CHAPTER 16

Integrated Economic and Climate Modeling

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Abstract

This survey examines the history and current practice in integrated assessment models (IAMs) of the economics of climate change. It begins with a review of the emerging problem of climate change. The next section provides a brief sketch of the rise of IAMs in the 1970s and beyond. The subsequent section is an extended exposition of one IAM — the DICE/RICE family of models. The purpose of this description is to provide readers an example of how such a model is developed and what the major components are. The final section discusses major important open questions that continue to occupy IAM modelers. These involve issues such as the discount rate, uncertainty, the social cost of carbon, the potential for catastrophic climate change, algorithms and fat-tailed distributions. These issues are the ones that pose both deep intellectual challenges as well as important policy implications for climate change and climate change policy.

Keywords

Integrated assessment models, climate change, economics of climate change, social cost of carbon, large-scale economic modeling, energy models

JEL classification codes

Q5, Q54, 6

16.1 INTRODUCTION

16.1.1 Integrated assessment models

Many areas of the natural and social sciences involve complex systems that link together multiple physical or intellectual sectors. This is particularly true for environmental problems, which are intrinsically ones having strong roots in the natural sciences, and require social and policy sciences to solve in an effective and efficient manner. A good example, which will be the subject of this survey, is climate change science and policy, which involve a wide variety of sciences such as atmospheric chemistry and climate sciences, ecology, economics, political science, game theory, and international law.

As understanding progresses across the different fronts, it is increasingly necessary to link together the different areas to develop effective understanding and efficient policies. In this role, integrated assessment analysis and models play a key role. Integrated

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assessment models (IAMs) can be defined as approaches that integrate knowledge from two or more domains into a single framework. These are sometimes theoretical, but are increasingly computerized dynamic models of varying levels of complexity.

The present survey provides a roadmap to developments in IAMs for climate change over the last quarter century. It is constructed in the following sequence. We begin in this section with a review of the emerging problem of climate change. This is necessary to lay the background and motivation for why so many social and natural scientists are spending so much of their time on this issue.

Section 16.1.6 provides a brief sketch of the rise of IAMs in the 1970s and beyond. It is relatively brief because earlier surveys have covered much of the ground in an admirable fashion.

Section 16.2 is an extended exposition of one IAM – the DICE/RICE family of models. The purpose here is to provide readers with an example of how such a model is developed and what the components are. Other IAMs will have different structures, algorithms and assumptions, but the underlying modeling philosophy of integrating modules from different disciplines is common to virtually all IAMS. The development of the modeling is followed in Section 16.3 by a set of illustrative results from the RICE-2010 model. This is used to illustrate the kind of questions that IAMs can address.

Section 16.4 discusses major important open questions that continue to occupy IAM modelers. These involve issues such as: the discount rate, uncertainty, the social cost of carbon (SCC), the potential for catastrophic climate change and fat-tailed distributions. These issues are ones that pose both deep intellectual challenges as well as important policy implications for climate change and climate change policy.

16.1.2 Emerging problems of climate change

Before getting into modeling details, it will be useful to sketch the scientific basis for concerns about global warming, as reviewed by the Intergovernmental Panel on Climate Change (IPCC)'s *Fourth Assessment Report* (IPCC, 2007) with updates from other sources. As a result of the buildup of atmospheric greenhouse gases, it is expected that significant climate changes will occur in the coming decades and beyond. The major industrial greenhouse gases are carbon dioxide (CO_2), methane, ozone, nitrous oxides and chlorofluorocarbons (CFCs). The most important greenhouse gas is CO_2, whose emissions have risen rapidly in recent decades.

The atmospheric concentration of CO_2 of 390 parts per million (p.p.m.) in 2011 far exceeds the range over the last 650,000 years, estimated to be between 180 and 300 p.p.m. (current estimates of CO_2 concentrations at Mauna Loa are available at ftp://ftp.cmdl.noaa.gov/ccg/co2/trends/co2_mm_mlo.txt.).

Current calculations from climate models are that doubling the amount of CO_2 or the equivalent in the atmosphere compared with preindustrial levels will, in equilibrium, lead to an increase in the global surface temperature of 2−4.5°C, with a best estimate of about 3°C. The suite of models and emissions scenarios used by the IPCC produce a range of temperature change over the twenty-first century of between 1.8 and 4.0°C. Other projected effects are increases in precipitation and evaporation, an increase in extreme events such as hurricanes, and a rise in sea levels of 0.2−0.6 m over this century. Some models also predict regional shifts, such as hotter and drier climates in mid-continental regions, including the US Midwest. Climate monitoring indicates that actual global warming is occurring in line with scientific predictions.

The agreed framework for all international climate change deliberations is the UN Framework Convention on Climate Change, which took force in 1994. That document stated, "The ultimate objective … is to achieve … stabilization of greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system" (United Nations, 2009). The Framework Convention was implemented in the Kyoto Protocol in 1997, in which both high-income countries and countries in transition from central planning agreed to binding emissions limits for the 2008−2012 period. The framework for implementing the Protocol is most solidly institutionalized in the EU's Emissions Trading Scheme (ETS), which covers almost half of Europe's CO_2 emissions.

Notwithstanding its successful implementation, the Kyoto Protocol is widely seen as a troubled institution. Early problems appeared with the failure to include the major developing countries, the lack of an agreed-upon mechanism to include new countries and an agreement that is limited to a single budget period. The major blow came when the US withdrew from the Treaty in 2001. Whereas 66% of 1990 world emissions were included in the original Protocol, that number declined to about one-third in 2010 with the withdrawal of the US and strong economic growth in developing countries (Nordhaus, 2010). Strict enforcement of the Kyoto Protocol is likely to be observed primarily in those countries and industries covered by the EU ETS, but their emissions today account for only about 8% of the global total. If the current Protocol is extended at current emissions levels, models indicate that it will have little impact on global climate change (see the several studies in Weyant and Hill, 1999).

16.1.3 Copenhagen Accord

The 2009 Copenhagen Conference of the Parties was designed to negotiate a successor agreement for the post-Kyoto period. Owing to deep divisions about costs and about the distribution of emissions reductions, the meeting concluded without a binding agreement. However, it did lead to an agreement known as the "Copenhagen Accord" (United Nations, 2009). The accord adopts a target of limiting the increase in global mean temperature and states that the target is set "recognizing the scientific view that the increase … should be below 2 degrees Celsius." Developing countries did sign on to the

Accord. A close look reveals, however, that countries committed themselves to very little. They agreed to "communicate" their "nationally appropriate mitigation actions seeking international support efforts," but no binding targets for countries were set.

The reality behind the accord is not encouraging. To begin with, even if the high-income countries fulfilled their commitments, these would probably not achieve anything close to the 2°C target, as is shown below. Meanwhile, progress on reaching a more binding agreement has stalled. At present, a global agreement is waiting for the US to take credible legislated steps. At present (2011), there are no active plans for legislation in the US and instead there are proposals to roll back current plans to regulate greenhouse gases required under the US 1970 Clean Air Act.

16.1.4 Climate change as a global public good

Climate change is a polar case of economic phenomena known as global public goods (Samuelson, 1954). Public goods are activities for which the cost of extending the service to an additional person is zero and for which it is impossible or expensive to exclude individuals from enjoying. Global public goods are ones whose influences are felt around the world rather than in one nation, town, or family. What makes global public goods different from normal economic activities is that there are at best weak economic and political mechanisms for resolving these issues efficiently and effectively.

The economic theory of public goods has been extensively discussed in many contexts (e.g. Oakland, 1987). For this reason, this review limits the discussion to the application of public-goods theory to climate change and modeling in this area.

16.1.5 Economic modeling of climate change

Most economic studies of climate change, including most IAMs, integrate geophysical stocks and flows with economic stocks and flows. The major difference between IAMs and geophysical models is that economic measures include not only quantities but also valuations, which for market or near-market transactions are prices. The essence of an economic analysis is to convert or translate all economic activities into monetized values using a common unit of account and then to compare different approaches by their impact on total values or a suite of values.

There are different ways of creating a standardized unit of account. The most satisfactory is to use a common "purchasing power parity" (PPP) exchange rate across different regions. For example, I will use the unit of 2005 international US$ below. However, the values are not really money. Rather, they represent a standard bundle of goods and services (such as $1000 worth of food, $3000 of housing, $900 of medical services, and so on). Thus, we are really translating all activities into the number of such standardized bundles. Both translation of different currencies into a common currency and conversion of values over time into a present value using a discount rate are deep issues in economics and we will review the latter in Section 16.4 on open problems.

To illustrate the economic approach, suppose that an economy produces only corn. We might decide to reduce corn consumption today and store it for the future to offset the damages from climate change on future corn production. In weighing this policy, we consider the economic value of corn both today and in the future in order to decide how much corn to store and how much to consume today. In a complete economic account, "corn" would represent all economic consumption. It would include all market goods and services as well as the value of non-market and environmental goods and services. That is, economic welfare – properly measured – should include everything that is of value to people, even if those things are not included in the marketplace.

The central questions posed by economic approaches to climate change are the following: how sharply should countries reduce CO_2 and other greenhouse gas emissions? What should be the time profile of emissions reductions? How should the reductions be distributed across industries and countries?

There are also important and politically divisive issues about the instruments that should be used to impose cuts on consumers and businesses. Should there be a system of emissions limits imposed on firms, industries, and nations? Or should emissions reductions be primarily induced through taxes on greenhouse gases? Should we subsidize green industries? What should be the relative contributions of rich and poor households or nations? Are regulations an effective substitute for fiscal instruments?

In practice, an economic analysis of climate change weighs the costs of slowing climate change against the damages of more rapid climate change. On the side of the costs of slowing climate change, this means that countries must consider whether, and by how much, to reduce their greenhouse gas emissions. Reducing greenhouse gases, particularly deep reductions, will require taking costly steps to reduce CO_2 emissions. Some steps involve reducing the use of fossil fuels; others involve using different production techniques or alternative fuels and energy sources. Societies have considerable experience in employing different approaches to changing energy production and use patterns. Economic history and analysis indicate that it will be most effective to use market signals, primarily higher prices on carbon fuels, to give signals and provide incentives for consumers and firms to change their energy use and reduce their carbon emissions. In the longer run, higher carbon prices will also provide incentives for firms to develop new technologies to ease the transition to a low-carbon future.

On the side of climate damages, our knowledge is very meager. For most of the time span of human civilizations, global climatic patterns have stayed within a very narrow range, varying at most a few tenths of a degree Centigrade (°C) from century to century. Human settlements, along with their ecosystems and pests, have generally adapted to the climates and geophysical features they have grown up with. Economic studies suggest that those parts of the economy that are insulated from climate, such as air-conditioned houses and most manufacturing operations, will be little affected directly by climate change over the next century or so (see by reference IPCC, 2007b).

However, those human and natural systems that are "unmanaged," such as rain-fed agriculture, seasonal snow packs and river runoffs, and most natural ecosystems, may be significantly affected. While economic studies in this area are subject to large uncertainties, recent surveys of the literature on damages from future climate change indicate that the economic damages from climate change with no interventions will be in the order of 2–3% of world output per year by the end of the twenty-first century (for a recent review of damage estimates, see particularly Tol, 2009). The damages are likely to be most heavily concentrated in low-income and tropical regions such as tropical Africa and India. While some countries may benefit from climate change, there is likely to be significant disruption in any area that is closely tied to climate-sensitive physical systems, whether through rivers, ports, hurricanes, monsoons, permafrost, pests, diseases, frosts or droughts. Moreover, damage estimates cannot reliably include estimates of the costs of ecological impacts such as ocean acidification, species extinction, ecosystem disruption or of the dangers posed by tipping points in the earth systems.

16.1.6 Previous surveys of IAMs

A search on Google Scholar finds 3610 citations to "Integrated Assessment Models." However, the number of journal publications is much smaller at 175 over the period 1995–2011. The time trend for both is shown in Figure 16.1. Clearly, there is a major growth in research in this area, although the ratio of ISI publications to Scholar publications is low. One reason is that a great deal of the work is done in the "gray literature" rather than in standard journal publications.

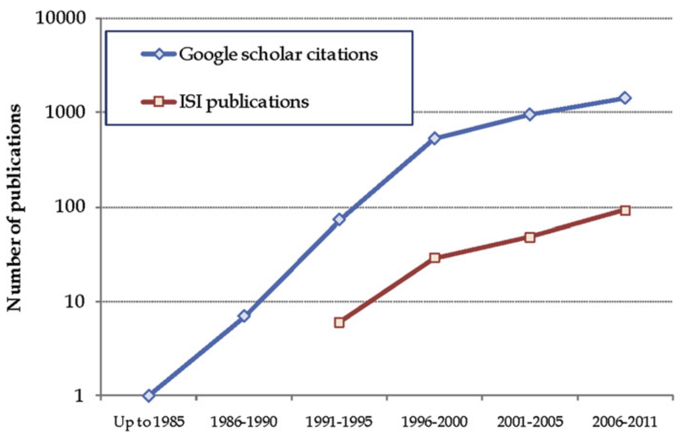

Figure 16.1 Time trend of publications citing "Integrated Assessment Models" from Google Scholar and ISI Citations.

Although IAMs have been increasingly used for two decades, there is relatively little literature that surveys the technical aspects of models. By contrast, there is a vast literature on the results as well as on applications of models.

An exemplary survey by Weyant *et al*. (1996) for the IPCC's *Second Assessment* examined a range of IAMs and provided a fine survey of the state of the art at that time. Unfortunately, that survey is not currently available on the internet, but it should be the starting point for those wishing to understand the state of the art as of the mid-1990s. Weyant *et al*. (1996) emphasized, as we will below, the importance of multiple approaches to development of IAMs because of the difficulty of encompassing all the important elements in a single model.

A more recent pair of surveys is by Kolstad (1998) and Kelly and Kolstad (1999). These surveys examine 21 IAMS, with dates from 1992 to 1996. The authors emphasized the important distinction between policy optimization and evaluation models. This distinction remains one of the central dividing lines among different models, although it is not clearly understood. Kolstad (1998) writes that "nearly all the results have come from the so-called policy optimization models, the top-down economy-climate models. Virtually no new basic understanding appears to have emerged from the policy evaluation models...." This strong challenge appears to have been largely lost on the modeling community.

Another issue that was emphasized by Kelly and Kolstad was the importance of uncertainty. The conclusion of this survey was the following:

> *The integrated assessment community has done an excellent job of analyzing, comparing, and contrasting the multitude of IAMs. Because of the analysis, IAMs give a remarkably consistent message. However, despite the consistent message and the large amount of government research money which has been spent, the message is not known far outside the integrated assessment community. The integrated assessment community must still do more to bring the results to the forefront of the debate on what to do about climate change.*

This has changed somewhat in recent years as models have been increasingly used by governments in their policy analyses.

16.1.7 Need for integrated modeling

The challenge of coping with global warming is particularly difficult because it spans many disciplines and parts of society. Ecologists may see it as a threat to ecosystems, marine biologists as a problem leading to ocean acidification, electric utilities as a debit to their balance sheets and coal miners as an existential threat to their livelihood. Businesses may view global warming as either an opportunity or a hazard, politicians as a great issue as long as they do not need to mention taxes, ski resorts may view it as a mortal danger to their already-short seasons, golfers as a boon to year-round recreation, and poor countries as a threat to their farmers as well as a potential source of financial and technological aid. This many-faceted nature also poses a challenge to natural and social scientists who must incorporate a wide variety of geophysical, economic and political disciplines into their diagnoses and prescriptions.

The task of integrated modeling is to pull together the different aspects of a problem so that a decision or analysis can consider all important endogenous variables that operate simultaneously. Figure 16.2 shows schematically the important modules in the case of climate change. A complete analysis must consider emissions, concentrations, climate change and impacts. The last arrow in the process links the impacts and policies back to emissions, thus closing the loop.

16.1.8 Essential simultaneity of economic decisions, geophysical reactions, impacts and economic policy

It must be emphasized that a complete integrated assessment is not necessary for all parts of the climate change challenge. Each of the different boxes in Figure 16.2 is in fact an entire discipline, with many talented scientists pursuing questions at the frontier of

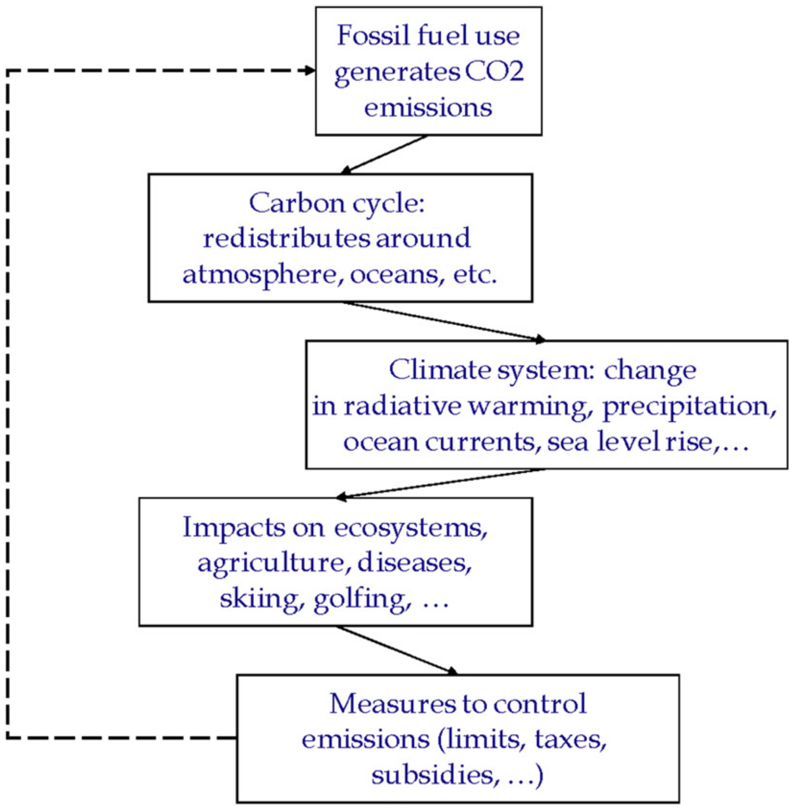

Figure 16.2 Schematic flow chart of a full IAM for climate change science, economics and policy.

modern natural and social science. For example, the "climate system" box would represent the work of dozens of teams in many countries, building models, calibrating the models to data, and the like. Indeed, much of the 1000-page reports of the IPCC on science are built on scientists studying "the climate system." Similar teams are at work in the other areas.

The point emphasized in IAMs is that we need to have *at a first level of approximation* models that operate all the modules simultaneously. The climate models, for example, use stylized emissions as inputs to their simulations. In the most recent round of model results (the IPCC *Fourth Assessment Review*), the inputs were a set of scenarios generating several years earlier in the *Special Report on Emissions Scenarios* study (IPCC, 2000). There is no linkage from the climate models to the economy and then back to emissions. It is exactly this linkage that is the purpose of integrating the different parts of the climate change nexus in IAMs.

