5) is drawn independently from the same probability distribution.[33] Imagine that $(1+r)$ is a random draw from an urn in which, in each period, $\ln(1+r)$ is distributed independently, identically, and normally, with mean μ and variance σ^2. We take it that μ and σ^2 are known.

Let $\bar{r}$ be the expected value of $\tilde{r}$. Assume $\bar{r} > \delta$. Obviously, $\bar{r}$ is a function of μ and σ; as is the variance of $\tilde{r}$.[34] Assume that $\eta \geq 1$. Levhari and Srinivasan showed that the optimum saving rate, $(K_t - C_t)/K_t$, is

$$s^{**} = (1+\bar{r})^{-(\eta-1)/\eta}(1+\delta)^{-1/\eta}\exp\left[(\eta-1)\sigma^2/2\right], \tag{12}$$

provided the parameters $\bar{r}$, σ, η, and δ assume values for which $s^{**} < 1$. For the moment, let us suppose they do.[35]

[33] The subsequent, asset-pricing literature (e.g., Brock 1982) has explored models that are more general than the one studied by Levhari and Srinivasan (1969). I use the Levhari-Srinivasan formulation to illustrate my points because of its simplicity and because its findings are directly comparable to those discussed in textbooks on asset pricing (e.g., Cochrane 2005), where asset prices are taken to be exogenous stochastic variables.

[34] It is easy to show that,

$$1+\bar{r} = \exp(\mu + \sigma^2/2), \tag{F4}$$

$$\textit{and} \quad \mathrm{var}(1+\tilde{r}) = \mathrm{var}(\tilde{r}) = \left(\exp(\sigma^2) - 1\right)\exp(2\mu + \sigma^2). \tag{F5}$$

[35] Notice that $s^{**} = s^{*}$ (equation (8)) if $\sigma = 0$.

Let $\widetilde{s}$** be the saving-output ratio.[36] Assume that $\overline{r}$ and δ are both small. A computation identical to the one that yielded approximation (8a) reduces Eq. 12 to the approximation

$$\widetilde{s}^{**} \approx (\overline{r} - \delta)/\eta\overline{r} + (\eta - 1)\sigma^2/2\overline{r}. \tag{12a}$$

If the unit interval of time is made to smaller and smaller, approximation (12a) becomes better and better. Notice that if $\eta > 1$, uncertainty in future productivity is a reason for saving more (the precautionary motive for saving); but that if $\eta = 1$, uncertainty has no effect on s^{**}. $\eta = 1$ is a bad assumption in this model.

Consider a sample consumption sequence $(\widetilde{C}_t)$. Using Eq. 4 we know that the *stochastic consumption discount rate*, $\widetilde{\rho}_t$, satisfies the equation

$$1 + \widetilde{\rho}_t = (1 + \delta)\left(1 + g\left(\widetilde{C}_t\right)\right)^{\eta}. \tag{13}$$

If δ and $g(\widetilde{C}_t)$ are both small, Eq. 13 reduces to the approximation,

$$\widetilde{\rho}_t \approx \delta + \eta g\left(\widetilde{C}_t\right). \tag{13a}$$

Imagine now that the social evaluator wants to avoid working with random variables, and so replaces perturbations to random consumptions by their expected values. The rate she should use to discount sure changes to those consumptions if she is to avoid making an error in computing optimum investment policies is called the *risk-free* rate in the finance literature.[37] Let that rate be $\overline{\rho}_t$, and let $\widetilde{C}_t$** be the optimum uncertain consumption at *t*. It can be shown that, provided δ and $\overline{r}$ are small,[38]

$$\overline{\rho}_t \approx \overline{\rho} = \delta + \eta E\left[g\left(\widetilde{C}_t^{**}\right)\right] - \eta^2 \operatorname{var}\left[g\left(\widetilde{C}_t^{**}\right)\right]/2, \tag{14}$$

where $E[g(\widetilde{C}_t^{**})]$ is the expected value of $g(\widetilde{C}_t^{**})$ and $\operatorname{var}[g(\widetilde{C}_t^{**})]$ is the variance of $g(\widetilde{C}_t^{**})$. As $E[g(\widetilde{C}_t^{**})]$ and $\operatorname{var}[g(\widetilde{C}_t^{**})]$ are both constants, the risk-free rate is a constant, not hyperbolic.

