What's wrong with infinity – A note on Weitzman's dismal theorem

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Abstract. We discuss the meaning of Weitzman's (2008) dismal theorem. We show that an infinite expected marginal willingness to substitute between today's and future's consumption does not jeopardize cost-benefit-analysis. The optimal level of this willingness to transfer wealth is finite if a safe way of transfering wealth into the future exists. For investments with uncertain returns, we demonstrate that marginal expected rate of substitution is no meaningful indicator for cost-benefit analysis: cost-benefit analysis under uncertainty must jointly consider marginal willingness and the options/technologies of transfering income. (JEL: D81, Q54)

Keywords: decisions under uncertainty, cost-benefit-analysis, climate policy

1 Introduction

In a recent paper, Weitzman (2008) laid out a dismal theorem which states a potential problem for applying cost-benefit analysis (CBA) in the realm of large structural uncertainty. Using the example of climate change, he derives theoretically that the rate at which society would be willing to exchange today's consumption for future's consumption might very well be infinity. The implicit discount factor for consumption might therefore be dominated by such structural uncertainty.

In this note, we question the applicability of this theorem for practical cost benefit analysis. More explicitly, we argue that the pure value of infinity does not prevent us from applying CBA in a meaningful way. We show that Weitzman's result applies only to situations in which it is not possible to transfer certain wealth to the future. When such transfers are possible, and because a situation with the ability to transfer is the only economically interesting one, a standard cost-benefit problem arises and an interior solution exists.

In the context of climate change, no investment option may exist whose returns are not

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subject to climate uncertainty as well. That is, no action we can take may improve, in an absolute sense, our future outcomes should the worst realizations of climate change occur. It is not surprising that cost-benefit analysis should fail in a context where no improvement is possible. However, even if investments are jeopardized in the worst cases, it can be expected that they pay off should climate change turn out to be less severe. When we consider such a situation in which actions that ameliorate climate change are indeed available, the dismal theorem does not apply and the choice of an optimal action is a well-defined, solvable problem.

In this case, however, we show that the expected marginal rate of substitution between consumption today and in the future does not represent a meaningful indicator for cost-benefit analysis. We demonstrate that a meaningful cost-benefit analysis must jointly consider marginal willingness to substitute between consumption today and tomorrow and the options/technologies of transfering income. In this sense, a separate determination of the benefits from investing and the costs of doing so is impossible. A second part of Weitzman's article, a demonstration that there are fat tails in probabilities of future outcomes, remains valid and indeed skews us toward taking stronger actions that we would under thinner tailed distributions. But this fat tail problem does not preclude a cost-benefit framework.

The concept of infinite willingness-to-pay at the margin as suggested by Weitzman should be familiar to the reader: in many theoretical models, Inada conditions are being made in order to avoid corner solutions. In this sense, Weitzman's infinity corresponds to a justification of such Inada-like conditions for the demand of transfers from today to future generations. It shows that the *first* unit of wealth should be transferred at *any* cost. Once we have transferred this first unit, however, there is a finite benefit to climate-change control actions that can be weighed against the costs.

2 The basic points

Analogously to Weitzman (2008), we consider C as the consumption, with $C_0 = 1$ denoting the (normalized) present gross consumption. Consumption generates some utility which we assume to be of CRRA (constant relative risk aversion) form

$$U(C) = \frac{C^{1-\eta}}{1-\eta} \tag{1}$$

with $\eta > 0$. The marginal utility is given by $U'(C) = C^{-\eta}$. Future consumption C_1 is random and depends on a state of nature $s \in S$.

Differently from Weitzman, we consider investment decisions explicitly. That is, C is not assumed to implicitly include all adaptation and mitgation options. Let p be investment in period 0, yielding net consumption $1-p$. Investment generates a future return of $R(p,s)$, yielding net consumption $C_1(s)+R(p,s)$. For simplicity, we denote $R'(p,s) = \partial R(p,s)/\partial p$. We assume that $R(0,s) = 0$. Furthermore, we assume $R''(p,s)) = \partial R(p,s/\partial p^2 \leq 0$ and that $R'(p,s) \leq \bar{R}$ for all p,s. That is, the marginal returns from investment are bounded. This specification explicitly allows that the returns from investment could be zero or even negative in some states of nature. Note, however, that with the choice of the utility function, consumption is normalized to satisfy $C_1(s) + R(p,s) > 0$.

The standard discounted expected utility in this two-period model from an investment at level p is then given by:

$$U(1-p) + \beta E[U(C_1(s) + R(p,s))] \tag{2}$$

where $\beta > 0$ denotes the pure rate of time-preference.

We first consider a situation in which a technology exists which generates such a safe return to investment, i.e. $R(p,s) = \hat{R}(p)$ with $\hat{R}'(0) > 0$. An interior solution to the maximization of (2) then requires:

$$\beta \frac{E[U'(C_1(s) + \hat{R}(p))]}{U'(1-p)} = \frac{1}{\hat{R}'(p)} \tag{3}$$

Here, the left hand side describes the marginal rate at which the agent would be willing to give up present consumption to obtain one sure unit of consumption in the future while the right hand side gives the marginal costs of doing so.