16.1.9 Background in energy modeling and early approaches

IAMs of climate change grew organically from energy models. One of the earliest careful comparisons of energy models was the Modeling Resource Group (MRG) analysis of different models (MRG, 1978). This project, chaired by economist Tjalling Koopmans, formed one of the study groups of the larger National Academy of Sciences Study of Nuclear and Alternative Energy Systems (CONAES, 1978). The MRG analyzed a number of energy models that projected energy demands and technologies over a long time horizon. The earlier work of Koopmans on the linear programming approach to production as well as the Samuelson principle of "markets as maximization" (Samuelson, 1949) formed the intellectual core of much of the energy modeling starting at that time and proceeding to the present.

It is notable that the even though the CONAES study identified climate change as a key long-term issue, none of the energy models used in the study or reviewed by the MRG explicitly included CO_2 emissions or climate change in their analyses. Work of Nordhaus extending the MRG modeling approach to include a climate module was undertaken in parallel with the CONAES study and was published in Nordhaus (1977, 1979). This approach, which built on a highly disaggregated partial equilibrium model of the world energy system, was abandoned in favor of more aggregated approaches (the DICE and RICE models discussed later).

Several of the current IAMs grew out of the energy models of the 1970s and 1980s. Particularly important were the studies of Alan Manne. In a series of studies from the 1960s through the 1990s, his work on mathematical programming, integer programming, learning and integration of energy and environmental modules served as landmarks and inspiration for later models (see Manne (1962, 1974, 1976) and well as in joint work with Richard Richels discussed later).

The first IAMs in climate change were basically energy models with an emissions model included, and later with other modules such as a carbon cycle and a small climate model. Nordhaus's early approaches (Nordhaus, 1975, 1977, 1978) were partial equilibrium energy models with exogenous output. One of the important landmarks in development of IAMs was Manne's ETA-Macro model, which was the first to imbed an energy system in a full economic growth model (Manne, 1977). The earliest versions of the DICE and RICE models in Nordhaus (1992, 1994a) moved to a growth-theoretic framework similar to the Manne and Manne-Richels models (Manne and Richels, 1991, 1992).

16.1.10 Current scene

It is not possible to make a comprehensive list of IAMs as of mid-2011. One indication of the richness of the landscape is the participation in the IAM Consortium (see http://iamconsortium.org/), which lists 42 different organizations. Table 16.1 shows the sectoral distribution of members of the Consortium (which does not map one-to-one to models, but is indicative).

IAMs are increasingly used in analyses by national governments and international assessments. Particularly important have been the intermodal comparisons undertaken by the Energy Modeling Forum (EMF) headed by John Weyant. Exemplary in this respect is the EMF-22 (Clarke, 2009), which used 17 models and compared a range of scenarios including a reference (uncontrolled) scenario along with several scenarios that constrained radiative forcings. These studies are extremely valuable because they provide a range of projections so that scientists and decision makers can understand the uncertainties of the projections.

The next section presents the DICE/RICE models as an example of an IAM. These models are discussed largely because they are small and transparent. For many scientific and policy purposes, more detailed IAMs will be necessary. Three IAMs that are widely used in the US are the NEMS (National Energy Modeling System) model (developed by the Energy Information Administration of the US government, see NEMS, 2011), the IGEM (Intertemporal General Equilibrium Model) model (developed and maintained

Table 16.1 Organizations sponsoring IAMs

Sector	Number
Universities	17
Research institutes	12
Government institutes	9
Business or consulting	4
Total	42

Compiled by the author from http://iamconsortium.org as of March 2011.

by Dale Jorgenson and his colleagues, see IGEM, 2011) and the MIT EPPA (Economics, Emissions and Policy Cost) model (developed by a team of researchers currently led by John Reilly, see EPPA, 2011).

There are several other important models that have been widely used in both the scholarly and policy circles. For example, 17 models participated in the EMF-22 model comparison study. These models were PACE, IMAGE, MRN-NEEM, GTEM, MiniCAM, SGM, IGSM, WITCH, ADAGE, GEMINI, POLES, IGEM, MESSAGE, FUND, ETSAP-TIAM, MERGE and DART. Descriptions of the models are beyond the scope of this survey. For a description of the models, with references, see Clarke *et al.* (2009).

The larger IAMs tend to be very detailed. I will use IGEM to illustrate the complexity of large models. IGEM has about 4000 endogenous variables per period (year) and the solution works by backward induction from 2130. Policy variables include taxes on commodities, marginal and average taxes on factors, tax credits on investment, a consumption-only tax, tariffs on imports, taxes on carbon, and technology mandates. The program is written in Fortran and C codes, with a total of about 40,000 lines. According to its primary developer, IGEM is proprietary, and being too complicated to modify by outsiders, has never been transferred to another entity. Without going into the details of the larger models, it will be useful to note that such models can investigate questions at a much higher level of resolution than the smaller models. For example, such models have done important studies of the impacts of climate change policies on the distribution of income; the impacts of a specific set of policies, such as the American Clean Energy and Security Act of 2009, the impact of climate policy on US aviation, and the international leakage involved when policies are not harmonized. The larger models play a central role in policy analysis but are more difficult to use than the smaller models and, as noted above, are often difficult to transfer to other users.

16.2 DICE AND RICE MODELS AS EXAMPLES OF IAMS

16.2.1 Purpose of this section

In this section, I present an extended description of the DICE and RICE IAMs. The purpose is primarily to show the way such a model is constructed and to provide details on the components. Most IAMs have a similar analytical structure, although they vary greatly in their detail, coverage, data and algorithmics. The last part (Section 16.2.9) reviews some of the oversimplifications in IAMs.

16.2.2 Introduction to the models

The DICE (Dynamic Integrated model of Climate and the Economy) and RICE (Regional Integrated model of Climate and the Economy) models have gone through several revisions since their first development around 1990. The latest published versions are the RICE-2010 and DICE-2010 model, and this exposition will follow that model structure. This is very similar to the 2007 version fully documented in Nordhaus (2008). We begin with a description of the DICE-2010 model, after which we provide the detailed equations. In a Section 16.2.6, we discuss the RICE model. This section draws heavily on Nordhaus (1994a, 2008, 2010) and Nordhaus and Boyer (2000).

The DICE model is a globally aggregated model. The RICE-2010 model is essentially the same except that output and abatement have regional structures for 12 regions. The discussion will use the term "DICE model" and for most modules the analysis applies equally to the RICE model. The differences will be described later.

The DICE model views the economics of climate change from the perspective of neoclassical economic growth theory. In this approach, economies make investments in capital, education and technologies, thereby reducing consumption today, in order to increase consumption in the future. The DICE model extends this approach by including the "natural capital" of the climate system as an additional kind of capital stock. In other words, it views concentrations of greenhouse gases as negative natural capital, and emissions reductions as investments that raise the quantity of natural capital (or reduce the negative capital). By devoting output to emissions reductions, economies reduce consumption today but prevent economically harmful climate change and thereby increase consumption possibilities in the future.

16.2.3 Objectives or goals of the IAM

IAMs can be divided into two general classes − policy optimization and policy evaluation models (Weyant *et al.*, 1996). Policy evaluation model generally are recursive or equilibrium models that generate paths of important variables but do not optimize an economic outcome. Policy optimization models have an objective function or welfare function that is maximized and can be used to evaluate alternative paths or policies. In models that have an economic structure, the objective function is generally a measure of economic welfare. This would typically be a set of utility functions in general equilibrium models or consumer and producer surplus in partial equilibrium models. These are not as different as might be supposed, as policy optimization models can be run in a non-policy mode, while policy evaluation models can compare different policies. However, there is often a difference in the solution algorithm as recursive models are often much simpler to solve computationally than are optimization models.

The DICE/RICE models are primarily designed as policy optimization models, although they can be run as simple projection models as well. In both, the approach is to maximize an economic objective function. The objective function represents the goal implicit in the problem. For the DICE/RICE models, the objective function refers to the economic well-being (or utility) associated with a path of consumption. As will be emphasized below, the use of optimization can be interpreted in two ways: they can be seen both, from a positive point of view, as a means of simulating the behavior of a system of competitive markets and, from a normative point of view, as a possible approach to comparing the impact of alternative paths or policies on economic welfare.

In the DICE and RICE models, the world or individual regions are assumed to have well-defined preferences, represented by a social welfare function, which ranks different paths of consumption. The social welfare function is increasing in the *per capita* consumption of each generation, with diminishing marginal utility of consumption. The importance of a generation's *per capita* consumption depends on the size of the population. The relative importance of different generations is affected by two central normative parameters, the pure rate of social time preference ("generational discounting") and the elasticity of the marginal utility of consumption (the "consumption elasticity" for short). These two parameters interact to determine the discount rate on goods, which is critical for intertemporal economic choices. In the modeling, we set the parameters to be consistent with observed economic outcomes as reflected by interest rates and rates of return on capital – a choice that will be central to the results and is further discussed in Section 16.4.5 on discounting.

The DICE/RICE models assume that economic and climate policies should be designed to optimize the flow of consumption over time. It is important to emphasize that consumption should be interpreted as "generalized consumption," which includes not only traditional market goods and services like food and shelter but also non-market items such as leisure, health status and environmental services.

The mathematical representation of this assumption is that policies are chosen to maximize a social welfare function, W, that is the discounted sum of the population-weighted utility of *per capita* consumption, where c is *per capita* consumption, L is population and $R(t)$ is the discount factor, all of which are discussed as we proceed. Equation (16.1) is the mathematical statement of the objective function. This representation is a standard one in modern theories of optimal economic growth (see Ramsey, 1928; Koopmans, 1965; Cass, 1965):

$$W = \sum_{t=1}^{T_{\max}} U[c(t), L(t)]R(t). \tag{16.1}$$

There are a number of further assumptions underlying this choice of an objective function. First, it involves a specific representation of the value or "utility" of consumption. The DICE/RICE models assume that utility is represented by a constant elasticity utility function, as shown in Equation (16.2):

$$U[c(t), L(t)] = L(t)[c(t)^{1-\alpha}/(1-\alpha)]. \tag{16.2}$$

This form assumes a constant elasticity of the marginal utility of consumption, α. The elasticity is a parameter that represents to extent of substitutability of the consumption of different years or generations. If α is close to zero, then the consumptions of different generations are close substitutes; if α is high, then the consumptions are not close substitutes. Often, α will also be used to represent risk aversion, but these are strictly speaking quite distinct concepts and should not be confused (see Epstein and Zin, 1989, 1991). Additionally, the elasticity is distinct from the *personal* behavioral characteristics, as will be emphasized below. We calibrate α in conjunction with the pure rate of time preference, as is discussed below.

Second, this specification assumes that the value of consumption in a period is proportional to the population. In the RICE model, the presence of multiple agents will lead to major issues of interpretation and computation discussed below.

Third, this approach applies a discount on the economic well-being of future generations, as is defined in Equation (16.3):

$$R(t) = (1+\rho)^{-t}. \tag{16.3}$$

In this specification, $R(t)$ is the discount factor, while the pure rate of social time preference, ρ, is the discount rate which provides the welfare weights on the utilities of different generations.

We should add a note of interpretation of the equilibrium in the DICE model. We have specified the baseline or no-controls case so that, from a conceptual point of view, it represents the outcome of market and policy factors as they currently exist. In other words, the baseline model is an attempt to project from a positive perspective the levels and growth of major economic and environmental variables as would occur with no climate change policies. It does not make any case for the social desirability of the distribution of incomes over space or time of existing conditions, any more than a marine biologist makes a moral judgment on the equity of the eating habits of whales or jellyfish. This point will be further discussed in Section 16.4.4.

We can put this point differently in terms of welfare improvements. The calculations of the potential improvements in world welfare from efficient climate change policies examine potential improvements within the context of the existing distribution of income and investments across space and time. There may be other improvements — in environmental policies, in military policies, in tax or transfer programs, or in international aid programs — would improve the human condition, and might improve it even more than the policies we consider, but these are outside the scope of this analysis.

16.2.4 Economic variables

The economic sectors are standard to the economic growth literature. The main difference from standard analysis is the very long timeframe that is required for climate change modeling. While most macroeconomic models run for a few years, or in the development context a few decades, climate change projects necessarily must run a century or more. The result is that many of the projections and assumptions are based on very thin evidence.

We begin with the standard neoclassical decisions about capital accumulation and then consider the geophysical constraints. The DICE/RICE models are simplified relative to many models because they assume a single commodity, which can be used for either consumption or investment. Consumption should be viewed broadly to include not only food and shelter, but also non-market environmental amenities and services.

It is useful to consider the multiregion RICE version, because in reality the DICE model is built up from regional aggregates. Each region is endowed with an initial stock of capital and labor and an initial and region-specific level of technology. Population growth and technological change are region-specific and exogenous, while capital accumulation is determined by optimizing the flow of consumption over time for each region. Regional outputs and capital stocks are aggregated using PPP exchange rates (although this has been controversial, see IPCC, 2007c; Nordhaus, 2007b).

The next set of equations determines the evolution of world output over time. Population and the labor force are exogenous. These are simplified to be logistic-type equations. The growth of population in the first decade is given, and the growth rate declines so that total world population approaches a limit of 10.3 billion in 2100. These numbers have been revised upward in line with the most recent UN projections and are about 20% higher than the 2007 DICE/RICE model estimates, (a fine recent review is Lee (2011) and other articles in the same issue).

Output is produced with a Cobb–Douglas production function in capital, labor and energy. Energy takes the form of either carbon-based fuels (such as coal) or non-carbon-based technologies (such as solar or geothermal energy or nuclear power). Technological change takes two forms: economy-wide technological change and carbon-saving technological change. Carbon-saving technological change is modeled as reducing the ratio of CO_2 emissions to output. Carbon fuels are limited in supply. Substitution from carbon to non-carbon fuels takes place over time as carbon-based fuels become more expensive, either because of resource exhaustion or because policies are taken to limit carbon emissions.

Production is represented by a modification of a standard neoclassical production function. The underlying population and output estimates are aggregated up from a 12-region model. Outputs are measured in PPP exchange rates using the International Monetary Fund (IMF) estimates (IMF, 2007). Total output for each region is projected using a partial convergence model, and the outputs are then aggregated to the world total. The regional and global production functions are assumed to be constant-returns-to-scale Cobb–Douglas production functions in capital, labor, and Hicks-neutral technological change. Global output is shown in Equation (16.4):

$$Q(t) = [1 - \Lambda(t)]A(t)K(t)^{\gamma}L(t)^{1-\gamma}/[1 + \Omega(t)]. \tag{16.4}$$

In this specification, $Q(t)$ is output net of damages and abatement, $A(t)$ is total factor productivity and $K(t)$ is capital stock and services. The additional variables in the production function are $\Omega(t)$ and, $\Lambda(t)$ which represent climate damages and abatement costs, shown in Equations (16.5) and (16.6):

$$\Omega(t) = f_1[T_{AT}(t)] + f_2[SLR(t)] + f_3[M_{AT}(t)]. \tag{16.5}$$

$$\Omega(t) = \Psi_1 T_{AT}(t) + \Psi_1[T_{AT}(t)]^2. \tag{16.5$'$}$$

Equations (16.5) and (16.5′) involve the economic impacts of climate change, which is the thorniest issue in climate change economics. These estimates are indispensable for making sensible decisions about the appropriate balance between costly emissions reductions and climate damages. However, providing reliable estimates of the damages from climate change over the long run has proven extremely difficult. The present study relies on estimates from earlier syntheses of the damages, with updates in light of more-recent information. The basic assumption is that the damages from gradual and small climate changes are modest, but that the damages rise non-linearly with the extent of climate change. These estimates also assume that the damages are likely to be relatively larger for poor, small and tropical countries than for rich, large and mid-latitude countries.

The damage function in (16.5) is the complete damage function in the RICE-2010 model. The aggregate damage curve is built up from estimates of the damages for the 12 regions, including assumed sectoral change and underlying income elasticities of different outputs. It includes estimated damages to major sectors such as agriculture, the cost of sea-level rise, adverse impacts on health, non-market damages, as well as estimates of the potential costs of catastrophic damages. The functions include damages from temperature change (T_{AT}), specific damages from sea-level rise (SLR) and the impacts of CO_2 fertilization, which are a function of atmospheric concentrations of CO_2 (M_{AT}).

To a first approximation, the damages are quadratic in temperature over the near term, and these are represented in Equation (16.5′). In some model simplifications, (16.5′) can be used instead of (16.5). Figure 16.3 shows the results of the Tol (2009) survey on damages, the IPCC assessment and the assumption in the 2010 vintage of the DICE-RICE models as a function of global mean temperature increase.

A warning about the functional form in Equation (16.4) for damages should be noted when using for large temperature increases. The damage function has been calibrated for damage estimates in the range of 1−4°C. The evidence is very limited for higher warming. Note also that the functional form in (16.4), which puts the damage ratio in the denominator, is designed to ensure that damages do not exceed 100% of output, and this limits the usefulness of this approach for catastrophic climate change. The damage

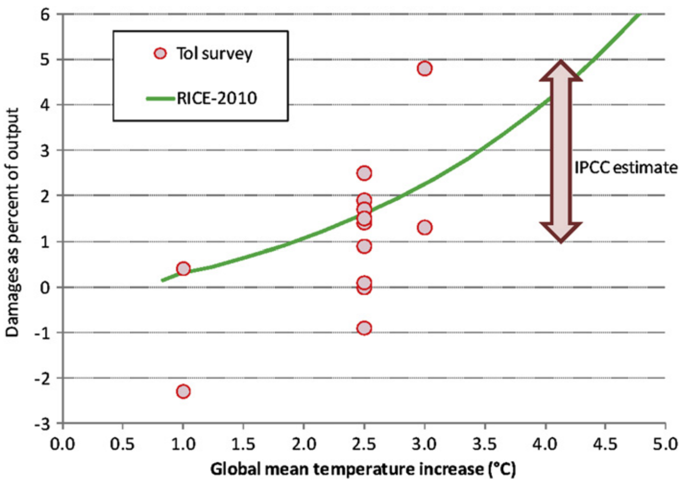

Figure 16.3 Estimates of the impact of global warming on the global economy. This shows a compilation of studies of the aggregate impacts or damages of global warming for each level of temperature increase (dots from Tol, 2009). The solid line is the estimate from the RICE-2010 model. The arrow is from the IPCC (2008).

function needs to be examined carefully or respecified in cases of higher warming or catastrophic damages:

$$\Lambda(t) = \Psi(t)\theta_1(t)\mu(t)^{\theta_2}. \tag{16.6}$$

The abatement cost Equation in (16.6) is a reduced-form type model in which the costs of emissions reductions are a function of the emissions reduction rate, $\mu(t)$. The abatement cost function assumes that abatement costs are proportional to output and to a polynomial function of the reduction rate. The cost function is estimated to be highly convex, indicating that the marginal cost of reductions rises from zero more than linearly with the reductions rate.

A new feature of the DICE-2007 and RICE-2010 models is that they explicitly include a backstop technology, which is a technology that can replace all fossil fuels. The backstop technology could be one that removes carbon from the atmosphere or an all-purpose environmentally benign zero-carbon energy technology. It might be solar power, or carbon-eating trees or windmills, or some as-yet undiscovered source. The backstop price is assumed to be initially high and to decline over time with carbon-saving technological change. In the full regional model, the backstop technology replaces 100% of carbon emissions at a cost of between \$230 and \$540 per ton of CO_2 depending upon the region in 2005 prices. The backstop technology is introduced into the model by setting the time path of the parameters in the abatement-cost Equation (16.6) so that the marginal cost of abatement at a control rate of 100% is equal to the backstop price for each year.