The third term on the right hand side of Eq. 14 shows that an increase in risk reduces the risk-free rate, other things being equal.[39] This feature of $\overline{\rho}$ is related to summary point (b) of Section 3.1: an increase in risk raises the downside risk that the economy will hit very low consumption levels in the future, and that lowers the risk-free rate.

In Dasgupta (2007) I argued that Stern and his co-authors ought to have tested the sensitivity of their recommendations to the choice of η on grounds that without running tests, it isn't possible to tell whether η in the range 2–4 would have led them to recommend a greater immediate concern for global climate change (i.e., do more

[36] That is, $\widetilde{s}^{**} = [s^{**} - (1 + \overline{r})^{-1}]/[1 - (1 + \overline{r})^{-1}]$. See footnote 24.

[37] Alternatively, we could call it the "certainty-equivalent rate".

[38] The proof, which makes use of the assumption that $(1+r)$ is drawn from a lognormal distribution, is similar to the one that was used to arrive at equation 4a. The equation is familiar in the theory of finance (Cochrane 2005: p. 10). Notice that if $\sigma = 0$, equation (14) reduces to equation 4a.

[39] Approximation (12a) summarized a related finding that the optimum saving rate increases with increasing risk.

now to ease the problem than would be recommended by $\eta=1$) or a less immediate concern (i.e., do less now to ease the problem than would be recommended by $\eta=1$). As they did not conduct such a test, it will be useful to summarise what s^{**} (Eq. 12) tells us.

Proposition 3 *η is not only an index of inequality aversion, it is also an index of risk aversion. At the saving rate s^{**}, future generations can be expected to be richer than the present generation. Because of the growth effect, larger values of η recommend earlier generations to save less for the future (the equity motive). However, as future productivity is uncertain, larger values of η recommend earlier generations to save more (the precautionary motive). The combined effect depends on the parameters η, δ, $\bar{r}$ and σ.*

Economists working on climate change have tended to set $\eta=1$. We found that $\eta=2$ or 3 yield more reasonable recommendations about saving rates in the deterministic version of the pure capital model of Section 3.2. Consider $\eta=3$. Equation 12 says that whether society ought to save more for the future or less if $\eta=3$ than it ought to if $\eta=1$ depends on whether σ^2 is greater or less than $2\ln((1+\bar{r})/(1+\delta))/3$. That $\bar{r}$ and σ contribute to the answer in opposition to one another is what intuition should have told us.

It will prove instructive to experiment with values of η higher than 1. As before, assume $\bar{r}$ = 4% a year and δ=0.1% a year. Assume also that $\sigma/\mu=1$. From approximation (12a) we find that the optimum saving-output ratio is 52% if $\eta=2$, and 38% if $\eta=3$. On comparing these figures with the corresponding numbers we arrived at for $\sigma=0$ (Section 3), we note that if $\eta=2$, the risk in future productivity of capital is a reason for raising the saving-output ratio from 49% to 52%; whereas, if $\eta=3$, that same risk is a reason for raising the saving-output ratio from 32% to 38%. The precautionary motive for saving is noticeable even if the risk is relatively small.

The precautionary motive would seem to increase rapidly with uncertainty. For example, suppose $\sigma/\mu=2$. If $\eta=3$, the optimum saving-output ratio is 48%, which is a considerable increase from the figure of 32% in the absence of risk.

5 Large uncertainties

All that said, we shouldn't believe any model that explicitly models risk when the horizon extends 100–200 years into the future. We simply don't know what the probabilities are. If we were to acknowledge this in the Levhari–Srinivasan model, we would say that $\bar{r}$ and σ are themselves unknown. Estimating them from observations raises the problem that we are required to make a forecast of future realizations of $\tilde{r}$ over the indefinite future, but have data on only a finite number of its past realizations. Worse, we will continue to observe only a finite number of realizations. Pesaran et al. (2007) and Weitzman (2007a, b) have shown that the probability distributions over the uncertain $\bar{r}$ and σ can plausibly have a *thick* lower tail, implying that a long, long run of low realizations of $\tilde{r}$ would not be improbable. In the context of global climate change, this reasoning becomes relevant, because we have little-to-no *usable* record from a world where the mean global temperature was, say, 3°C above the current level.

In the Levhari–Srinivasan model, however, $\bar{r}$ and σ are assumed to be known. So, with one hand tied behind our back, let us interpret the econometrician's message as being that σ is "large". By assumption, $\ln(1+\tilde{r})$ is normally distributed. But that implies that the distribution is *thin*-tailed. So we want to identify the implications for the optimum saving rate when the risk is thin-tailed but "large". In Section 4 it was assumed that the values of the parameters, δ, η, $\bar{r}$, and σ fall within a range for which s^{**} is less than 1. However, Eq. 12 says that $s^{**} \geq 1$ if

$$\sigma^2/2 \geq \ln(1+\delta)/\eta(\eta-1) + \ln(1+\bar{r})/\eta. \tag{14}$$

As $s^{**} \geq 1$ is nonsensical, we can summarise the finding as

Proposition 4 *If σ satisfies inequality (14), no optimum policy exists.*

How large must the uncertainty be for inequality (14) to hold? Let σ^* be the value of σ at which (14) is an equality; implying that, for values of σ in excess of σ^*, inequality (14) holds strictly. Suppose, as earlier, that $\delta=0.1\%$ a year, $\eta=3$, and $\bar{r}=4\%$ a year. Routine computations show that $\sigma^* \approx 0.17$. Now, when $\bar{r}=4\%$, the value of μ that corresponds to $\sigma^* \approx 0.17$ is approximately 0.027; which implies a coefficient of variation, σ^*/μ, of a bit over 7. This is large, but perhaps not remarkably so. And yet, no optimum policy exists. Suppose $\eta=2$ instead. We should expect σ^*/μ to be larger than the previous figure. This is indeed so. If $\eta=2$, $\sigma^* \approx 0.20$ and $\mu \approx 0.019$, which implies that σ^*/μ is a little over 10.

Proposition 4 holds that if σ satisfies (14), for any saving rate there is a higher saving rate for which the expected value of intergenerational well-being is higher. But at 100% saving rate no one ever consumes anything. We therefore have a contradiction. Another way to interpret Proposition 4 is to say that if σ satisfies inequality (14), the problem of optimum saving, when formulated in terms of expected well-being over an infinite horizon, is so inadequately posed as to defy an answer. To put it crudely, every saving policy yields an infinitely awful outcome. So, consumption discount rates cannot be defined and social cost–benefit analysis of projects becomes meaningless. To be sure, for any value of σ, no matter how large, one can always choose η to be sufficiently close to 1 to ensure that inequality (14) does *not* hold. But as values of η close to 1 carry with them serious ethical deficiencies, choosing a figure for η close to 1 would not be a legitimate way out of the dilemma. To do so would be a technical fix, nothing more. So we search for more defendable escape routes from the ethical dilemma.

Integrated assessment models consider only a finite number of scenarios, implying that the downside risks associated with climate change are bounded. In the context of our model here, we could ensure the existence of an optimum programme by truncating the normal distribution of $\ln(1+r)$ on the left. But there is no ready recipe for determining where we should perform the truncation.

Another escape route would be to abandon the assumption that $U(C)$ is unbounded below (i.e., $\eta \geq 1$, for very low consumption levels) and assume instead that no matter how greatly the economy were to be hit by bad luck, the loss in well-being people would suffer from is bounded. But we economists have very limited experience of working with U's for which the elasticity of $U'(C)$ is less than one for low Cs and greater than one for high Cs.

So we turn to two assumptions underlying expression (2) that are surely very questionable: the constant hazard rate (δ) for Humanity's extinction and an infinite horizon. One way to ensure that the ethical framework we invoke doesn't have contradictions no matter how high σ is would be to abandon the infinite time horizon. But the choice of a terminal date would at best be arbitrary. That is why economists have avoided working with finite time horizon models.

Another possible way out would be to continue to postulate an infinite time horizon, but formalise Humanity's extinction process in terms of a hazard rate that increases in an unbounded fashion over time at a sufficiently high rate. The problem is that we have little intuition on how to formulate that in a way that is scientifically reasonable.