The next three equations are standard accounting equations. Equation (16.7) states that output includes consumption plus gross investment. Equation (16.8) defines *per capita* consumption. Equation (16.9) states that the capital stock dynamics follows a perpetual inventory method with an exponential depreciation rate:

$$Q(t) = C(t) + I(t) \tag{16.7}$$

$$c(t) = C(t)/L(t) \tag{16.8}$$

$$K(t) = I(t) - \delta_{\mathrm{K}} K(t-1). \tag{16.9}$$

CO_2 emissions are projected as a function of total output, a time-varying emissions-output ratio and an emissions-control rate. The emissions-output ratio is estimated for individual regions and is then aggregated to the global ratio. The emissions-control rate is determined by the climate change policy under examination. The cost of emissions reductions is parameterized by a log-linear function, which is calibrated to recent studies of the cost of emissions reductions.

Early versions of the DICE and RICE models used the emissions control rate as the control variable in the optimization because it is most easily used in linear-program algorithms. In recent versions, we have also incorporated a carbon tax as a control variable. This can be accomplished using an Excel SOLVER version with a modified Newton method to find the optimum. Using the carbon price is advantageous when modeling uncertainty or using price-type administrative regimes, although the solutions are identical in deterministic cases.

The final two equations in the economic block are the emissions equation and the resource constraint on carbon fuels. Uncontrolled industrial CO_2 emissions in Equation (16.10) are given by a level of carbon intensity, $\sigma(t)$, times output. Actual emissions are then reduced by one minus the emissions-reduction rate, $\mu(t)$, described above. The carbon intensity is taken to be exogenous and is built up from emissions estimates of the 12 regions, whereas the emissions-reduction rate is the control variable in the different experiments:

$$E_{\mathrm{Ind}}(t) = \sigma(t)[1-\mu(t)]A(t)K(t)^{\gamma}L(t)^{1-\gamma}. \tag{16.10}$$

Equation (16.11) is a limitation on total resources of carbon fuels, given by *CCum*. The model assumes that incremental extraction costs are zero and that carbon fuels are efficiently allocated over time by the market, producing the optimal Hotelling rents on carbon fuels:

$$CCum \geq \sum_{t=1}^{T_{\max}} E_{\mathrm{Ind}}(t). \tag{16.11}$$

16.2.5 Geophysical sectors

The major differentiating feature of the DICE/RICE models is the inclusion of several geophysical relationships that link the economy with the different forces affecting climate change. These relationships include the carbon cycle, a radiative forcing equation, climate change equations and a climate-damage relationship. A key feature of IAMs is that the modules operate in an integrated fashion rather than taking inputs as exogenous inputs from other models or assumptions.

The next Equations (16.12)−(16.18) link economic activity and greenhouse gas emissions to the carbon cycle, radiative forcings and climate change. These relationships have proven a major challenge because of the need to simplify what are inherently complex dynamics into a small number of equations that can be used in an integrated economic−geophysical model. As with the economics, the modeling philosophy for the geophysical relationships has been to use parsimonious specifications so that the theoretical model is transparent and so that the optimization model is empirically and computationally tractable.

In the DICE/RICE-2010 models, the only greenhouse gas that is subject to controls is industrial CO_2. This reflects the fact that CO_2 is the major contributor to global warming and that other greenhouse gases are likely to be controlled in different ways (the case of the CFCs through the Montreal Protocol being a useful example). Other greenhouse gases are included as exogenous trends in radiative forcing; these include primarily CO_2 emissions from land-use changes, other well-mixed greenhouse gases and aerosols.

Recall that Equation (16.10) generated industrial emissions of CO_2. Equation (16.12) then generates total CO_2 emissions as the sum of industrial and land-use emissions. CO_2 arising from land-use changes are exogenous and are projected based on studies by other modeling groups:

$$E(t) = E_{\text{Ind}}(t) + E_{\text{Land}}(t). \tag{16.12}$$

The carbon cycle is based upon a three-reservoir model calibrated to existing carbon-cycle models and historical data. We assume that there are three reservoirs for carbon. The variables $M_{AT}(t)$, $M_{UP}(t)$ and $M_{LO}(t)$ represent carbon in the atmosphere, carbon in a quickly mixing reservoir in the upper oceans and the biosphere, and carbon in the deep oceans. Carbon flows in both directions between adjacent reservoirs. The mixing between the deep oceans and other reservoirs is extremely slow. The deep oceans provide a finite, albeit vast, sink for carbon in the long run. Each of the three reservoirs is assumed to be well-mixed in the short run. Equations (16.13)−(16.15) represent the equations of the carbon cycle:

$$M_{AT}(t) = E(t) + \phi_{11} M_{AT}(t-1) + \phi_{21} M_{UP}(t-1) \tag{16.13}$$

$$M_{UP}(t) = \phi_{12} M_{AT}(t-1) + \phi_{22} M_{UP}(t-1) + \phi_{32} M_{LO}(t-1) \tag{16.14}$$

$$M_{LO}(t) = \phi_{23} M_{UP}(t-1) + \phi_{33} M_{LO}(t-1). \tag{16.15}$$

The parameters ϕ_{ij} represent the flow parameters between reservoirs. Note that emissions flow into the atmosphere.

The next step concerns the relationship between the accumulation of greenhouse gases and climate change. The climate equations are a simplified representation that includes an equation for radiative forcing and two equations for the climate system. The radiative forcing equation calculates the impact of the accumulation of greenhouse gases on the radiation balance of the globe. The climate equations calculate the mean surface temperature of the globe and the average temperature of the deep oceans for each time-step. These equations draw upon and are calibrated with large-scale general circulation models of the atmosphere and ocean systems from the IPCC *Fourth Assessment Report* (IPCC, 2007a, 2007b, 2007c).

On the whole, existing climate research models are much too complex to be included in economic models, particularly ones that are used for optimization. Instead, we employ a small structural model that captures the basic relationship between greenhouse gas concentrations, radiative forcing, and the dynamics of climate change.

Accumulations of greenhouse gases lead to warming at the earth's surface through increases in radiative forcing. The relationship between greenhouse gas accumulations and increased radiative forcing is derived from empirical measurements and climate models, as shown in Equation (16.16):

$$F(t) = \eta\{\log_2[M_{AT}(t)/M_{AT}(1750)]\} + F_{EX}(t). \tag{16.16}$$

$F(t)$ is the change in total radiative forcings of greenhouse gases since 1750 from anthropogenic sources such as CO_2, $F_{EX}(t)$ is exogenous forcings and the first term is the forcings due to CO_2. The equation uses estimated carbon in the year 1750 as the preindustrial equilibrium. The major part of warming is due to CO_2, while the balance is exogenous forcing from other long-lived greenhouse gases, aerosols, ozone, albedo changes and other factors. The DICE model treats other greenhouse gases and forcing components as exogenous either because these are relatively small and their control is exogenous (as the case of CFCs) or because they are poorly understood (as with cloud albedo effects).

Higher radiative forcing warms the atmospheric layer, which then warms the upper ocean, gradually warming the deep ocean. The lags in the system are primarily due to the diffusive inertia of the different layers. The latest version of the models adjusted the climate sensitivity to the center of the IPCC range of 3.2°C for an equilibrium CO_2 doubling. The dynamics are determined so that the transient temperature sensitivity is the same as the average of the atmosphere–ocean global circulation models reviewed in the IPCC *Fourth Assessment Report*:

$$T_{AT}(t) = T_{AT}(t-1) + \xi_1\{F(t)\text{-}\xi_2 T_{AT}(t-1) - \xi_3[T_{AT}(t-1) - T_{LO}(t-1)]\}. \tag{16.17}$$

$$T_{LO}(t) = T_{LO}(t-1) + \xi_4\{T_{AT}(t-1) - T_{LO}(t-1)]\}. \tag{16.18}$$

$T_{AT}(t)$ and $T_{LO}(t)$ represent respectively the mean surface temperature and the temperature of the deep oceans. Note that the equilibrium temperature sensitivity is given by $\Delta T_{AT} = \Delta F(t)/\xi_2$.

This completes the description of the DICE model. We now turn to describe the difference between the DICE and RICE models.

16.2.6 RICE-2010 model

The RICE model is a regionalized version of the DICE model. It has the same basic economic and geophysical structure, but contains a regional elaboration.

The general structure of the RICE model is similar to the DICE model with disaggregation into regions. However, the specification of preferences is different because it must encompass multiple agents (regions). The general preference function is a Bergson–Samuelson social welfare function over regions of the form $W = W(U^1, \ldots, U^N)$., where U^I is the preference function of the Ith region. The model is specified using the Negishi approach in which regions are aggregated using time- and region-specific weights subject to budget constraints, yielding:

$$W = \sum_{t=1}^{T_{\max}} \sum_{I=1}^{N} \psi_{I,t} U^I[c^I(t), L^I(t)]]R^I(t). \tag{16.19}$$

In this specification, the $\psi_{I,t}$ are the "Negishi weights" on each region and each time period. Each region has individual consumption and population. In principle, they may have different rates of time preference, although in practice the RICE model assumes that they are all equal. The Negishi algorithm in the RICE model sets each of the weights so that the marginal utility of consumption is equal in each region and each period, which ensures that the requirement for maximization as market simulation principle holds. We elaborate below on the Negishi approach, which is widely used in IAMs for climate change, in Section 16.2.7 on "Computational and algorithmic aspects." The RICE-2010 model divides the world into 12 regions. These are US, EU, Japan, Russia, Eurasia (Eastern Europe and several former Soviet Republics), China, India, Middle East, Sub-Saharan Africa, Latin America, Other high-income countries (OHI) and Other developing countries. Note that some of the regions are large countries such as the US or China; others are large multicountry regions such as the EU or Latin America.

Each region is assumed to produce a single commodity, which can be used for consumption, investment or emissions reductions. Each region is endowed with an initial stock of capital and labor and with an initial and region-specific level of technology. Population data are from the UN, updated with more recent estimates through

2009, with projections using the United UN estimates to 2300. Output is measured as standard GDP in constant prices, and the GDPs of different countries are converted into constant US international prices using PPP exchange rates. Output data through 2009 are from the World Bank and the IMF, with projections to 2014 from the IMF. CO_2 emissions data are from the US Energy Information Administration and Carbon Dioxide Information Analysis Center, and are available in preliminary form through 2008.

The population, technology, and production structure is the same as in the DICE model. However, each region has its own levels and trends for each variable. The major long-run variable is region-specific technological change, which is projected for a frontier region (the US), and other countries are assumed to converge partially to the frontier. For convenience, both carbon-energy inputs and industrial emissions are measured in units of carbon weight. Economic growth rates for the different regions are provided in Table 16.2.

The RICE-2010 model calibrates the energy-related parameters using data on historical GDP and CO_2 emissions for the period 1960−2008. The model uses a cost function for CO_2 emissions reductions that is drawn from more detailed models at the national and regional levels from the IPCC *Fourth Assessment Report* (IPCC, 2007c) and the EMF-22 report (Clarke *et al.*, 2009). Figure 16.4 shows historical rates of decarbonization. It further divides the growth between the weighted growth of countries and the composition effect from the increasing weight of high-emissions-intensity countries. The composition effect has added nearly one percentage point per year to the growth of CO_2 emissions in recent years.

Table 16.2 Growth of net national income by region, baseline run: RICE-2010 model (% by year, logarithmic)

Growth of net national income	2005−2055	2055−2105	2105−2205
US	2.04	1.08	0.29
EU	1.84	0.88	0.28
Japan	1.02	0.71	0.30
Russia	1.73	0.85	0.36
Eurasia	2.65	1.44	0.40
China	3.90	1.25	0.30
India	4.29	1.87	0.37
Middle East	3.59	1.69	0.27
Africa	4.99	2.33	0.30
Latin America	3.16	1.47	0.33
OHI	1.99	0.79	0.27
Other developing Asia	4.10	2.10	0.37
World	*2.79*	*1.45*	*0.32*

Net national income equals consumption plus the growth in the net capital stock, including climate damages in baseline run.

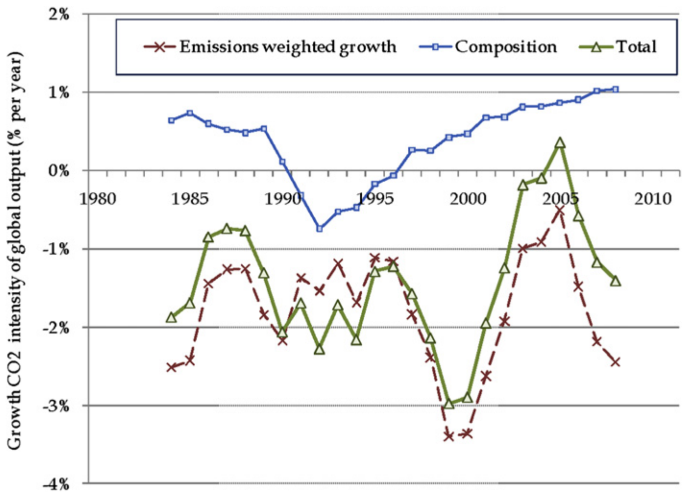

Figure 16.4 Global rates of decarbonization: the weighted growth of emissions, composition effect and total growth in emissions. Weighted growth takes the chain weighted grow of regions where the weights are the share of global total emissions. *(*Source*: Nordhaus (2010).)*

The supply curve for carbon fuels allows for limited albeit very large long-run supplies of carbon fuels. In the optimal-growth framework, energy resources are efficiently allocated across time, which implies that low-cost carbon resources have scarcity prices (called Hotelling rents) and that carbon-energy prices rise over time.

The geophysical sectors are basically the same as the DICE model. The only difference is that there are region-specific land-use CO_2 emissions, but these are exogenous and have little effect on the outcomes.

The objective function used in the RICE model differs from that in the DICE model. Each region is assumed to have a social welfare function, and each region optimizes its consumption, greenhouse gas policies and investment over time. The parameters for each region are calibrated to ensure that the real interest rate in the model is close to the average real interest rate and the average real return on capital in real-world markets in the specific region. It is here that the interpretation of optimization models as "markets as maximization algorithms" (see Section 16.3 below) becomes important. We do not view the solution as one in which a world central planner is allocating resources in an optimal fashion. Rather, output and consumption is determined according to the initial endowments of technology. "Dollar votes" in the RICE model may not correspond to any ethical norms but instead reflects the laws of supply and demand. To put this in terms of standard welfare economics, the outcome is optimal in the sense of both efficient and fair if the initial endowments are ethically appropriate, but without that assumption we can only label the outcome as Pareto efficient.

16.2.7 Computational and algorithmic aspects

As we discuss in Section 16.4.3, IAMs are generally computationally complex compared to physical science models, such as climate models, that use recursive time-stepped algorithms. The DICE model is relatively simple because it is a straightforward non-linear optimization problem. The DICE model traditionally was solved using the CONOPT or NLP solver in the GAMS modeling system (see Brooke *et al.*, 2005). This is based on the generalized reduced gradient (GRG) algorithm. The basic approach is to embed a linear programming algorithm inside an algorithm that linearizes the non-linear equations. While this algorithm does not guarantee that the solution is the global optimum, our experience over the years has not suggested any solutions other than those found by the algorithm. In the latest round of models, we have used the Excel Solver (using the Risk Solver Platform or other premium product). This has the major advantage that optimization can be performed over prices, which is a natural approach for global warming economics. (It is very difficult to implement a solution using prices as a decision variable in a standard linear programming algorithm.) Using Excel Solver is also much easier to understand and to detect programming errors.

By contrast, the RICE model (with multiple optimizing agents in equilibrium) is conceptually a fixed point problem. Most IAMs today use a Negishi algorithm to solve this and this is the approach followed in the RICE solutions. The origins of the Negishi approach date from work of Takashi Negishi, Peter Dixon, Victor Ginsberg, Jean Waelbroeck, Thomas Rutherford and Alan Manne. The Negishi theorem is essentially an application of the second theorem of welfare economics. Several authors implemented this in the mid-1990s, particularly Nordhaus and Yang (1996) in the first version of the RICE model, although the actual implementations were and continue to differ among IAMs.

The RICE-2010 model has been implemented only in the Excel format. The baseline RICE-2010 model can be used by researchers and students in the Excel format and need not rely upon Solver. However, the optimization requires the advanced proprietary versions of Solver. It should also be noted that Solver is unable to solve the largest version of the RICE model in a reliable fashion, and errors sometimes occur when using Solver. For example, when using the Solver to optimize the solution for reaching a global optimum for limiting temperature to 2°C, different starting points yield optimal carbon prices that differ by as much as 0.005% for the first few periods when tolerances are set at their maximum. In some circumstances, Solver simply stops and cannot find a solution, and sometimes it finds a wildly incorrect solution. The major advantage of using the Excel Solver approach is the ability to optimize using prices as control variables, which is much more difficult using standard mathematical programming algorithms (such as Rutherford, 1995).

16.2.8 Other solution approaches

This survey cannot present a comprehensive review of algorithms that are used to solve IAMs. (This section is based on a survey of modelers by the author and Zhimin Li.) Most IAMs have a relatively high degree of computational complexity. They generally involve solutions of simultaneous equations, such as multimarket supply and demand functions, and often are general equilibrium problems that in principle are fixed-point problems. Either as Nash equilibria or as general equilibrium problems, the models are in a complexity class between P and NP known as PPAD (polynomial parity arguments on directed) graphs (see Papadimitriou, 1994).

Up to now, only a limited number of approaches have been widely used. Most IAMs use constrained optimization, e.g. using the GAMS software described above. These include DICE, RICE, MERGE, FUND and WITCH. A small number of models, such as DART and MRN-NEEM, have used complementarity algorithms (CP). While the Scarf fixed-point algorithm can find a fixed point solution (Scarf, 1973), it is generally computationally infeasible for the current generation of IAMs. As in many areas, the complexity of the models has grown as fast as computer speeds, and there is no prospect short of quantum computing of solving the large models using fixed-point techniques.

The common solvers used in optimization IAMs include: CONOPT in GAMs (e.g. DICE, MERGE and WITCH); PATH in GAMs (e.g. MRN-NEEM); Premium Risk Platform in Excel (e.g. current version of RICE). There has been little experience in the IAM community of other approaches, such as genetic algorithms.

16.2.9 Simplifications in IAMs

This sketch of a pair of IAMs in the DICE and RICE models makes it clear that they are highly simplified representations of the complex economic and geophysical realities. While small and comprehensive models have many advantages (as is discussed in later sections), they also have major shortcomings because of their simplifications. I discuss those related to production, taxation and functional forms as examples.

One example of simplification is the use of a single commodity to represent all consumption, investment, and public goods and services. The use of a single commodity is particularly restrictive in the context of international trade, where the essence of trade is heterogeneity across regions. This point is particularly important in the question of whether to use market exchange rates or PPP exchange rates in measuring relative national outputs. A study by McKibbin *et al.* (2007) investigated this issue using the G-Cubed multicountry model. It showed that, under one scenario, emission projections based on convergence assumptions defined in market exchange rates terms are 40%

higher by 2100 than emission projections based on PPP comparisons of income differentials.