6 Avoiding misplaced concreteness

In this paper I have offered a fairly complete account of the idea of consumption discount rates as applied to public policy analysis. Sections 1–2 introduced the language we economists use to formalize the notion of "social" discount rates. The background of the discussion in the paper has been the global climate change that accompanies an accumulation of "greenhouse" gases in the atmosphere. Because of the externalities generated by individuals' emissions of those gases, observed rates of return on investment are very likely overestimates of social rates of return. So, using even the rates on offer in government bonds for discounting long lived projects is most likely to be misleading. In Section 3, where I presented a simple deterministic model of consumption and saving, I argued that consumption discount rates are neither ethical primitives nor observables as market rates of return on investment, but that they ought instead to be derived from economic forecasts and society's conception of distributive justice concerning the allocation of goods and services across personal identities, time, and events. The welfare theory was developed in the context of three empirical studies on the economics of global climate change: Cline (1992), Nordhaus (1994), and Stern (2006). One striking feature of the ethics underlying intergenerational welfare economics that all three protagonists have adopted is that it is characterized by only *two* parameters: δ (the time discount rate) and η (the elasticity of marginal felicity). If, as many moral philosophers advise us, the appropriate value for δ in a deterministic world is approximately zero, the whole weight of the ethics regarding the distribution of consumption across the generations is borne by η. That's an awful lot of work for a single number to do adequately. Curiously, both Nordhaus and Stern have assumed $\eta=1$. I have shown that in classroom models a combination of $\delta\approx 0$ and $\eta=1$ can together prescribe absurdly high saving ratios. If we are to maintain the assumption that the elasticity of $U'(C)$ is a constant, we should work with higher values of η, perhaps in the region, 1.5 to 3. But the assumption that η is independent of C has only tractability to commend it. It seems that many of the ethical puzzles thrown up by intergenerational welfare economics and reviewed in this paper have been due to that assumption. It may be time that economists experiment with Us for which the elasticity of $U'(C)$ is an increasing function of C.

The standard precautionary motive for saving was reviewed in the case where future uncertainties are not large (Section 4). It was shown that uncertainty in future consumption does not per se lead to hyperbolic discounting, but that, to generate the latter requires the state space to be artificially confined. In a natural generalization of the pure capital model of Section 3, where the gross return on saving is generated by an i.i.d process involving a log normal distribution, it was shown that the risk-free consumption discount rate along the optimum saving rule is constant, not hyperbolic.

In Section 5 we found that if the uncertainties associated with climate change losses are large, the formulation of intergenerational well-being we economists have grown used to could lead to ethical paradoxes even when the uncertainties are thin-tailed: an optimum policy may not exist (Proposition 4). Various modelling avenues that offer a way out of the dilemma were discussed. It was shown that none of them is entirely satisfactory.

The (linear) model economy we have worked with in this paper is utterly simple. Nevertheless it has yielded several insights that are relevant to the study of the economics of climate change. The concentration of carbon dioxide in the atmosphere is currently 385 p.p.m. (parts per million), a figure which ice cores in Antarctica have revealed to be in excess of the maximum that had been reached during the past 650,000 years and more. Since the late 1970s, climate change has been taken seriously by those who have studied the science. Even the now-famous "hockey-stick", displayed by time series of carbon concentration in the atmosphere, appeared some time ago (Bolin 1989: fig. 5). Moreover the Earth system is driven by interlocking non-linear processes running at differing speeds and operating at different spatial scales. Doing little about climate change would involve Earth crossing an unknown number of tipping points (formally, separatrices) in the global climate system.[40] We have no data on the consequences if Earth were to cross those tipping points. They could be good in some places, disastrous in others. And even if we did have data, they would probably do us little good, because Nature's processes are irreversible. One implication of Earth system's deep non-linearities is that estimates of climatic parameters based on observations from the recent past are unreliable for making forecasts about the state of the world at concentration levels of 560 p.p.m. or more. The uncertainties are therefore enormous.

Proposition 4 exposes the limitation of overly formal analyses of the economics of climate change. (We should add to that the economics of biodiversity loss.) Because advancements in global sequestration technologies and technologies using alternative sources of energy may prove to be harder to realise than is currently hoped, it is possible to believe that Humanity should invest a lot more in reducing climate change than the 2% of the GDP of rich countries proposed by Stern (2006). One can hold such a belief even while being unable to justify it from formal modelling.

Intergenerational welfare economics raises more questions than it is able to answer satisfactorily.

[40] See Lenton et al. (2007).

References

Arrow, K. J. (1965). *Aspects of the theory of risk-bearing*. Helsinki: Yrjö Jahnssonin säätiö.

Arrow, K. J. (1963). *Social choice and individual values* (2nd ed.). New York: Wiley (1951).

Arrow, K. J. (1999). Discounting, morality, and gaming. In P. R. Portney, & J. P. Weyant (Eds.), *Discounting and intergenerational equity*. Washington, DC: Resources for the Future.

Arrow, K. J., et al. (1996). Intertemporal equity, discounting, and economic efficiency. In J. P. Bruce, H. Lee, & E. F. Haites (Eds.), *Climate Change 1995: Economic and social dimensions of climate change, contribution of working group III to the second assessment report of the intergovernmental panel on climate change*. Cambridge: Cambridge University Press.

Arrow, K. J., & Dasgupta, P. (2007). Conspicuous Consumption, Inconspicuous Leisure. Mimeo, Faculty of Economics, University of Cambridge.

Arrow, K. J., & Kurz, M. (1970). *Public investment, the rate of return and optimal fiscal policy*. Baltimore: Johns Hopkins University Press.

Barrett, S. (2003). *Environment and statecraft: The strategy of environmental treaty-making*. Oxford: Oxford University Press.

Bolin, B. (1989). Changing climates. In L. Friday, & R. Laskey (Eds.), *The fragile environment*. Cambridge: Cambridge University Press.

Brock, W. A. (1982). Asset prices in a production model. In J. McCall (Ed.), *The economics of information and uncertainty*. Chicago: University of Chicago Press.

Broome, J. (1992). *Counting the cost of global warming*. Cambridge: White Horse.

Broome, J. (2008). The Ethics of Climate Change, Scientific American June, 69-73.

Cline, W. R. (1992). *The economics of global warming*. Washington, DC: Institute for International Economics.

Cline, W. R. (1993). Give Greenhouse Abatement a Fair Chance, Finance and Development March, 3-5.

Cochrane, J. H. (2005). *Asset pricing* (2nd ed.). Princeton, NJ: Princeton University Press.

Dasgupta, P. (2001). *Human well-being and the natural environment*. Oxford: Oxford University Press.

Dasgupta, P. (2005). What do economists analyze and why: values or facts? *Economics and Philosophy, 2* (2), 221–278.

Dasgupta, P. (2007). Commentary: The Stern Review's economics of climate change. *National Institute Economic Review, 199*, 4–7.

Dasgupta, P., & Heal, G. (1979). *Economic theory and exhaustible resources*. Cambridge: Cambridge University Press.

Dasgupta, P., Maler, K. G., & Barrett, S. (1999). Intergenerational equity, social discount rates, and global warming. In P. R. Portney, & J. P. Weyant (Eds.), *Discounting and intergenerational equity*. Washington, DC: Resources for the Future.

Dasgupta, P., Marglin, S. A., & Sen, A. (1972). *Guidelines for project evaluation*. New York: United Nations.

Dasgupta, P., & Maskin, E. (2005). Uncertainty and hyperbolic discounting. *American Economic Review, 95*(4), 1290–1299.

Hall, R. E. (1988). Intertemporal substitution in consumption. *Journal of Political Economy, 96*(2), 339–357.

Harsanyi, J. C. (1955). Cardinal welfare, individualistic ethics and interpersonal comparisons of utility. *Journal of Political Economy, 63*(3), 309–321.

Heal, G. M. (1998). *Valuing the future: Economic theory and sustainability*. New York: Columbia University Press.

Heal, G. M. (2007). Climate Change Economics: A Meta-Review and Some Suggestions. Mimeo, Columbia Business School, Columbia University.

Hoel, M., & Sterner, T. (2007). Discounting and Relative Prices. Climate Change, doi:10.1007/s10584-007-9255-2.

Koopmans, T. C. (1960). Stationary ordinal utility and impatience. *Econometrica, 28*(2), 287–309.

Koopmans, T. C. (1972). Representation of preference orderings over time. In C. B. McGuire, & R. Radner (Eds.), *Decision and organization*. Amsterdam: North Holland.

Layard, R. (1980). Human satisfaction and public policy. *Economic Journal, 90*(4), 737–750.