Another important set of important issues concerns taxation. The simplest models ignore the structure of the tax system. This is particularly important for energy and capital taxes, which have large effects on energy use and on the rates of return used in making long-term decisions in the energy sector. Some of the more detailed IAMs, such as the IGEM, NEMS and EPPA discussed above, include more realistic detail on the US tax and regulatory system, but they oversimplify or ignore the issues raised by international tax systems. The structure of tax systems is particularly important for estimation of the optimal level of carbon pricing or taxation because of the need to consider the interaction of carbon pricing with the structure of pre-existing tax and regulatory distortions (see particularly the several important studied collected in Goulder, 2002).

Many simplifications are also buried in the functional forms of models. For example, the RICE and DICE models rely on the Cobb–Douglas function to represent the production process. This is likely to overestimate substitution in some areas and underestimate it in others. Additionally, it may suggest a degree of smoothness in substitution that is not present when there are only a small number of processes, in which case an activity analysis framework would be preferable (such as is used in the several components in the NEMS model and in parts of the energy sector of the MERGE model, see Manne and Richels, 2004).

We must put these concerns about oversimplification in the context of the questions that are being asked. The purpose of models is not to be an exact replica of real-world processes. Aside from the impossibility of achieving that goal, greater detail would actually be less valuable for many purposes. Instead, models are used for insights about key questions. For example, if we are concerned about the long-run intertemporal tradeoffs between consumption today and consumption in the future, a relatively simple model can illustrate the issues. Similarly, to determine the uncertainties associated with future climate change, the model must be sufficiently small and manageable so that the uncertainties can be estimated and Monte Carlo or other techniques can be used to capture all the major uncertainties. However, for many other questions, such as the impact of changes in tax policies or international trade or carbon leakage, more detail is needed to capture the international and sectoral reactions to policy changes.

A useful analogy here is to the animal kingdom. Each model is like an animal that has its useful niche in the policy ecosystem. Small models can be fleet and can adapt easily to a changing environment, while large models take many years to mature but are able to handle much larger and more complex tasks. There is room for all in the world of climate change science.

16.3 ILLUSTRATIVE MODEL RESULTS: THE COPENHAGEN ACCORD

16.3.1 Model outputs

IAMs have a wide variety of applications. These were comprehensively reviewed in Weyant *et al.* (1997). Among the most important applications are the following:

- Making consistent projections, i.e. ones that have consistent inputs and outputs of the different components of the system (so that the GDP projections are consistent with the emissions projections).
- Calculating the impacts of alternative assumptions on important variables such as output, emissions, temperature change and impacts.
- Tracing through the effects of alternative policies on all variables in a consistent manner, as well as estimating the costs and benefits of alternative strategies.
- Estimating the uncertainties associated with alternative variables and strategies.
- Calculating the effects of reducing uncertainties about key parameters or variables, as well as estimating the value of research and new technologies.

Different IAMs are like different animals in terms of comparative strengths and weaknesses in tackling the different questions listed above. Comprehensive models can do a full cost—benefit analysis, but they are likely to be weak on the regional or industrial detail. The larger species of models provide great detail, but may be unable to trace out impacts and damages, and they may be less transparent and be unable to do full uncertainty analyses. Some models are able to trace through the impacts of policies on land use; others can investigate a wide range of technologies, a few have full damage functions, while others include a limited number of technologies and economic variables. This modeling diversity allows many questions to be answered, and most questions can be addressed by a large group of models and thereby provide tests of the consensus or dissensus across models.

16.3.2 Modeling the Copenhagen Accord

IAMs are useful devices to improve our understanding of tradeoffs, costs, benefits and uncertainties. They are not truth machines, although they sometimes can be helpful in rooting out obvious inconsistencies and errors. With these objectives in mind, this section presents illustrative results from studies using the RICE-2010 model. There are many other models that can be examined, and a particularly valuable review of comparative results is that in the EMF-22 study (Clarke *et al.*, 2009).

For this illustration, I will focus on an analysis of the Copenhagen Accord and similar policies that have been discussed in policy circles in the current period. We reviewed some of the history of international agreements on climate change above as well as the results of the Copenhagen Accord. Given the current state of these agreements, it is useful to review the prospects for climate change and the economic implications, both for the case where controls are implemented as envisioned by the

Copenhagen Accord and for the case where the present stalemate continues. This report presents the results of an updated version of the RICE model, denoted the RICE-2010 model. These results use the modeling results presented in Nordhaus (2010) with some small modifications.

16.3.3 Policy scenarios

One advantage of IAMs is that they can compare the economic and climate trajectories associated with different policy approaches. For this discussion of the implications of current policy and the Copenhagen Accord, I consider five different policy options:

- *Baseline:* No climate change policies are adopted.
- *Optimal:* Climate change policies maximize economic welfare, with full participation by all nations starting in 2010 and without climatic constraints.
- *Temperature-limited:* The optimal policies are undertaken subject to a further constraint that the global temperature does not exceed 2°C above the 1900 average.
- *Copenhagen Accord:* High-income countries implement deep emissions reductions similar to those included in the current US proposals, with developing countries following in the next two to five decades. It is assumed that implementation is through a system of national emission caps with full emissions trading within and among countries (although a harmonized carbon tax would lead to the same results).
- *Copenhagen Accord with only rich countries:* High-income countries implement deep reductions as in last scenario, but developing countries do not participate until the twenty-second century.

The baseline can be interpreted as complete inaction and stalemate on climate policies. The "optimal" scenario assumes the most efficient climate change policies; in this context, efficiency involves a balancing of the present value of the costs of abatement and the present value of the benefits of reduced climate damages. Although unrealistic, this scenario provides an efficiency benchmark against which other policies can be measured. The "temperature-limited" scenario is a variant of the optimal scenario that builds in a precautionary constraint that a specific temperature increase is not exceeded.

The "Copenhagen Accord" scenario assumes that the announced emissions-reduction policies for high-income countries for the near term are implemented. It then extends these to OHI countries to parallel the US-proposed reductions. Developing countries are assumed to follow within a few decades. Table 16.3 shows the base and commitment years for different regions. The fifth scenario is the same as the Copenhagen Accord scenario, but developing countries do not participate until well into the twenty-second century. For this scenario, the high-income participants are the US, the EU, Japan, Russia and a group of other high-income countries.

Table 16.3 Participation rates in Copenhagen Accord: RICE−2010 model

Capping region	Date of participation	Base year	Commitment year	Fraction of base year in commitment year	Further reductions tied to
US	2015	2005	2015	0.84	House bill
EU	2005	1990	1995	0.80	US
Japan	2005	1990	1995	0.94	US
Russia	2005	1990	2005	1.00	US
Eurasia	2020	1990	2020	1.00	US
China	2030	2030	2030	1.00	US
India	2040	2040	2040	1.00	US
Middle East	2050	2050	2050	1.00	US
Africa	2070	2070	2070	1.00	US
Latin America	2030	2030	2030	1.00	US
OHI	2015	2015	2015	1.00	US
Other non-OECD Asia	2040	2040	2040	1.00	US

The table shows the assumed dates at which different regions join the protocol, the base year used to index emissions, the commitment year (the year in which the emissions are limited), the fraction of emissions covered for the commitment year and the indexation plan. The "House bill" is the legislation passed by the US House of Representatives in 2009 (HR 2454).

16.3.4 Major results

The results presented here should be viewed as only suggestive and illustrative. They come from a single model and modeling perspective, and most of the relationships are subject to large uncertainties. They are presented to show the kinds of results that can be obtained using IAMs. Similar results are found in the report in Rogelj *et al.* (2010).

Figure 16.5 shows global CO_2 emissions under each of the five policy scenarios. Unrestrained emissions are estimated to grow very rapidly. Emissions under the optimal and temperature-limited scenarios are essentially flat for the next two to six decades and then decline. The optimal path imposes a cut in global emissions of 50% from 2005 in 100 years, and the temperature-limited path requires zero emissions at about 2075.

Atmospheric concentrations of CO_2 rise sharply under the baseline path, reaching 793 p.p.m. by 2100 (see Figure 16.6 and Table 16.4). The optimal and temperature-limited paths show some rise of concentrations from current levels, peaking between 500 and 600 p.p.m. (note that these refer to CO_2, not to CO_2-equivalent, concentrations). Radiative forcings (Table 16.4) peak at 4.4 W/m^2 in the optimal path and at 3.2 W/m^2 in the temperature-limited path. These forcings include those from other greenhouse gases as well as estimates of other anthropogenic forcings such as from sulfates (note that many current studies include only long-lived greenhouse gases and therefore will generally overstate current radiative forcings relative to those estimated by the IPCC in its *Fourth Assessment Report*).

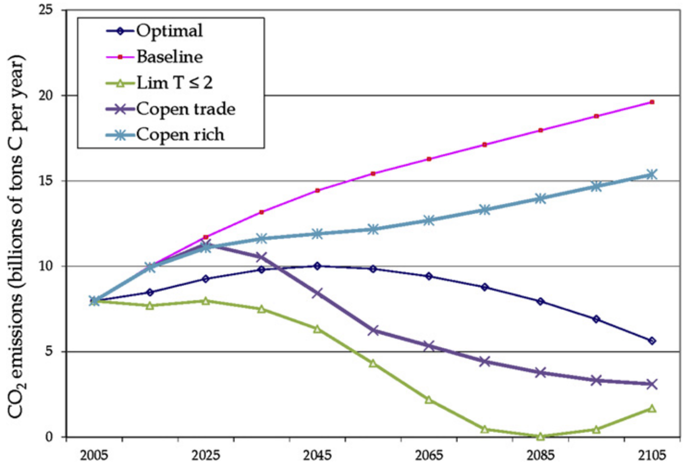

Figure 16.5 Projected emissions of CO_2 under alternative policies: RICE-2010 model. Projected emissions of industrial CO_2 associated with different policies. Policies are explained in the text. Note that other greenhouse gases are taken to be exogenous in the projections.

Global temperature projections, shown in Figure 16.7 and Table 16.4, rise sharply under the baseline, with increases of 3.5°C in 2100, 5.7°C in 2200 and a peak (not shown) at 6.7°C, all relative to 1900. The optimal and temperature-limited paths rise in the early twenty-first century because of the momentum of past emissions. They then

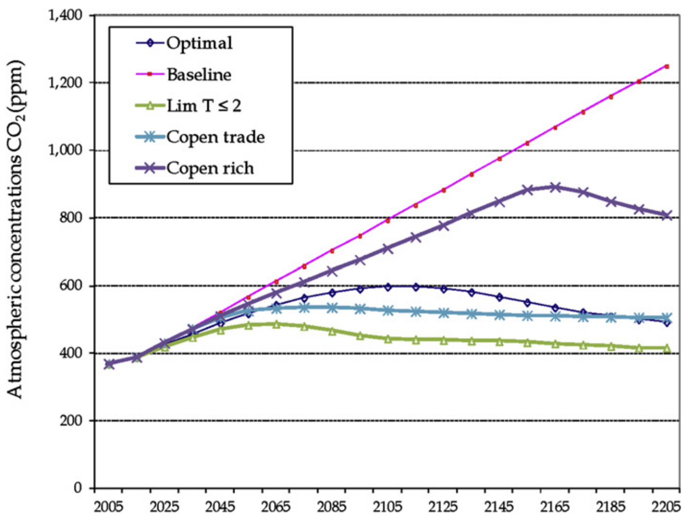

Figure 16.6 Atmospheric concentrations of CO_2 under alternative policies: RICE-2010 model. Projected atmospheric concentrations of CO_2 associated with different policies. The concentrations include emissions from land-use changes. Policies are explained in text.

Table 16.4 Results for climate variables for different runs: RICE-2010 model

	2000	2100	2200	2300
CO_2 concentrations (p.p.m)				
Base	369.5	748.0	1250.0	1227.6
Optimal	369.5	591.7	493.2	455.6
Limit $T < 2°C$	369.5	453.7	417.4	398.4
Copenhagen: full trade	369.5	532.9	506.4	474.2
Copenhagen: no trade	369.5	530.8	483.2	463.6
Copenhagen: rich only	369.5	676.4	808.9	726.6
Radiative forcings (W/m^2)				
Base	1.60	5.99	8.50	8.41
Optimal	1.60	4.42	3.41	2.97
Limit $T < 2°C$	1.60	2.83	2.49	2.23
Copenhagen: full trade	1.60	3.77	3.55	3.19
Copenhagen: no trade	1.60	3.74	3.29	3.06
Copenhagen: rich only	1.60	5.38	6.12	5.48
Temperature (°C from 1900)				
Base	0.83	3.51	5.72	6.56
Optimal	0.83	2.77	2.71	2.41
Limit $T < 2°C$	0.83	2.00	1.92	1.82
Copenhagen: full trade	0.83	2.49	2.64	2.58
Copenhagen: no trade	0.83	2.48	2.51	2.43
Copenhagen: rich only	0.83	3.20	4.52	4.37

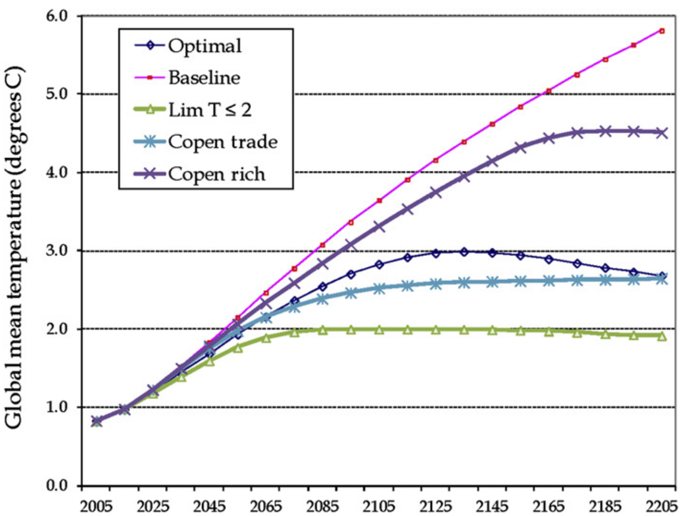

Figure 16.7 Global temperature increase (°C from 1900) under alternative policies: RICE-2010 model. Projected global mean temperature paths associated with different policies.

Table 16.5 Carbon prices (2005 $ per tC) in the different runs: RICE-2010 model

	2005	2010	2015	2020	2025	2055	2105
Optimal	0.00	28.93	37.70	49.13	64.02	153.45	430.59
Limit $T < 2°C$	0.00	58.80	78.79	105.57	141.46	529.64	723.42
Copenhagen: full trade	0.00	0.10	0.39	1.51	5.79	358.37	593.10
Copenhagen: rich only	0.00	0.07	0.39	2.21	12.40	64.11	27.68

The carbon prices are the market prices that are required to attain the policy objectives. These assume full trading and participation in all regions that are in the policy regime.

bend downward as emissions are reduced, peaking at 2.0°C (obviously) for the temperature-limited path and 3.0°C for the optimal path. Two important results are that the optimal path has a relatively low maximum temperature, and that the temperature increase for this path averaged over 2100–2300 is 2.7°C.

Perhaps the most important outputs of integrated economic models of climate change are the near-term "carbon prices." This is a concept that measures the marginal costs of reductions of emissions of greenhouse gases. In a market environment, such as a cap-and-trade regime, the carbon prices would be the trading price of carbon emission permits. In a carbon-tax regime, these would be the harmonized carbon tax among participating regions. We can also judge different policies against benchmarks by examining their near-term carbon prices, which are shown for the different scenarios in Table 16.5, in 2005 dollars. A graphical comparison is shown in Figure 16.8. Carbon prices in the baseline scenario, equal to the Hotelling rents on carbon fuels, are essentially

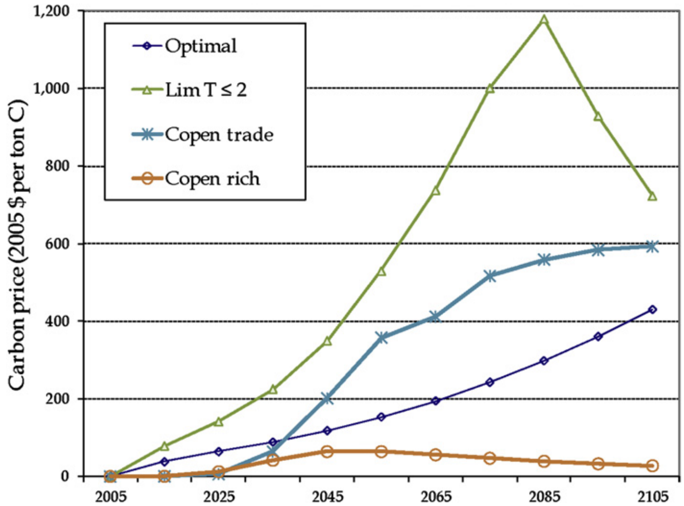

Figure 16.8 Globally averaged carbon prices in different policy runs: RICE-2010 model. Note the sharp decrease in carbon price in the limit case. This occurs as temperature hits the ceiling.

zero and are therefore not depicted. Prices under the optimal and temperature-limited scenarios at first rise to \$38 and \$79 per ton of carbon, respectively, by 2015. Prices under the optimal scenario then continue to rise sharply until they reach the projected backstop price (all prices in this review are in tons of carbon weight in 2005 international US\$; note that prices per ton of CO_2 are 3.67 times smaller).

Global average carbon prices under the two Copenhagen Accord scenarios are much lower than under the previous scenarios for the first two decades of the projections, reflecting the gradual introduction of policy interventions as well as incomplete participation. Note that the effective carbon price today (around \$1 per ton carbon) is well below that required under either the optimal or the temperature-limited scenario. Numerical values for carbon prices for the different scenarios are reported in Table 16.5 and those for the Copenhagen Accord with no trading in Table 16.6. Table 16.7 presents the associated emissions control rates for the Copenhagen Accord with full trading.

Table 16.8 shows the large stakes involved in climate change policies as measured by aggregate costs and benefits. Using the model discount rates, the optimal scenario raises the present value of world income by \$9.1 trillion, or 0.35% of discounted income. This is equivalent to an annuity of \$454 billion per year at a 5% annual discount rate. Imposing the 2°C temperature constraint has a significant economic penalty, reducing the net benefit by almost half, because of the difficulty of attaining that target with so much inertia in the climate system. The Copenhagen Accord with phased-in participation of developing countries has substantial net benefits, but lack of participation in the "rich only" case reduces these substantially. Figure 16.9 shows the path of net costs as a percentage of income for seven major regions. Costs rise gradually over the coming decades and reach around 1% of national income for the high-income countries in the mid-twenty-first century.