Layard, R. (2005). *Happiness: Lessons from a new science*. New York: Penguin.

Lenton, T. M. et al. (2007). Tipping Elements in the Earth System. Proceedings of the National Academy of Sciences (under review).

Levhari, D., & Srinivasan, T. N. (1969). Optimal savings under uncertainty. *Review of Economic Studies, 36*(2), 153–163.

Lind, R. C. (Ed.) (1982). In *Discounting for time and risk in energy planning*. Baltimore: Johns Hopkins University Press.

Little, I. M. D., & Mirrlees, J. A. (1969). *Manual of industrial project analysis in developing countries: Social cost benefit analysis*. Paris: OECD.

Marglin, S. A. (1963). The opportunity costs of public investment. *Quarterly Journal of Economics*, *77*(2), 274–289.

Mirrlees, J. A. (1967). Optimum growth when technology is changing. *Review of Economic Studies*, *34*(1), 95–124.

Mirrlees, J. A. (1971). An exploration in the theory of optimal income taxation. *Review of Economic Studies*, *38*(1), 175–208.

Nordhaus, W. D. (1994). *Managing the global commons: The economics of climate change*. Cambridge, MA: MIT.

Nordhaus, W. D. (2007). The Stern Review on the economics of climate change. *Journal of Economic Literature*, *45*(3), 686–702.

Nordhaus, W. D., & Boyer, J. (2000). *Warming the world: Economic modeling of global warming*. Cambridge, MA: MIT.

Pesaran, H., Pettenuzzo, D., & Timmermann, A. (2007). Learning, structural instability, and present value calculations. *Econometric Reviews*, *6*(2-4), 253–288.

Phelps, E. S., & Pollak, R. (1968). Second-best national savings and game equilibrium growth. *Review of Economic Studies*, *35*(2), 185–199.

Portney, P. R., & Weyant, J. P. (Eds.) (1999). In *Discounting and intergenerational equity*. Washington, DC: Resources for the Future.

Ramsey, F. P. (1928). A mathematical theory of saving. *Economic Journal*, *38*(4), 543–549.

Rawls, J. (1972). *A theory of justice*. Oxford: Oxford University Press.

Ryder, H. E., & Heal, G. M. (1973). Optimal growth with intertemporally dependent preferences. *Review of Economic Studies*, *40*(1), 1–31.

Scheffler, S. (1992). *Human morality*. Oxford: Oxford University Press.

Schelling, T. C. (1999). Intergenerational discounting. In P. R. Portney, & J. P. Weyant (Eds.), *Discounting and intergenerational equity*. Washington, DC: Resources for the Future.

Solow, R. M. (1974). Intergenerational equity and exhaustible resources. *Review of Economic Studies*, *41* (Symposium Issue), 29–45.

Sozou, P. D. (1998). On hyperbolic discounting and uncertain hazard rates. *Proceedings of the Royal Society of London (Series B)*, *265*, 2015–2020.

Stern, N. H. (1976). On the specification of models of optimum income taxation. *Journal of Economic Theory*, *6*(1-2), 123–162.

Stern, N. H., et al. (2006). *The Stern Review of the economics of climate change*. Cambridge: Cambridge University Press.

Stern, N. H. (2008). The economics of climate change. *American Economic Review*, *98*(2), 1–37.

Sterner, T., & Persson, U. M. (2008). An Even Sterner Review: Introducing Relative Prices into the Discounting Debate. *Review of Environmental Economics and Policy*, *2*(1), 61–76. doi:10.1093/reep/rem024.

Viscusi, W. K. (2007). Rational discounting for regulatory analysis. *University of Chicago Law Review*, *74* (1), 209–246.

Weitzman, M. L. (2001). Gamma discounting. *American Economic Review*, *91*(1), 260–271.

Weitzman, M. L. (2007a). The Stern Review of the economics of climate change. *Journal of Economic Literature*, *45*(3), 703–724.

Weitzman, M. L. (2007b). The Role of Uncertainty in the Economics of Catastrophic Climate Change. Discussion Paper, Department of Economics, Harvard University.

World Bank (2007). *World development indicators*. Washington, DC: World Bank.

Yaari, M. (1965). Uncertain lifetime, life insurance, and the theory of the consumer. *Review of Economic Studies*, *32*(2), 137–150.