Table 16.6 Carbon price (2005 \$ per tC) for Copenhagen Accord with no trading: RICE-2010 model

	2015	2025	2035	2045	2055	2105
US	34.55	82.64	229.22	397.39	523.12	592.74
EU	0.00	74.04	196.79	476.89	706.78	878.72
Japan	184.87	198.12	352.71	532.04	708.96	836.46
Russia	0.00	0.00	0.00	39.13	141.26	289.42
Eurasia	0.00	0.00	0.00	11.79	114.17	297.16
China	0.00	0.00	69.54	201.85	317.52	417.29
India	0.00	0.00	0.00	213.96	467.48	711.35
Middle East	0.00	0.00	0.00	0.00	168.95	553.48
Africa	0.00	0.00	0.00	0.00	0.00	417.91
Latin America	0.00	0.00	210.00	528.51	757.51	897.81
OHI	0.00	20.82	160.61	361.20	527.40	648.55
Other	0.00	0.00	0.00	269.48	566.65	821.90
Global average	*10.13*	*22.28*	*80.29*	*209.41*	*354.51*	*570.71*

Table 16.7 Emissions control rate, Copenhagen Accord with full trading: RICE-2010 model (% of baseline)

	2005	2015	2025	2035	2045	2055	2065	2075	2085	2095	2105
US	0.0	2.1	9.7	28.7	49.8	64.3	73.3	77.1	82.5	86.6	89.5
EU	0.0	1.6	7.6	22.5	38.9	50.3	57.3	60.3	64.6	67.7	70.0
Japan	0.0	1.6	7.6	22.5	38.9	50.3	57.3	60.3	64.6	67.7	70.0
Russia	0.0	2.6	12.2	36.0	62.3	80.5	91.8	96.5	103.4	108.5	112.1
Eurasia	0.0	0.0	12.2	36.0	62.3	80.5	91.8	96.5	103.4	108.5	112.1
China	0.0	0.0	0.0	33.0	57.2	73.9	84.2	88.6	94.9	99.6	102.9
India	0.0	0.0	0.0	0.0	44.5	57.5	65.5	68.9	73.8	77.5	80.0
Middle East	0.0	0.0	0.0	0.0	0.0	60.6	69.1	72.7	77.8	81.7	84.4
Africa	0.0	0.0	0.0	0.0	0.0	0.0	0.0	68.9	73.8	77.5	80.0
Latin America	0.0	0.0	0.0	23.4	40.6	52.4	59.7	62.8	67.3	70.6	72.9
OHI	0.0	1.8	8.7	25.7	44.5	57.5	65.5	68.9	73.8	77.5	80.0
Other developing Asia	0.0	0.0	0.0	0.0	42.4	54.8	62.4	65.7	70.3	73.8	76.3
Global	0.0	0.8	3.4	17.7	35.7	53.0	56.2	69.8	73.4	75.7	77.9

The results of the RICE-2010 model highlight the spatial asymmetry between winners and losers among countries. The trajectory of net costs for selected countries is shown in Figure 16.9 and the numerical net costs in 2055 are shown in the last column of Table 16.9. The regions designated to undertake the largest emissions reductions under the Copenhagen Accord are the US, China and the EU; the price tag for these regions totals more than $1 trillion in discounted costs through 2055. Several other regions, particularly Russia, can expect net benefits in a trading regime because they have been

Table 16.8 Present value of global consumption, different policies: RICE-2010 model (scaled to consumption in US international dollars, 2005 prices)

	PV utility	Difference		Annualized[a]
	trillions of 2005 $	trillions of 2005$	percent of base	billions of $ per year
Base	2301.5	0.00	0.00	0
Optimal	2309.6	8.07	0.35	403
Limit $T < 2°C$	2305.9	4.41	0.19	220
Copenhagen: full trade	2307.8	6.26	0.27	313
Copenhagen: no trade	2307.1	5.63	0.24	281
Copenhagen: rich only	2304.1	2.55	0.11	128

The estimates are the present value of global consumption equivalent for the entire period. This is equivalent to the present value of utility in consumption units. The difference in numerical column 2 shows the difference between the control run and the no-policy or baseline run. Incomes of countries are calculated using ppp exchange rates and are discounted using an international interest rate that is the capital-weighted average of the real interest rates for different regions.
[a]Annual value of consumption at an annuitization rate of 5% per year.

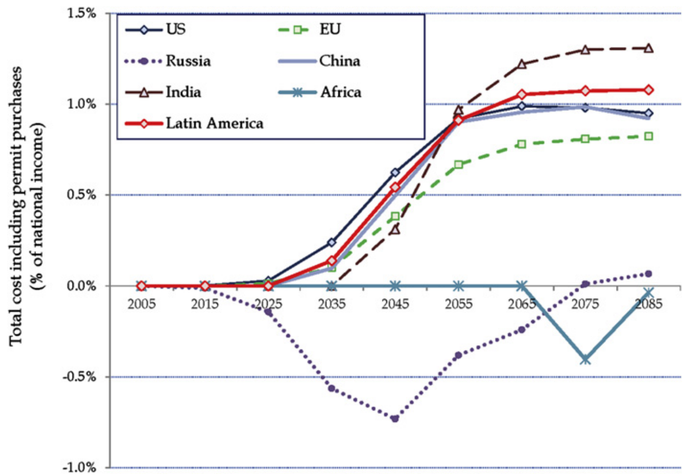

Figure 16.9 Total costs of compliance as a percentage of national income: RICE-2010 model. The total costs equal the abatement costs plus the net purchases of emissions permits from other regions under full participation and full trading. These are then divided by net national income for the region.

allocated excess emissions permits under the Kyoto Protocol and are assumed to continue those allocations in its successor. Although poor countries can present reasoned arguments why rich countries should take the major emissions cuts, rich countries will weigh their own costs and attempt to spread the burden more widely. This asymmetry reinforces the tendency of countries to move to their non-cooperative equilibrium, resulting in an "*après vous*" syndrome in which no country takes substantial steps.

Table 16.9 Costs and benefits of Copenhagen Accord through 2055: RICE-2010 model

Region	Change in damages	Abatement costs	Permit purchases	Net costs
US	−51	328	228	505
EU	−56	160	171	276
Japan	−12	44	64	96
Russia	−5	92	−176	−89
Eurasia	−4	62	−150	−92
China	−52	655	−268	335
India	−54	185	−1	130
Middle East	−47	123	−134	−57
Africa	−41	0	0	−41
Latin America	−33	127	154	248
OHI	−18	96	48	126
Other	−42	188	64	209
World	*−413*	*2060*	*0*	*1647*

The table illustrates the regional asymmetry of the Copenhagen Accord. The estimates take the present value of abatement costs and averted damages using the capital-weighted international real interest rate. The last column is the sum of the first three columns. OHI, other high income. Figures are in billions of 2005 international US$.

There are many conclusions that can be drawn from the present modeling effort. One important result is that, even if countries meet their ambitious objectives under the Copenhagen Accord, global temperatures are unlikely to keep within the 2°C objective. This conclusion is reinforced if developing countries delay their full participation beyond the 2030–2050 timeframe.

16.3.5 Comparisons with other studies

The results here can be compared with those of earlier versions of the RICE model as well as those of other modeling groups. The temperature projections of the RICE-2010 model are close to those of the earliest vintages (Figure 16.10). The damage ratio (the ratio of climate damage to output) is similar to that found in earlier versions for the first century, but the latest version projects higher damage ratios in the more distant future because the projected temperature rise is larger (see SI in Nordhaus, 2010). The optimal carbon price in the near term is substantially higher than in earlier versions (see SI in Nordhaus, 2010). For example, that price for 2015 is \$38 per ton of carbon, whereas in the early vintages the optimal carbon price was in the range of \$12–15 per ton of carbon, all in 2005 US\$. The major factors accounting for the increase in the optimal carbon price are a major upward revision of global output, particularly those associated with adoption of PPP income measurement, a higher assumed temperature sensitivity, and a lower discount rate on goods (Nordhaus, 2007b).

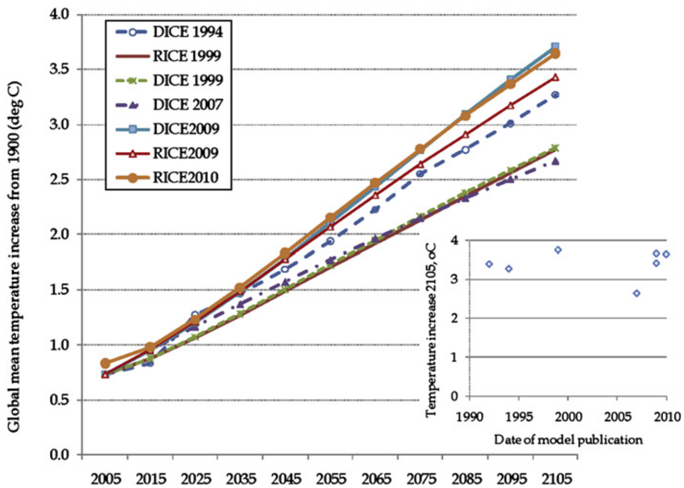

Figure 16.10 Comparison of temperature projections, baseline runs, alternative vintages of RICE/DICE models. This shows the temperature projections from the first DICE model in 1994 through RICE-2010. The inset graph shows the calculated temperature for 2105 plotted against the date of the model publication.

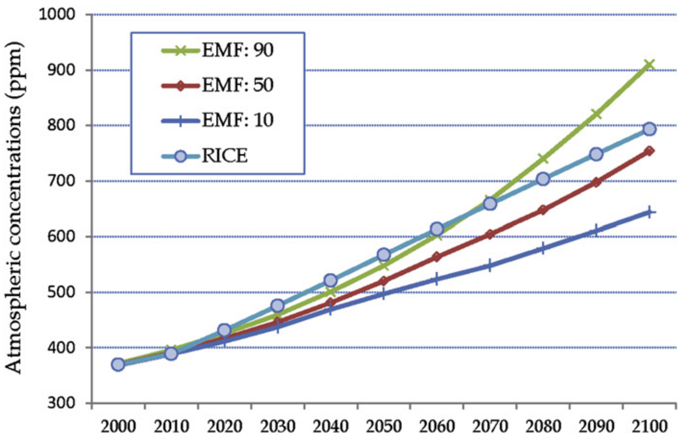

Figure 16.11 Comparison of CO_2 concentrations between RICE-2010 and EMF-22 models. *(Source: Clarke* et al. *(2009) and spreadsheet with results provided by Leon Clarke.)*

The results can also be compared with the latest round of model comparisons done for EMF-22 (Clarke *et al.*, 2009). The closest comparison is the path of CO_2 concentrations for the 2000–2100 period for the RICE baseline and the EMF reference path. The RICE concentrations path is above the median of the 10 models with complete data. For the terminal year of 2100, the 10th, 50th and 90th percentiles of CO_2 concentrations for EMF-22 are 643, 754 and 910 p.p.m., whereas the RICE projection for 2100 is 793 p.p.m. (see Figure 16.11 for a more detailed comparison). The EMF projections also indicate the difficulty of attaining the 2°C objective.

Note that the optimal carbon prices in the RICE model are well below those in studies with very low discount rates, particularly those in the *Stern Review* (2007). Discussions about discounting involve unresolved issues of intergenerational fairness, aversion to inequality, projections about future technological change and population growth, as well as the appropriateness of the utilitarian framework used in the Ramsey model. We review these issues in the section on discounting below.

The conclusion here about the Copenhagen accord parallels that of Rogelj *et al.* (2010), who conclude, "If nations proceed on the basis of the few pledges they have made for 2050, the Copenhagen Accord will almost certainly miss its own 2°C goal. Our model shows a greater than 50% chance that warming will exceed 3°C by 2100."

16.3.6 Qualifications with the results

Analyses using integrated assessment economic models present an unrealistically smooth picture of the functioning of economic and political systems, in much the same way that global climate models abstract from the turbulence of weather systems. The major difficulties with all IAMs are the problems associated with estimation and validation of

Table 16.10 Illustration of the impact of uncertainty about TSC on the optimal carbon price

TSC	Optimal carbon price, 2015 (2005 $)
1.0	8.64
2.0	22.45
3.0	35.34
3.2[a]	*37.70*
4.0	46.41
5.0	55.77
6.0	63.70

This uses a variant of DICE-2010 and the results are slightly different from the RICE-2010 results shown in other tables. Results are price per ton of carbon in 2005 $ international prices.
[a]Baseline assumption.

the models. As the models make projections well into the future, it is difficult to find a reliable approach to estimating the relationships from appropriate historical or cross-sectional data. Additionally, some of the elements, such as the optimization structure, have no obvious empirical counterpart.

Experience has shown two facts that should provide major cautions to users: (i) Different vintages of the same model often show dramatic changes in the results. The examples cited in the above for the DICE/RICE models are interesting in this respect because they have a long history of published projections on which to base comparisons. (ii) As can be seen with the EMF-22 results, different models have widely varying projections of future conditions. For example, the projections for CO_2 emissions for 2100 from the different EMF-22 models in the baseline range from 43 to 131 billion tons of CO_2 per year.

At the same time, it is critically important to recognize that the key issue about the uncertainty about long-term projections is whether they have a large impact upon current policies, such as the current optimal carbon tax or emissions control rate. Table 16.10 illustrates the impact of one of the most important parameters on the near-term carbon prices. The elasticity of the carbon price with respect to the temperature sensitivity coefficient (TSC) is close to unity. The elasticities for of other parameters differ considerably, with that of the fossil fuel supply being close to 0, that of the cost of the backstop technology being 0.26 and that of the rate of decarbonization being −0.008. The elasticity of current policy with respect to other parameters is an important subject of research for determining the importance of uncertainties of different variables.

16.4 SOME MAJOR ISSUES FOR RESEARCH IN IAM

16.4.1 Introduction

Here, I review some of the major issues that arise in the construction, design and interpretation of IAMs. Some of these are local to climate change, while others pertain more broadly to integrated modeling involving economics across different fields. This is necessarily an incomplete treatment and one that reflects the experience, tastes, and knowledge of the author. However, most of the issues discussed here are ones that have been major sources of concern and even controversy among modelers and users of IAMs.

16.4.2 Social cost of carbon

A new and important concept emerging from IAMs is the "social cost of carbon" (SCC; the first reference is apparently Pearce, 2003). This concept represents the economic cost caused by an additional ton of carbon dioxide emissions (or more succinctly carbon) or its equivalent. In a more precise definition, it is the change in the discounted value of the utility of consumption denominated in terms of current consumption per unit of additional emissions. In the language of mathematical programming, the SCC is the shadow price of carbon emissions along a reference path of output, emissions and climate change.

In an optimized climate policy (abstracting away from the deadweight losses of other taxes and the complications due to tax or regulatory distortions), the SCC will equal the carbon price or the carbon tax. In an uncontrolled regime, the SCC will generally exceed the (zero) carbon price. There is some confusion about the path along which the SCC should be calculated. For most purposes, it should refer to the marginal damages along the *actual* path of emissions and output (or some distribution of that in a stochastic framework).

Estimates of the SCC are a critical ingredient in climate change policy. They provide policy makers a guidepost to aim for if they are seeking an economically efficient policy for carbon pricing. Another application is for rulemaking where countries do not have comprehensive policies covering all greenhouse gases. In this context, regulators might use the SCC in a calculation of social costs and benefits of policies involving energy or climate-affecting decisions. For example, the US government has undertaken rule-making proceedings to determine the SCC for use in such areas as subsidies for the installation of low carbon energy sources, regulations requiring energy efficiency standards in buildings and motor vehicles, and rebates for home insulation materials [see the discussion by the US Working Group (2010) in US Regulatory Impact Analysis 2010 and also discussed in Greenstone *et al.* (2011)].

There have been many estimates of the SCC in different models (for reviews, see Tol, 2005, 2009). Tol has undertaken a systematic research synthesis (inaccurately called a meta-analysis) and has calculated a (subjectively determined) quality-weighted mean of the results of the different estimates. The most recent estimate finds a SCC of $36 per ton carbon (for the median of the Fisher–Tippett kernel density for peer-reviewed estimates with a 3% pure rate of time preference, without equity weights, adjusted to 2005 and 2005 US$). Another study was undertaken by the US Working Group on the SCC (US Working Group, 2010; Greenstone *et al.*, 2011).

Table 16.11 Estimates of the SCC (2005 $ per tC) from different sources

Year of discounting and emission	Discount rate on goods					
	US Working Group 2010			Tol survey median		RICE-2010
	2.5%	3.0%	5.0%	3.0%	5.0%	5.50%
2010	123	75	16	131	42	35
2020	134	83	24			52

Estimates from US Working Group are central deflated to 2005 prices. RICE-2010 are from Nordhaus (2011b). Tol estimates use all references that contain estimates of the consumption discount rate and take unweighted medians of the SCC for the given discount rate. The Tol estimates are in 1995 and 1995 prices. This are reflated to 2005 prices using the GDP price index and moved to 2010 using an assumed real rate of increase of the SCC equal to the real discount rate.

We have collected three different estimates for the SCC in Table 16.11. These are from the RICE-2010 model, from the Tol database and from the US Working Group. They have been grouped according to the discount rate on goods or consumption. The estimates for a 5% per year discount rate are similar for the Tol data and the RICE-2010 model. The US Working Group estimates are lower than the SCC estimates from the other two sources. The major difference among studies is usually due to different treatments of the discount rate, as shown in Table 16.11 (also see Nordhaus, 2011b). Also see Table 16.12.

The work on the SCC has proven an important application of IAMs. It is important to better understand the assumptions underlying the estimates if they are to be used for policy purposes.

Table 16.12 Calculated carbon prices (2005 $ per tC) in Nash equilibrium: RICE-2010 model

	2005	2015	2025	2035
US	0.00	4.28	6.07	8.17
EU	0.00	5.55	7.75	10.40
Japan	0.00	1.69	2.26	2.81
Russia	0.00	0.10	0.10	0.10
Eurasia	0.00	0.53	0.75	0.98
China	0.00	6.81	9.87	13.77
India	0.00	5.05	7.42	10.55
Middle East	0.00	2.94	4.21	5.77
Africa	0.00	4.17	7.00	10.90
Latin America	0.00	2.85	4.28	6.00
OHI	0.00	2.71	3.63	4.72
Other	0.00	2.44	4.22	6.88
Global (emissions weighted)	0.00	4.17	6.02	8.31

16.4.3 Complexity and transparency

One of the major issues in all IAMs is the problem of transparency. Models are generally either scientifically acceptable and opaque — or highly simplified and relatively transparent. This problem is seen in the great difficulty most researchers have had in exporting their models to other groups. I will use the example of the DICE/RICE models to illustrate the difficulties.

Even though the DICE and RICE models are extremely simplified in many areas, they remain complex non-linear systems with several poorly determined relationships. The DICE model shown above has 18 dynamic equations which contain 44 non-trivial parameters (omitting straightforward initial conditions such as world population, output and global mean surface temperature anomaly). Some of these parameters are relatively inconsequential (such as the capital elasticity in the production function). Others are central (such as the temperature sensitivity for CO_2 doubling or the rate of growth of total factor productivity). Additionally, the structural equations are invariably aggregates of complicated non-linear spatial and temporal relationships, and they are likely to be difficult to determine exactly and are probably misspecified.

Even though it is one of the simplest of the models with regional resolution, the RICE-2010 model is very complicated. In the Excel version, each of the 12 regions has 118 variables (including identities), and the global calculations and calibration add around another 1000 variables, for a total of approximately 2400 variables. Owing to the need to solve the model using the Negishi algorithm, the RICE model requires an Excel macro to solve for the Negishi weights. This means that it is difficult for users other than the model developers to actually use such complex models. As a result of its difficulty, the RICE model has been adopted by at most a single-digit number of groups and in most cases there was substantial recoding necessary.

These difficulties are representative of other models. Small and transparent models are sometimes adopted by other researchers or used by students, but the large models are very seldom transferable. The DICE model is sufficiently simple that many researchers have used it. It has been recoded in GAMS and has been coded in different modeling languages. After about two decades of experience, it seems likely that the modeling output is correct even if the assumptions of the model are subject to debate.

The problems with complex models are illustrated with the example of the OECD GREEN model. When the MIT EPPA model was in the design phase, it was decided to begin with the GREEN model. GREEN was coded in C+ and the printed source codes were a large volume consisting of tens of thousands lines of code. It was found to be next to impossible to change model structure and design meaningful counterfactual scenarios. The MIT team decided to recode the model in GAMS in MPSGE, which required only a few pages of code. As the recoding proceeded, a problem with the price-determination algorithm was discovered and the GREEN results were never replicated in GAMS. It seems unlikely that the mistake would ever have been uncovered if it had not been recoded.

This example is not intended to disparage the GREEN model or to discourage more detailed models. Rather, it illustrates some of the difficulties that arise with the use of large computerized modeling structures. The difficulties of transferring and validating models are not uncommon for large computerized systems in a wide variety of fields. The lesson is that the major way in which large models can be tested and validated is through construction of alternative models by other research groups.

16.4.4 Positive versus normative models

One of the issues that pervades the use of IAMs is whether they should be interpreted as normative or positive. In other words, should they be seen as the recommendations of a central planner, a world environmental agency, or a disinterested observer incorporating a social welfare function? Or are they meant to be a description of how economies and real-world decision makers (consumers, firms, and governments) actually behave? This issue arises particularly in the analysis of the discount rate that we review in the Section 16.4.5.

For most simulation models, such as general circulation climate models, the interpretation is clearly that these are meant to be descriptive. The interpretation of optimization models is more complex, however. In some cases, the purpose is clearly normative. For example, the *Stern Review* represented an attempt to provide normative guidance on how to cope with the dangers raised by climate change. In other cases, such as baseline projections, these are clearly meant to be descriptive.

The ambiguity arises particularly because many models use optimization as a technique for calibrating market outcomes in a positive approach. This is the interpretation of "market mechanisms as maximization or minimization devices." The question was addressed in the MRG report chaired by Tjalling Koopmans, "The use of optimization in these models should be seen as a means of simulating, as a first approximation, the behavior of a system of interacting competitive markets" (MRG, 1978, p. 5, emphasis added).

This point was elaborated at length in the integrated assessment study of copper by Gordon *et al*. (1987, with minor edits to simplify and emphasis added):

> *We can apply this result to our problem of exhaustible resources as follows: if each firm is faced with the same market prices for its inputs and outputs, and if each firm chooses its activities so as to maximize the firm's discounted profits, then the outcome will be economically efficient. In more precise language, such an equilibrium will be economically efficient in the sense that (1) each firm will provide its share of the market at minimum discounted cost; and (2) the requirements of the market will be met by producers in a manner that satisfies total demand at minimum discounted total cost to society.*
>
> *Examining these two conditions, we see that our competitive equilibrium has indeed solved a minimization problem of sorts — it has found a way of providing the appropriate array of services at lowest possible costs. But this minimization is exactly the objective of a linear-programming*

> *problem as well. Consequently, we can mimic the outcome of the economic equilibrium by solving the LP problem that minimizes the same set of cost functions subject to the same set of technical constraints. Put differently, given the appropriate quantities of resources available and the proper demand requirements, by solving a cost-minimizing LP problem we can determine the equilibrium market prices and quantities for all future periods. We call this lucky analytical coincidence the correspondence principle:* determining the prices and quantities in a general economic equilibrium and solving the embedded cost-minimization problem by linear programming are mathematically equivalent.

This discussion implies that we can interpret optimization models as a device for estimating the equilibrium of a market economy. As such, it does not necessarily have a normative interpretation. Rather, the maximization is an algorithm for finding the outcome of efficient competitive markets. Particularly if the distribution of endowments across individuals, nations, or time is ethically unacceptable, then the "maximization" is purely algorithmic and has no compelling normative properties.

This approach has another subtle requirement that is often overlooked. To use optimization as a solution technique, it is necessary that the solution cannot be improved when there are zero external effects. For example, with zero externalities, it should not be possible to improve the outcome by changes in savings rates or energy use. If this condition is not met, then the solution to the optimization *with* externalities may find an incorrect policy that arises from the initial deviation of the solution from the optimum rather than from appropriately responding to the externality.

This point is shown in Figure 16.12. We show two objective functions, $W(u)$ and $V(u)$, which are functions of an environmental control variable, u. The function $W(u)$ assumes that there is no externality (no damages from greenhouse gases in our example); while $V(u)$ assumes there is an externality, so the market solution produces an inefficient outcome. In the $W(u)$ case, all spillovers are internalized, while in the $V(u)$ case some spillovers are not internalized. In the no-externality case of $W(u)$, an appropriate algorithm would find the optimum at $u = 0$ (reflecting the efficiency of the market equilibrium). With a negative externality, as in the case of $V(u)$, an increase in the control variable would initially increase economic welfare. Hence, $W'(0) = 0$ while $V'(0) > 0$.

A correctly calibrated model would ensure that $W'(0) = 0$, i.e. the algorithmic maximum would come at $u = 0$. If the model is incorrectly calibrated, so that $W'(0) \neq 0$, then the value of the policy for externality correction is incorrectly calculated. Put differently, the model should find that welfare is unaffected by changes in optimized variables (such as changes in mitigation in a situation where mitigation perfectly internalizes the externalities). Such a mistake might occur if the model were constructed with parameters that implied that the competitive equilibrium was not optimal. This might lead to an incorrect global warming policy to correct a non-global-warming defect. This illustrates why in the DICE/RICE models, the models are calibrated without damages so that $W'(0) = 0$ for all control variables.

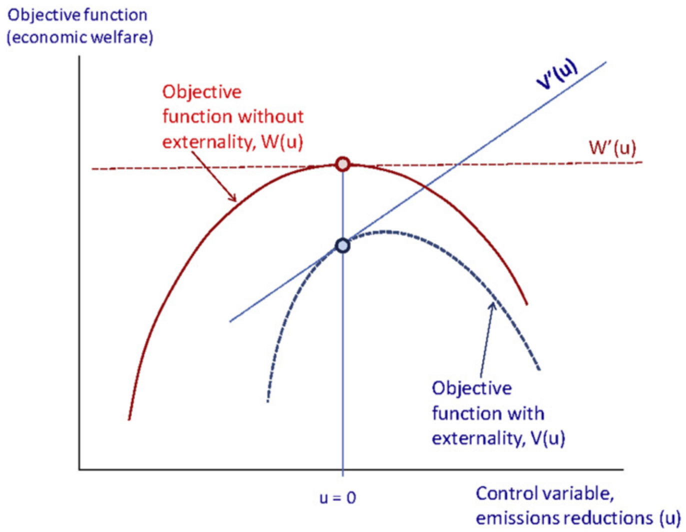

Figure 16.12 Marginal value of control variable is zero in optimization IAM with zero damages. Economic models should be designed like *W*(*u*) with no externalities. This implies that objective function (say, economic welfare) is maximized at zero level of the control variable when all market failures are corrected. If an uncorrected externality is added, as in *V*(*u*), then a change in the control variable will increase objective function and economic welfare.

16.4.5 Discount rate

Controversies involving the discount rate have been central to global warming models and policy for many years. The economic theory of discounting, which has been a relatively obscure topic in public finance and project analysis, assumes great prominence in climate change IAMs because of the long delays between investments in abatement and returns in averted damages. However, notwithstanding the extensive discussions, discounting is just as contentious as it was at the dawn of the studies in this area. I will review some of the issues in this context.

Discounting involves two related and often confused concepts. One is the idea of a *discount rate on goods*, which is a market-based concept that measures a relative price of goods at different points of time. This is also called the real return on capital, the real interest rate, the opportunity cost of capital and the real return. The real return measures the yield on investments corrected by the change in the overall price level. In principle, this is observable in the marketplace, although the exact numbers differ on the risk characteristics of the return involved. For example, the real return 10-year US Treasury securities over the period 1960–2000 averaged 3.0% per year. Similarly, the real pretax return on US corporate capital (a risky investment) over the same four decades has averaged about 6.6% per year. Estimated real returns on human capital range from 6% per year to more than 20% per year depending upon country and time period. In the studies used for the DICE/RICE models, I generally use a benchmark real return on capital of around 6% per year. Since taxes are excluded in the DICE/RICE models, this is the real discount rate on consumption as well.

The second important discount concept involves the relative weight of the economic welfare of different households or generations over time. This is called the pure rate of social time preference, but I will denote it the *generational discount rate* for clarity. It is calculated in percent per unit time, like an interest rate, but refers to the discount in future welfare, not in future goods or dollars. A zero generational discount rate means that future generations into the indefinite future are treated symmetrically with present generations; a positive generational discount rate means that the welfare of future generations is reduced or "discounted" compared to nearer generations. Philosophers and economists have conducted vigorous debates about how to apply generational discount rates in areas as diverse as economic growth, climate change, energy policy, nuclear waste, major infrastructure programs such as levees and reparations for slavery (see, e.g. Arrow *et al.*, 1996; Portney and Weyant, 1999).

While the concept of discounting is a very broad philosophical and ethical question, most analyses of the discounting issue in the economic and IAM literature use the approach of the Ramsey–Koopmans–Cass model of optimal economic growth (Ramsey, 1928; Cass, 1965; Koopmans, 1965). This is precisely the model of growth underlying the DICE model and will be used in this discussion. The major point to recognize is that the economic units in the economy are generations or cohorts. Similarly, the key parameters are α (the elasticity of utility with respect to a generation's consumption, or consumption elasticity) and ρ (the generational discount rate). We suppress the details of the decision making of the generation such as the time profile of consumption, lifespan, working and leisure, as well as individual preferences such as personal risk aversion and time preference as distinct elements not specifically related to the social choices.

One of the major confusions about discounting is whether the variables apply to the welfare of different generations or to individual preferences. In the DICE/RICE framework, the relationships emphatically concern generations. The individual rate of time preference, risk preference, and utility functions do not enter directly into the concepts. An individual may have high time preference, or perhaps hyperbolic discounting, but this has no necessary connection with how social decisions should weight different generations. Similar cautions apply to the consumption elasticity, which relates to the social valuation of inequality across different generations and not to individual risk preference.

Optimizing the social welfare function with a constant population, no risk or taxes, and a constant rate of growth of consumption across different generation, $g*$, yields the standard equation for the relationship between the equilibrium real return on capital, $r*$, and the other parameters, $r* = \rho + \alpha g*$. This is usually called the Ramsey Equation.

The Ramsey Equation shows that in a welfare optimum under simplified conditions, the rate of return on capital is determined by the generational discount rate, the consumption elasticity and the rate of growth of generational *per capita* consumption.

There are two ways of using the Ramsey equation as a framework for discounting in global warming or other long run questions. One is the *prescriptive view*, in which analysts argue for particular values of the ethical parameters, ρ and α, and from this derive the ethically appropriate discount rate on goods. This is the approach taken in Cline (1994) and the *Stern Review* (2007). The latter argues that it is indefensible to make long-term decisions with a positive generational discount rate: "[Our] argument … and that of many other economists and philosophers who have examined these long-run, ethical issues, is that [a positive generational discount rate)] is relevant only to account for the exogenous possibility of extinction." The generational discount rate used in the *Stern Review* is 0.1% per year, which is justified by estimates of the probability of extinction. The *Stern Review* further assumes a consumption elasticity of $\alpha = 1$ and a long-run growth rate of $g* = 1.3\%$ per year, which leads to a real interest rate (discount rate on goods) of 1.4% per year. A similar approach was endorsed by Cline (1994).

A second approach is the *descriptive approach*, advocated by Lind and Ruskin (1982), Lind (1995), and Nordhaus (1994a), and which is the approach in the DICE/RICE models. This approach assumes that investments to slow climate change must compete with investments in other areas. The benchmark for should therefore reflect the opportunity cost of investment. If we interpret the IAMs in the framework of "markets as maximization simulations" as discussed above, then the real interest rates in the model (as with other prices and outputs) are calculated to reflect market prices. In this interpretation, there is no ethical presumption that these are the correct prices or interest rates, but they should reflect market realities. It is inefficient, in the descriptive view, to accept investments in climate mitigation with a yield of 1.4% per year if there are available investments in education or capital with yields of 6% per year.

The need to consider opportunity costs can be seen starkly in considering the appropriate discount rate to use in 2011 in countries that are severely constrained such as Greece or Spain. It hardly seems appropriate to use the idealized normative approach for a country that cannot even finance its schools and public services.

In the descriptive view, the relevant equation is still the Ramsey Equation, but the primitives are the rate of return ($r*$) and the growth rate ($g*$), and the other two parameters must be calibrated to be consistent with observed market realities. The calibration for DICE-2010 is slightly different from these equilibrium calculations because of population growth and changing consumption growth, but we can use the equilibrium calculations to give the flavor of the results. In the baseline empirical model, I adopt a generational discount rate of 1.5% per year with a consumption elasticity of 1.5. These yield an equilibrium real interest rate of 5% per year with the consumption growth that is projected over the next century by the model.

Most of the debate about discounting has concentrated on the ethical concerns with using a positive generational discount rate. However, the goods discount rate in the normative Ramsey framework is determined by two ethical parameters and one economic parameter. If we start with the fundamental Ramsey Equation, $r* = \rho + \alpha g*$, note that we have two observable parameters (r and g) and two unobservable ethical parameters (ρ and α). A low real interest rate in the prescriptive view cannot be justified by a zero generational discount rate alone, but also depends upon the consumption elasticity, the growth rate of consumption, and in a world of non-zero population growth that influence it as well. Similarly, observations on the real interest and growth rates are insufficient to determine the generational discount rate in the descriptive view. In both, there is one free parameter. This implies that they are observationally equivalent in a steady state. This point has been widely ignored in the debates over discounting.

The point about the insufficiency of generational discounting alone can be illustrated using the DICE model calibration. Table 16.13 shows how different combinations of α and ρ lead to alternative real interest rates and climate change policies as reflected in the SCC. The top line shows the standard DICE/RICE calibration, which leads to a SCC of $55 per ton carbon and a real interest rate of 5.3% per year. The *Stern Review* assumptions in the DICE model lead to a much higher SCC, but also a much lower interest rate. If we use the low generational discounting and calibrate to match the DICE model real interest rate, the required consumption elasticity needed to calibrate goes from the DICE value of $\alpha = 1.5$ to around $\alpha = 2.0$. The SCC rises in the recalibration, but note that it is less than 1/10th of the values implicit in the *Stern Review*'s low generational discount *and* low consumption elasticity. The main point of this example is that the generational discount rate is not sufficient to determine the goods discount rate. The exact calibration can have a major impact on the SCC and therefore on the optimal climate policy. Table 16.13 shows as well how sensitive policy is to the discounting assumptions.

Table 16.13 Alternative assumptions for discounting calibration

Model	Consumption elasticity	Generational discount rate	Social cost of carbon, 2015 (2005 $)	Real return on capital (2015)
DICE/RICE	1.5	0.015	55	5.3
Stern Review	1.0	0.001	1518	2.6
	1.5	0.001	381	3.9
Recalibrated	2.0	0.001	125	5.2
RICE/DICE	2.5	0.001	86	6.5
	3.0	0.001	52	7.8

Estimates of the SCC and the real return on capital are from the DICE-2010 model, which differ slightly from RICE-2010 model. For description, see text.

One further complication is the need to consider risk in the context of climate investments. Should the discount rate on abatement include a risk premium and if so how large should it be? This issue has been largely ignored in the IAM literature. If we assume that the equity premium is determined in markets as a systematic and justifiable premium on risky assets, then we would need to investigate the risk characteristics of investments in climate change. This is the subject of a vast literature on the consumption capital asset pricing model and the equity premium (see Mehra, 2008). Analysis in Nordhaus (2008, 2011b) indicates that the returns on abatement investments share the risk characteristics of consumption, so it would appear that a discount rate appropriate to risky investments would be appropriate for abatement investments. In other words, the discount rate for climate investments should include the equity premium.

The ethical attractiveness of very low generational discounting continues to dominate the debate on the appropriate discount rate in climate change. There is little *a priori* appeal to approaches that explicitly discriminate against future generations. However, research on the properties of zero discounting has uncovered deep paradoxes that remain unanswered today (Koopmans, 1965). Zero discount rates lead to incomplete preference structures. The paradox of low discounting can be illustrated with a "wrinkle experiment." Suppose that scientists discover a wrinkle in the climate system that will cause damages equal to 0.1% of net consumption starting in 2200 and continuing at that rate forever after. How large a one-time investment would be justified *today* to remove the wrinkle that starts only after *two centuries*? Using a near-zero discount rate of the kind proposed by the *Stern Review*, the answer is that we should pay a substantial fraction of a year's consumption today to remove the wrinkle (see Nordhaus, 2008).

This result is a reminder of the warning Tjalling Koopmans made in his path breaking analysis of discounting in growth theory half a century ago: "[T]he problem of optimal growth is too complicated, or at least too unfamiliar, for one to feel comfortable in making an *entirely a priori* choice of [a generational discount rate) before one knows the implications of alternative choices" (Koopmans, 1965). This conclusion applies with even greater force in global warming models, which have much greater complexity than the simple, deterministic, stationary models that Koopmans analyzed.

16.4.6 Uncertainty for thin-tailed distributions

If global warming is the mother of all public goods, it may also be the father of decision making under uncertainty. In terms of a model structure, every equation (except for the identities) contains major unresolved questions. Among the important uncertainties are the pace of economic growth in different regions, the damages in different regions, the pace at which developing countries move their labor forces and economies out of agriculture, future tastes for environmental goods and services, and the potential for competitive, low-carbon energy source. There are major differences among scientists and economists on the answers to these questions, and it seems fair to conclude that there are unlikely to be definitive answers in the next few years. Moreover, we do not know how fast these uncertainties will be resolved or what kinds of investments in learning would help resolve the uncertainties.

Since we cannot resolve these issues about the deep future, we can instead focus our research on what is after all the relevant question for environmental and economic policies − the impact of uncertainties on near-term policies (such as the control rate or the optimal tax on greenhouse gases). It is surely the case that we have a very imprecise estimate of recoverable carbon fuels and can estimate this only within an order of magnitude. Suppose, however, that uncertainty about the total fossil fuel resources has little influence on near-term policies. Then we can say that this variable is a less important uncertainty than one, say like the economic growth rate, which has a major effect on near-term policies.

There is a substantial literature on uncertainty in climate change. Major studies include Manne and Richels (1992), Peck and Teisberg (1993), Nordhaus and Popp (1997), Nordhaus and Boyer (2000), Webster (2002), and Hope (2006). IAMs differ in their approach to uncertainties. Some models (including the basic DICE and RICE models) take the first step of analyzing the economics of global warming under the assumption of perfect foresight or certainty equivalence. A certainty-equivalent approach provides the basic intuition about the economics of alternative approaches. It also provides a first approximation to a complete answer under certain conditions (such as where risk aversion is relatively low, functions are relatively linear, or risks are relatively small). Other models (such as the PAGE model as in Hope, 2006) emphasize uncertainties and spend considerable effort in modeling the structure of uncertainties.

For the present survey, it is useful to distinguish second-moment uncertainty (which is the subject of the present section) from higher-moment uncertainty (which is discussed in Section 16.4.6). Second-order uncertainty examines the impact of the second moment of distributions (dispersion around the mean) assuming that the distributions are normal or close to normal. This can be described as "thin-tail uncertainty" in the sense that it examines the effects of uncertainty assuming that the tails of the distribution do not dominate the effects of uncertainty. The most recent comprehensive study for the DICE model (Nordhaus, 2008) examined the implications of uncertainty about eight major variables on the optimal climate change policy and other variables assuming that the distribution of the variables was normal. This study found that the impact of second-order uncertainty was relatively small. In other words, best-guess or certainty-equivalent policy seemed a good approximation for the policy in which a full expected-utility framework is used. This finding is consistent with findings of other studies (see Cropper, 1976; Kolstad, 1996; Pizer, 1999; Yohe and Tol, 2010). However, as we will see in the next section, introducing non-linearities and more extreme parameter values can lead to completely different results.

One reservation to these results is that the studies in which thin-tailed uncertainty is relatively unimportant usually have structures in which there are no sharp discontinuities. If discontinuities are both sharp and relatively near-term, then this conclusion may not hold (see, e.g. Baranzini *et al.*, 2003).

16.4.7 Higher-moment uncertainty ("fat tails") and catastrophic climate change

Recent research has emphasized the issue of the potential for "fat tails" in the distribution of uncertain parameters and the risk of catastrophic climate change. We label this "higher-moment uncertainty" to indicate that it is concerned with "tail events." The issue arises because of the combination of outcomes that are potentially catastrophic in nature and have probability distributions with fat tails. The combination of these two circumstances may lead to situations in which our standard analyses needs to be modified or may even break down.

A tail event is an outcome that, from the perspective of the frequency of historical events and normal probability distributions, should happen only extremely infrequently. Statisticians have known for many years that events with fat tailed distributions may behave in an unintuitive way. Relatively little work has examined the implications of fat tails for economic modeling and policy. In a recent series of papers, Martin Weitzman (see especially Weitzman, 2009) has proposed a dramatically different conclusion from standard analysis in what he has called the Dismal Theorem. In the extreme case, the combination of fat tails, unlimited exposure and high risk aversion implies that the expected loss from certain risks such as climate change is infinite and we therefore cannot perform standard optimization calculations or cost–benefit analyses.

There has been virtually no work applying Weitzman's insights in empirical IAMs. The question is particularly demanding because it requires estimating the shape of the tails of distributions for events, such as damages to future consumption, where there is very sparse experience on which to estimate the distribution. Some analysts have used theoretical approaches (see Heal, 2008) or highly stylized models (see Weitzman, 2010).

A slightly different way of posing the issue is to examine the conditions under which extreme parameter values might produce catastrophic outcomes for climate change using a standard IAM. This section, based on Nordhaus (2011a, 2012), sketches such an approach. Begin with a definition of "catastrophic climate change." In this discussion, I define a catastrophic outcome as one in which world *per capita* consumption declines at least 50% below current levels for an extended period. This would represent a decline of at least 90% below a reference level for most assessments (*Stern Review*, 2007; Nordhaus, 1994b, 2010; and EMF-22). So by catastrophic, we mean damages from climate change far larger than what is envisioned in the direst of current IAM projections.

Designing scenarios which might lead to extreme outcomes is in itself a major research task. An important study by Frank Ackerman, Elizabeth A. Stanton and Ramón

Bueno (Ackerman *et al.*, 2010) shows the complicated interaction of catastrophes, fat tails and empirical analysis. They use a variant of the DICE-2007 model and examine distributions for two parameters, the TSC and the damage function exponent (DFE). The TSC is a function of the parameters in Equation (16.17) above, while the DFE is the coefficient "2" on the last term of Equation (16.6) above.

Their results show an interesting feature. They found that that uncertainty about either the temperature sensitivity or the damage exponent alone has little effect on the optimal abatement strategy. However, if both of the parameters take high values, then there is potential for catastrophic outcomes, and relying on the best-guess parameters can be very misleading. In extreme cases, very sharp increases in mitigation are necessary to prevent a major economic decline.

I have followed the approach used by Ackerman *et al.* (2010) to illustrate their points. Based on existing studies and several DICE model experiments, I settled on the following three conditions as important ingredients for producing extreme outcomes. A first condition is that the economic and geophysical systems lead to large climatic changes in the absence of effective policy measures. As in other studies, the simulations below examine a high temperature sensitivity coefficient as an example of unfavorable climatic conditions. A second ingredient is the potential for catastrophic damages at levels of climate change that might arise from the first condition. Most damage functions in the climate change literature would not lead to catastrophic damages as defined here for large temperature changes. A damage function that has sharp threshold effects would be required to lead to catastrophic outcomes.

A final requirement is a policy failure. This means either that scientists fail to understand the nature of the climate—society system in a timely fashion or that societies fail to take steps to reduce the threat of catastrophic climate change. If the threat is understood, then there seems little doubt that it is technologically and economically possible to reduce emissions to essentially zero in a short time period; the costs might be large, but would not be ruinous. In addition, we will examine the role of the generational discount rate because low discount rates have often been justified by the possibility of catastrophic climate change.

To combine these possibilities, I took the DICE model as described above and considered "extreme values" in four areas: (i) a much higher temperature sensitivity coefficient (TSC = 10), (ii) high convexity of the damage function at a threshold of 3°C (exponent or DFE = 6 for "convex"), (iii) a policy failures represented by inability to take actions that will reduce emissions ("no policy") and (iv) a near-zero pure rate of time preference ("low discounting").

For each of the parameters, we consider a "base value," which is the one used in the standard DICE model, along with an "extreme value," which is represented by one of the four cases just described. We do not attach any probabilities to the extreme outcomes. Rather, these might be considered the realization of a process which had fat tails and in which the probabilities of these outcomes is non-negligible. Table 16.14 shows the parameters considered in the runs below. We make runs for 600 years with different combination of parameters and policy assumptions.

The results for salient variables are shown in Table 16.15. The first numerical row shows the SCC for 2015. This is a useful indicator of the damages from additional carbon emissions. The first five columns show the results of taking each of the extreme values of the parameters *with policy*. The SCC ranges from \$42 per ton of carbon (\$/tC) in the standard case to \$350 in the most unfavorable case. The impact on economic welfare is large but not catastrophic, with a decline of around 2% of welfare or consumption annuity in the worst case. (The consumption annuity is the constant level of *per capita* consumption that gives the same level of utility as the case in question.).

The cases *without policy* are shown in the last four columns of Table 16.15. As in Ackerman *et al*. (2010), *either* high TSC *or* steep damage *plus* no policy are not sufficient to lead to the catastrophic results. High damages plus no policy (with a tipping point of 3°C) does lead to a very steep loss. However, the genuinely catastrophic results, in the sense used here, require all three conditions: high TSC, high convexity of the damage function and no policy, as shown in the last column. When all three of these conditions are met, the consumption annuity declines 96% relative to the baseline. The catastrophic nature of the extreme values is signaled by an initial SCC that is more than \$5100 per ton of carbon (this being indicative but unreliable because of computational difficulties).

A further important comparison is the column labeled "1 + 3 + 4" with "2 + 3 + 4." This shows the importance of policy to avoid the catastrophic outcomes where all parameters take their extreme value. Note as well that according to the DICE model structure, the world is not yet *irreversibly* on course for a catastrophic outcome even with the most unfavorable parameters. In all cases examined, a vigorous mitigation policy is able to prevent the world from going over the catastrophic threshold.

Table 16.14 Parameters in standard DICE-2010 runs and extreme values

Parameters	Base value	Extreme value
TSC[a]	3	10
Convex damage component[b]		
Intercept	0	0.1
Exponent	0	6
Tipping point (°C)	none	3
Policy begins[c]	2015	2255
Pure time discount rate[d]	0.015	0.001

[a]Equilibrium response of global mean temperature to a doubling of atmospheric CO_2 concentrations (°C).
[b]Term added to the DICE damage function that has tipping point at specified temperature increases.
[c]Indicates that there are no controls until that date, then controls are optimized after that date.
[d]Pure rate of social time preference per year.

Table 16.15 Results of alternative extreme values of parameters in DICE model

Variable	1 (Optimal)	1+3 (TSC=10 with policy)	1+4 (High damage with policy)	1+3+4 (All extreme with policy)	1+5 (Base parameters with low discounting with policy)	2 (Base parameters with no policy)	2+3 (TSC=10 with no policy)	2+4 (High damage with no policy)	2+3+4 (All extreme with no policy)
SCC 2015 (2005 $/tC)	42	92	80	350	102	44	105	551	5100
Per capita consumption (2005 $)									
Average 2000–2200	50,338	48,898	50,373	47,534	50,752	48,872	43,254	26,091	5966
Minimum 2000–2200	6801	6799	6799	6796	6799	6800	6800	6800	179
Consumption annuity *per capital*[c]									
Thousands 2005 $	17,765	17,641	17,723	17,441	—[b]	17,718	17,422	15,803	634
Percent decline (%)		0.7	0.2	1.8	—[b]	0.3	1.9	11.0	96.4
Objective function (consumption equivalent)									
Trillions 2005 prices	1391.1	1381.3	1387.8	1365.2	—[b]	1387.4	1363.7	1218.3	—[a]
Difference from optimal		−9.8	−3.3	−25.8	—[b]	−3.7	−27.3	−172.8	—[a]
Percent decline (%)		0.7	0.2	1.9	—[b]	0.3	2.0	12.4	—[a]

Cases:
1: Optimal Policy from 2015.
2: Hotelling rents on carbon until 2255, then optimal policy.
3: TSC = 10°C per CO_2 doubling.
4: Catastrophic damages at tipping point of 3°C.
5: Social discount rate at 0.1% per year.
[a]This value is a large negative because of non-linear objective funtion.Refer to consumption annuity.
[b]This value is not comparable to other runs because the discount rate is different from standard Cases.
[c]The consumption annuity is the level of constant consumption that yields the same discounted utility as the case under consideration.

These results are only suggestive because they examine only a small part of parameter space. They suggest, first, that none of the extreme parameter values taken singly produces catastrophic outcomes. Additionally, as long as there are no policy failures and mitigation policies are taken quickly for the catastrophic cases, no combination of extreme values examined here is sufficient to lead to catastrophic outcomes. Third, discounting is a second-order issue in the context of catastrophic outcomes. A high discount rate will slow mitigation, but it does not by itself produce policies that would lead to future catastrophes. If the future outlook is indeed catastrophic, and if that situation is understood, and if policies are taken, the discount rate has little effect on the estimate of the SCC or on the optimal mitigation policy.

This leads to the fourth and major finding of our investigation: all of the three extreme conditions must hold to obtain the catastrophic outcome. That is to say, there must be high temperature sensitivity *plus* catastrophic damages *plus* policy failure. The intuition is that a high TSC produces a steep temperature trajectory. The steep temperature trajectory produces catastrophic damages when the damage function is extremely convex. However, to these we must add that countries do not take steps to prevent the chain of catastrophic events.

In the end, the major result is the importance of "policy." As long as policy does not fail, the world economy can avoid catastrophic outcomes. However, we should not think of policy in a mechanical fashion as simply turning an emissions-control dial to the appropriate level and then going about the rest of our daily lives. Rather, policy involves a continuing series of difficult steps. It requires understanding the complicated geophysical and socioeconomic dynamics of climate change and economic growth over many decades; it requires solving the global public goods problem by gathering most nations together to take collective action; and it means designing a mechanism for ensuring that emissions-control policies are reasonably efficient and effective. None of these is easily accomplished, but taken together they are sufficient to overcome a set of outcomes that would otherwise be catastrophic for the human condition.

The issues raised by fat tails, or by extreme values and catastrophic outcomes, is one of the most difficult open questions in climate change science, economics and policy. The nature of extreme outcomes – both their rarity and their extremity – implies that it is virtually impossible to have a secure understanding of the likelihood and severity of extreme events.

16.4.8 Strategic considerations and the game-theoretic aspects of climate change policy

One of the central issues in climate change policy is the fact that it involves many countries for many time periods. No single country or generation can reduce emissions sufficiently to ensure that there are no dangerous interferences with the climate system.

The calls for cooperation and for meeting ambitious targets collide with the incentives of individual countries and generations. While current policy is often called myopic, a more appropriate diagnosis is that the world is locked into a non-cooperative equilibrium with no effective mechanism to break out.

A first issue arises because of the strategic relationship between costs of abatement (which are national) and avoidance of climate damage (which is a widely dispersed Samuelsonian public good). This structure of local costs and dispersed benefits leads to strong incentives to free riding: each country has little incentive to take action and will benefit greatly if everybody else abates.

This situation is analyzed using the concept of a Nash non-cooperative equilibrium from a game theory. A Nash non-cooperative equilibrium results when no player can find a strategy to improve its payoff assuming that the other players stick to their strategies (Nash, 1950). A Nash non-cooperative equilibrium does not rule out any climate change policies. Rather, non-cooperative behavior implies that countries take abatement actions only to the extent that they themselves benefit and the benefits to the rest of the world are not included.

There is a substantial literature investigating the nature of climate change policy using the non-cooperative framework (Carraro and Siniscalco, 1993; Barrett, 1994; Chander and Tulkens, 1995; Nordhaus and Yang, 1996; Peck and Tiesberg, 1999). We can use the RICE-2010 model to illustrate how a non-cooperative equilibrium can be calculated. This is achieved by assuming that each region maximizes its objective function taking into account only its own costs and damages. By assumption, there is no signaling or cooperation.

Earlier studies have found that a Nash non-cooperative equilibrium would lead to carbon prices and emissions reductions that are much lower than optimal (see particularly Nordhaus and Yang, 1996; Nordhaus, 2010). Similar results are found in RICE-2010. If we assume each of the 12 regions acts non-cooperatively, carbon prices are calculated to be approximately one-tenth of the efficient levels (see Table 16.12). This may actually overstate non-cooperative abatement because it assumes that countries within large regions such as Latin America coordinate their strategies. The strategic significance of this finding is 2-fold. First, the overall level of abatement in the non-cooperative equilibrium will be much lower than in the efficient (cooperative) strategy. A second and less evident point is that countries will have strong incentives to free ride by not participating, or not to comply fully with strong climate change agreements if they do participate. If they hide emissions or overstate reductions, their own economic welfare will improve even though others' welfare will deteriorate. This second point is seen in the Kyoto Protocol, where it seems likely that many countries outside of the EU will end up exceeding their allowable emissions.

The difficulty of escaping from a low-level non-cooperative equilibrium is amplified by yet another factor, the intertemporal tradeoff. The non-cooperative equilibrium shown in Table 16.12 may overstate the degree of cooperation because of the intertemporal structure of costs and benefits. Climate change policies require costly abatement in the near term to reduce damages in the distant future. The generational trade-off is shown in Table 16.9, which shows the intertemporal results for the Copenhagen Accord. The last line shows the difference in global discounted damages and discounted abatement costs through 2055 between the outcome under the Copenhagen Accord and that in the baseline scenario. Abatement costs are more than 5 times the averted damages in this early period. For the period after 2055 (not shown), however, the ratio is reversed: averted damages are more than 4 times abatement costs. If the players are generations for each nation rather than nations, then the non-cooperative equilibrium will lead to close to zero abatement because virtually all the benefits lie outside the lifetimes of a given generation. The delayed payoffs reinforce the incentives of the non-cooperative equilibrium, so the temptation is high to postpone taking costly steps to reduce emissions.

16.4.9 Modeling technological change

Most studies and models of environmental and climate change policy — indeed of virtually all aspects of economic policy — have sidestepped the thorny issue of endogenous technological change or induced innovation. These terms refer to the impact of economic activity and policy upon research, development, invention, innovation, and the diffusion of new technologies. Most IAMs assume that technological change is exogenous, that is, it proceeds with a rate and direction that is determined by fundamental scientific and technological forces, but is unaffected by higher carbon prices or tax and regulatory incentives.

This shortcoming has been recognized for many years (see, e.g. the discussion in the IAM review in Weyant *et al.*, 1996). The assumption of exogenous technological change is used both because of the lack of a firm empirical understanding of the determinants of technological change as well as because of the inherent difficulties in modeling economic processes with externalities and increasing returns to scale. While we suspect that we know the direction of the omission of induced innovation — to overestimate the cost of emissions reductions and the trend increase in climate change — IAMs have had difficulty assessing the magnitude of the effect or the importance of this omission. Would including induced innovation have a large or small impact on climate change and on climate change policies? This is a major open question.

There have been two approaches to including induced innovation — the research model and the learning model. The research model of induced innovation arose in the 1960s in an attempt to understand why technological change appears to have been largely labor saving. It emphasized that technological change is a public good that is produced by research, development and innovation (Nelson, 1959; Arrow, 1962). More recently, this approach has been integrated into a neoclassical economic growth theory in research such as Paul Romer (Romer, 1990). The thrust of the research model is to emphasize investment in knowledge-improving activities, where those activities have strong public goods qualities.

Virtually all studies of the research model have been theoretical. With few exceptions, they do not lay out a set of testable hypotheses or ones that can be used to model the innovation process at an industrial level. The difficulty of deploying well-specified and reliably estimated models has been a major impediment to widespread development of empirical models in the research-model tradition. There are but a handful of studies that incorporate the research-model approach. Work with the DICE model (Nordhaus, 2002), the ENTICE model (Popp, 2004) and the WITCH model (Bosetti *et al.*, 2009) have developed the research-model approach in the context of climate change. One of the major findings is that the omission of endogenous technological change has a major impact on welfare but has only a small effect on the temperature path or on the path of the optimal carbon price (Popp, 2004).

The alternative approach to modeling induced innovation is the learning by doing (LBD) model. This approach has become particularly widely used in recent years as models increase the granularity of the technological description down to individual technologies. It has also been attractive in policy studies because it can rationalize early investments in technologies that are presently uneconomical but have the promise, if they can "move down the learning curve," of being competitive in the future.

Models of learning and experience have a long history in studies of manufacturing productivity. Useful references in economics include Arrow (1961), and Jovanovic and Nyarko (1995), while a survey of the field can be found in Yelle (1979). As a result of their perceived successes in technological forecasting, they have recently been introduced in policy models of energy and global warming economics to make the process of technological change endogenous. There is a vast body of energy and climate change economics models using LBD; a useful survey is contained in Gillingham *et al.* (2008).

While LBD has the advantage of easy incorporation in models, it has serious dangers, as is recognized in Gillingham *et al.* (2008). They write:

> *The primary disadvantage to learning-induced TC [technological change] is its reduced-form nature. LBD can be inserted mechanically into many models, but it is difficult to identify the mechanisms behind LBD — or even be confident about the causality. Learning-induced TC does not have a theoretical structure analogous to the IPF [innovation possibility frontier, or the research model discussed above] on which R&D-induced TC is based. The ease with which learning curves can be estimated may give a false sense of comfort and precision that may belie the R&D or other resources that went into the technology development.*

In recent analyses on LBD in Nordhaus (2010), it is shown that there is a fundamental statistical identification problem in trying to separate learning from exogenous technological change and that the estimated learning coefficient (or learning curve slope) will generally be biased upwards.

This bias becomes particularly important in energy and global warming models which are designed to choose among different emerging technologies and where the technology is assumed to have an important learning component. For example, suppose that a policy calculation solves for future paths of solar and wind technologies based on current cost and different learning rates. Based on high learning rates, the model might suggest that a high-cost but immature technology is a good bet for research and development. However, this recommendation would be incorrect – that is, biased toward too high an investment in the rapid-learning technologies – if the learning coefficient is based on an upwardly biased estimate of learning rates.

The point to emphasize here is that, in analyses that pick technologies on the basis of total discounted cost of production (as is entirely appropriate), then an upward bias in the learning rate can have a major impact on the apparent benefit of technologies with learning outcomes. This danger is significant because of the tendency to estimate learning rates in bivariate relationships, which will generally lead to strong upward biases in the learning coefficient.

There is clearly much room for further development of models with endogenous technological change, but we should not underestimate the difficulties involved.

16.5 FINAL THOUGHTS

The present survey of IAMs of climate change shows the enormous progress that the field has made over the two decades since its emergence. The progress is made possible by the parallel developments in fundamental science and economics across a broad range of areas. These include development in public economics, game theory and environmental economics. However, development of the actual models has required improvement in computer hardware, software, algorithms, improved data, and the ability to access information and exchange ideas across long distances.

Perhaps the single most important set of results from IAMs has been the concepts and estimation of efficient paths of abatement and carbon pricing required for slowing climate change. There was essentially no awareness of the importance of carbon pricing two decades ago, and few would have hazarded an estimate of the appropriate carbon price. Today, in part because of developments in IAMs, carbon prices and estimates of the SCC are actually integrated into the regulatory decisions of major countries.

Looking forward, there is clearly much work remaining for modelers. Many of the topics discussed in Section 16.4 require further refinement and better modeling, particularly in issues surrounding uncertainty, technological change, and the need for mechanisms to break the non-cooperative trap of climate policy that is gripping the globe. There is much fruitful work that remains for future researchers.

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REFERENCES

Ackerman, F., Stanton, E.A., Bueno, R., 2010. Fat tails, exponents, extreme uncertainty: Simulating catastrophe in DICE. Ecol. Econ. 69, 1657–1665.

Arrow, K.J., 1962. Economic Welfare and the allocation of resources for invention. In: Nelson, R. (Ed.), The Rate and Direction of Inventive Activity. Princeton University Press for NBER, Princeton, NJ.

Arrow, K.J., Cline, W.R., Maler, K.-G., Munasinghe, M., Squitieri, R., Stiglitz, J.E., 1995. Intertemporal equity, discounting, and economic efficiency. In: Bruce, J., Lee, H., Haites, E. (Eds), Climate Change 1995: Economic and Social Dimensions of Climate Change, Contribution of Working Group III to the Second Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge, pp. 125–144.

Baranzini, A., Chesney, M., Morisset, J., 2003. The impact of possible climate catastrophes on global warming policy. Energy Policy 31, 691–701.

Barrett, S., 1994. Self-enforcing international environmental agreements. Oxf. Econ. Pap. 46, 878–894.

Barrett, S., 2003. Environment and Statecraft: The Strategy of Environmental Treaty-Making. Oxford University Press, Oxford.

Bosetti, V., Carraro, C., Tavoni, M., 2009. Climate change mitigation strategies in fast-growing countries: the benefits of early action. Energy Econ. 31, S144–S151.

Brooke, A., Kendrick, D., Meeraus, A., Raman, R., 2005. GAMS: A users guide. GAMS Development Corporation, Washington, DC.

Carraro, C., Siniscalco, D., 1993. Strategies for the international protection of the environment. J. Public Econ. 52, 309–328.

Cass, D., 1965. Optimum growth in an aggregative model of capital accumulation. Rev. Econ. Stud. 32, 233–240.

Chander, P., Tulkens, H., 1995. A core-theoretical solution for the design of cooperative agreements on trans-frontier pollution. Int. Tax Public Finan. 2, 279–294.

Clarke, L., Böhringer, C., Rutherford, T.F., 2009. International, US and EU climate change control scenarios: results from EMF 22. Energy Econ. 31 (2), S63–S306.

Cline, W., 1992. The Economics of Global Warming. Institute for International Economics, Washington, DC.

CONAES, 1980. Energy in Transition, 1985–2010: Final Report of the Committee on Nuclear and Alternative Energy Systems. National Research Council, Washington, DC.

Cropper, M., 1976. Regulating activities with catastrophic environmental effects. J. Environ. Econ. Manag. 3, 1–15.

EPPA, 2011. Economics, Emissions, and Policy Cost – The EPPA Model available at. http://globalchange.mit.edu/igsm/eppa.html.

Epstein, L., Zin, S., 1989. Substitution, risk aversion and the temporal behavior of consumption and asset returns: a theoretical framework. Econometrica 57, 937–969.

Epstein, L., Zin, S., 1991. Substitution, risk aversion and the temporal behavior of consumption and asset returns: an empirical analysis. J. Polit. Econ. 99, 263–288.

Gillingham, K., Newell, R.G., Pizer, W.A., 2008. Modeling endogenous technological change for climate policy analysis. Energy Econ. 30, 2734–2753.

Gordon, R.B., Koopmans, T., Nordhaus, W., Skinner, B., 1987. Toward a new Iron Age? Quantitative modeling of resource exhaustion. Harvard University Press, Cambridge, MA.

Goulder, L., 2002. Environmental Policy Making In Economies With Prior Tax Distortions. Edwin Elgar, Cheltenham.

Greenstone, M., Kopits, E., Wolverton, A., 2011. Estimating the social cost of carbon for use in US federal rulemakings: a summary and interpretation. NBER Working Paper 16913. NBER, Cambridge, MA.

Hansen, J., Sato, M., Ruedy, R., Lo, K., Lea, D.W., Medina-Elizade, M., 2006. Global temperature change. Proc. Natl. Acad. Sci. USA 103, 14288–14293.

Hope, C., 2006. The marginal impact of CO_2 from PAGE2002: an integrated assessment model incorporating the IPCC's five reasons for concern. Integrated Assess. 6, 19–56.

Hotelling, H., 1931. The economics of exhaustible resources. J. Polit. Econ. 39, 137–175.

IGEM, 2011. Documentation for Intertemporal General Equilibrium Model (IGEM) available at. http://www.igem.insightworks.com/.

IIASA World Population Program, 2007. Probabilistic Projections by 13 world regions, forecast period 2000–2100, 2001 Revision. Available at http://www.iiasa.ac.at/Research/POP/proj01/.

International Monetary Fund, 2007. World Economic and Financial Surveys, World Economic Outlook Database. IMF, Washington, DC. Available at http://www.imf.org/external/pubs/ft/weo/2006/02/data/index.aspx.

International Monetary Fund, 2009. World Economic and Financial Surveys, World Economic Outlook Database. IMF, Washington, DC.

IPCC, 1996. Climate Change 1995 – Economic and Social Dimensions of Climate Change. Cambridge University Press, Cambridge.

IPCC, 2000. Special Report on Emissions Scenarios. Cambridge University Press, Cambridge. Available at http://www.grida.no/publications/other/ipcc_sr/?src=/climate/ipcc/emission/.

IPCC, 2001. Contribution of Working Group I to the Third Assessment Report of the Intergovernmental Panel on Climate Change. Climate Change 2001: The Scientific Basis. Cambridge University Press, Cambridge.

IPCC, 2007a. Climate Change 2007: The Physical Science Basis, Contribution of Working Group I to the Fourth Assessment Report of the IPCC. Available at http://ipcc-wg1.ucar.edu/wg1/wg1-report.html.

IPCC, 2007b. Climate Change 2007: Impacts, Adaptation and Vulnerability, Working Group II Contribution to the Intergovernmental Panel on Climate Change. Summary for Policymakers Available at http://www.ipcc.ch/.

IPCC, 2007c. Climate Change 2007: Mitigation of Climate Change, Working Group III Contribution to the Intergovernmental Panel on Climate Change. Available at http://ipcc-wg1.ucar.edu/wg1/wg1-report.html.

Kelly, D.L., Kolstad, C.D., 1999. Integrated assessment models for climate change control. In: Folmer, H., Tietenberg, T. (Eds), International Yearbook of Environmental and Resource Economics 1999/2000: A Survey of Current Issues. Edward Elgar, Cheltenham.

Kolstad, C.D., 1996. Learning and stock effects in environmental regulation: the case of greenhouse gas emissions. J. Environ. Econ. Manag. 31, 1–18.

Koopmans, T., 1963. On the concept of optimal economic growth. Cowles Foundation Discussion Papers. Yale University, New Haven, CT. Available at http://cowles.econ.yale.edu/P/cd/d01b/d0163.pdf.

Koopmans, T.C., 1965. On the concept of optimal economic growth. Academiae Scientiarum Scripta 28, 1–75. Available at http://cowles.econ.yale.edu/P/au/p_koopmans.htm.

Lee, R., 2011. The outlook for population growth. Science 333, 569–573.

Lind, R.C., 1995. Intergenerational equity, discounting, and the role of cost–benefit analysis in evaluating global climate policy. Energy Policy 23, 379–389.

Lind, R.C., Ruskin, F.R. (Eds), 1982. Discounting for Time and Risk in Energy Policy. Resources for the Future, Washington, DC.

Manne, A.S., 1962. Product-mix alternatives: flood control, electric power, and irrigation. Int. Econ. Rev. 3, 30–59.

Manne, A.S., 1974. Waiting for the breeder. Rev. Econ. Stud. 41, 47–65. Symposium on the Economics of Exhaustible Resources.

Manne, A.S., 1976. ETA: a model for energy technology assessment. Bell J. Econ. 7, 379–406.

Manne, A.S., 1985. On the formulation and solution of economic equilibrium problems. Math. Program. Study 23, 1–23.
Manne, A., Richels, R., 1991. Global CO_2 emission reductions; the impacts of rising energy costs. Energy J. 12, 87–108.
Manne, A.S., Richels, R.G., 1992. Buying Greenhouse Insurance: The Economic Costs of Carbon Dioxide Emission Limits. MIT Press, Cambridge, MA.
Manne, A.S., Richels, R.G., 1999. The Kyoto Protocol: a cost-effective strategy for meeting environmental objectives? Energy J., 1–24 (Special Issue: The Costs of the Kyoto Protocol: A Multi-model Evaluation).
Manne, A.S., Richels, R.G., 2001. US rejection of the Kyoto Protocol: the impact on compliance costs and CO2 Emissions. AEI-Brookings Joint Center Working Paper 01-12. AEI/Brookings Institution, Washington, DC.
Manne, A.S., Richels, R.G., 2004. MERGE: An Integrated Assessment Model for Global Climate Change. Stanford University, Stanford, CA. Available at http://www.stanford.edu/group/MERGE/GERAD1.pdf.
McKibbin, W.J., Pearce, D., Stegman, A., 2007. Long term projections of carbon emissions. Int. J. Forecast. 23, 637–653.
Mehra, R. (Ed.), 2008. The Handbook of the Equity Risk Premium. Elsevier, Amsterdam.
MRG, 1978. Report of the Modeling Resource Group of the Committee on Nuclear and Alternative Energy Systems: Energy Modeling for an Uncertain Future. National Academy of Sciences Press, Washington, DC.
Nash, J., 1950. Equilibrium points in *n*-person games. Proc. Natl. Acad. Sci. USA 36, 48–49.
National Research Council, 1992. Committee on Science, Engineering, and Public Policy, Policy Implications of Greenhouse Warming: Mitigation, Adaptation, and the Science Base. National Academy Press, Washington, DC.
National Research Council, Committee on Abrupt Climate Change, 2002. Abrupt Climate Change: Inevitable Surprises. National Academy Press, Washington, DC.
Nelson, R.R., 1959. The simple economics of basic scientific research. J. Polit. Econ. 67, 297–306.
NEMS, 2011. The National Energy Modeling System: An Overview. Available at http://205.254.135.24/oiaf/aeo/overview/.
Nordhaus, W.D., 1977. Economic growth and climate: the carbon dioxide problem. Am. Econ. Rev. 67, 341–346.
Nordhaus, W.D., 1979. The Efficient Use of Energy Resources. Yale University Press, New Haven, CT.
Nordhaus, W.D., 1994a. Managing the Global Commons: The Economics of Climate Change. MIT Press, Cambridge, MA.
Nordhaus, W.D., 1994b. Expert opinion on climatic change. Am. Sci. 82, 45–51.
Nordhaus, W.D., 2002. Modeling induced innovation in climate change policy. In: Grübler, A., Nakicenovic, N., Nordhaus, W.D. (Eds), Technological Change and the Environment. Resources for the Future, Washington, DC, pp. 182–209.
Nordhaus, W.D., 2007a. The Stern Review on the economics of climate change. J. Econ. Lit. 45, 686–702.
Nordhaus, W.D., 2007b. Alternative measures of output in global economic–environmental models: purchasing power parity or market exchange rates? Energy Econ. 29, 349–372.
Nordhaus, W.D., 2008. A Question of Balance: Weighing the Options on Global Warming Policies. Yale University Press, New Haven, CT.
Nordhaus, W.D., 2010. Economic aspects of global warming in a post-Copenhagen environment. Proc. US Natl. Acad. Sci. USA 107, 11721–11726.
Nordhaus, W.D., 2011a. The economics of tail events with an application to climate change. Rev. Environ. Econ. Policy 5, 240–257.
Nordhaus, W.D., 2011b. Estimates of the Social Cost of Carbon: Background and Results from the RICE-2011 Model. Cowles Foundation, New Haven, CT.
Nordhaus, W.D., 2012. Economic policy in the face of severe tail events. J. Public Econ. Theory 14, 197–219.

Nordhaus, W.D., Boyer, J., 2000. Warming the World: Economic Modeling of Global Warming. MIT Press, Cambridge, MA.

Nordhaus, W.D., Popp, D., 1997. What is the value of scientific knowledge? An application to global warming using the PRICE model. Energy J. 18, 1–45.

Nordhaus, W.D., Yang, Z., 1996. A regional dynamic general-equilibrium model of alternative climate-change strategies. Am. Econ. Rev. 86, 741–765.

Oakland, W.H., 1987. Theory of public goods. In: Auerbach, A.J., Feldstein, M. (Eds), Handbook of Public Economics, Volume 2. North-Holland, Amsterdam, pp. 485–535.

Pearce, D., 2003. The social cost of carbon and its policy implications. Oxf. Rev. Econ. Policy 19, 362–384.

Papadimitriou, C., 1994. On the complexity of the parity argument and other inefficient proofs of existence. J. Comput. Syst. Sci. 48, 498–532.

Peck, S.C., Teisberg, T.J., 1993. Global warming uncertainties and the value of information: an analysis using CETA. Resource Energy Econ. 15, 71–97.

Peck, S.C., Teisberg, T.J., 1999. CO2 concentration limits, the costs and benefits of control and the potential for international agreements. In: Carraro, C. (Ed.), International Environmental Agreements on Climate Change. Kluwer, Dordrecht, pp. 37–57.

Pizer, W.A., 1999. Optimal choice of climate change policy in the presence of uncertainty. Resource Energy Econ. 21, 255–287.

Point Carbon, 2006. Historical prices available at. http://www.pointcarbon.com/ (accessed 9 November 2006 by subscription).

Popp, D., 2004. ENTICE: endogenous technological change in the DICE model of global warming. J. Environ. Econ. Manag. 48, 742–768.

Portney, P.R., Weyant, J.P. (Eds), 1999. Discounting and Intergenerational Equity. Resources for the Future, Washington, DC.

Ramsey, F.P., 1928. A mathematical theory of saving. Econ. J. 38, 543–559.

Rogelj, J., Nabel, J., Chen, C., Hare, W., Markmann, K., Meinshausen, M., Schaeffer, M., Macey, K., Höhne, N., 2010. Copenhagen Accord pledges are paltry. Nature 464, 1126–1128.

Romer, P., 1990. Endogenous technological change. J. Polit. Econ. 98, S71–S102.

Rutherford, T.F., 1995. Extension of GAMS for complementarity problems arising in applied economic analysis. J. Econ. Dynam. Control, Elsevier, 19 (8), 1299–1324, November.

Samuelson, P.A., 1954. The pure theory of public expenditure. Rev. Econ. Stat. 36, 387–389.

Samuelson, P.A., 1949. Market Mechanisms and Maximization, Rand Corporation, Santa Monica, CA.

Stern Review (2007) The Economics of Climate Change: The Stern Review. Cambridge University Press, Cambridge.

Tol, R.S.J., 2003. Is the uncertainty about climate change too large for expected cost–benefit analysis? Cli. Change 56, 265–289.

Tol, R.S.J., 2005. The marginal damage costs of carbon dioxide emissions: an assessment of the uncertainties. Energy Policy 33, 2064–2074.

Tol, R.S.J., 2008. The social cost of carbon: trends, outliers and catastrophes. Econ. Open-Access, Open-Assess. E-J. 2, 1–22.

Tol, R.S.J., 2009. The economic effects of climate change. J. Econ. Perspect. 23, 29–51.

United Nations, 1992. United Nations Framework Convention on Climate Change. United Nations, New York. Available at http://unfccc.int/resource/docs/convkp/conveng.pdf.

United Nations, 2004. World population to 2300. ST/ESA/SER.A/236. Department of Economic and Social Affairs, Population Division. United Nations, New York.

United Nations, 2009. Copenhagen Accord. United Nations, New York. available at. http://unfccc.int/resource/docs/2009/cop15/eng/l07.pdf.

US Working Group, 2010. Technical Support Document: Social Cost of Carbon for Regulatory Impact Analysis Under Executive Order 12866. Interagency Working Group on Social Cost of Carbon, US Government. Available at http://www.epa.gov/oms/climate/regulations/scc-tsd.pdf.

Webster, M.D., 2002. The curious role of learning: should we wait for more data? Energy J. 23, 97–119.

Weitzman, M.L., 2009. On modeling and interpreting the economics of catastrophic climate change. Rev. Econ. Stat. 91, 1–19.

Weyant, J. and J. Hill (Eds) (1999) The Costs of the Kyoto Protocol: A Multi-model Evaluation. Energy J., Special Issue.

Weyant, J., Davidson, O., Dowlatabadi, H., Edmonds, J., Grubb, M., Richels, R., et al., 1996. Integrated assessment of climate change. In: Bruce, J., Lee, H., Haites, E. (Eds), Climate Change 1995—Economic and Social Dimensions of Climate Change. Cambridge University Press, Cambridge, pp. 367–396.

World Bank, 2009. World Development Report, World Development Report Database. World Bank, Washington, DC.

Yelle, L.E., 1979. The learning curve: historical review and comprehensive survey. Decis. Sci. 10, 302–328.

Yohe, G., Richard, T., 2010. Precaution and a dismal theorem: implications for climate policy and climate research. In: Geman, H. (Ed.), Risk Management in Commodity Markets. Wiley, New York, pp. 91–99.