If a corner solution $p = 0$ were optimal, the first-order condition is[1]

$$\beta \frac{E[U'(C_1(s))]}{U'(1)} < \frac{1}{\hat{R}'(0)} \tag{4}$$

Weitzman demonstrates conditions under which the distribution of C_1 is such that

$$\beta E[U'(C_1(s))/U'(1)] = \infty.$$

From (4), we see however that this cannot correspond to an optimal choice of investment if there exists any technology which transfers a safe asset into the future at finite costs ($\hat{R}'(0) > 0$). Instead, we obtain that if there is such a technology, a positive investment $0 < p < 1$ implies $\beta E[U'(C_1(s))/U'(1)] < 1/\hat{R}'(0)$. For any positive $R(p) > 0$, however,

$$\beta E[U'(C_1(s) + \hat{R}(p))/U'(1-p)] < \beta U'(\hat{R}(p))/U'(1) = \hat{R}(p)^{-\eta}$$

such that the marginal rate of substitution is bounded away from infinity. We therefore obtain the following proposition:

Proposition 1 *If there is any investment option which generates a safe return on today's investment, the optimal expected marginal rate of substitution between current and future consumption is finite.*

Proposition 1 implies that Weitzman's dismal theorem does not apply at the optimal investment decision. The difference between our result and Weitzman is an assumption about whether C_1 can meaningfully include adaptation and mitigation. Weitzman claims that C_1 includes all such actions and therefore includes neither investments p nor their returns $R(p, s)$ in his set-up.

In general, however, it is doubtful if a safe asset exists as assumed so far. Instead, investments might reasonably be equally at risk from climate change. In the following, we therefore extend the analysis to capture situations in which the returns to investment are uncertain and can even be negative for some states of nature. Here, an interior solution to

[1] Note that the other potential corner solution, $p = 1$, cannot be optimal as $U'(0) = \infty$ and $U'(C_1(s) + \hat{R}(p)) < U'(R(p)) < \infty$ for $\hat{R}(p) > 0$.

this investment problem would require

$$-U'(1-p) + \beta E[U'(C_1(s) + R(p,s))R'(p,s)] = 0 \tag{5}$$

while it would be optimal to invest nothing ($p = 0$) if

$$\beta \frac{E[U'(C_1(s))R'(0,s)]}{U'(1)} < 1 \tag{6}$$

If indeed the distribution of C_1 already reflects the optimal choice of all mitigation and adaptation options, a corner solution $p = 0$ must be optimal. This would be consistent with Weitzman's infinity result, i.e. a situation where $\beta E[U'(C_1(s))/U'(1)] = \infty$ only if for all feasible investment options, condition (6) holds. That is, if $\beta E[U'(C_1(s))R'(0,s)]/U'(1) < 1$.

This finding thereby puts a word of caution on using expected marginal willingness to substitute between today's and tomorrow's consumption, i.e. a willingness to pay for investing into a safe asset, as an indicator for CBA. Instead, meaningful cost-benefit analysis must jointly consider marginal willingness to substitute between consumption today and tomorrow and the options/technologies of transfering income.

Indeed, (6) implies that even when the marginal willingness to shift consumption into the future is infinity at $p = 0$, society should not give up all consumption today in order to shift consumption into the future:

Proposition 2 *A society will never give up all its wealth in order to transfer it into some uncertain future.*

To demonstrate Proposition 2 formally, we reconsider the left hand side of (5) for p approaching 1. Note that if $p > p_0$ and $R'(p,s) > 0$, it also follows that $R(p,s) > R(p_0,s) > 0$. Using this inequality and $R'(p,s) \leq \bar{R}$, we obtain

$$\begin{aligned} & -U'(1-p) + \beta E[U'(C_1(s) + R(p,s))R'(p,s)] \\ < \; & -U'(1-p) + \beta \text{prob}[R'(p,s) > 0]E[U'(C_1(s) + R(p,s))R'(p,s)|R'(p,s) > 0)] \\ < \; & -U'(1-p) + \beta \text{prob}[R'(p,s) > 0)]E[U'(C_1(s) + R(p_0,s))R'(p_0,s)|R'(p,s) > 0)] \end{aligned}$$

Now note that $E[U'(C_1(s) + R(p_0,s))R'(p_0,s)|R'(p,s) > 0)]$ is bounded from above since

$C_1(s) + R(p_0, s) > 0$. On the other hand, $U'(1-p) \to \infty$ for $p \to 1$. We therefore obtain that the marginal benefit from investing a further unit must be negative for p sufficiently close to 1 which proves Proposition 2.

3 Conclusions

A cost-benefit framework for climate change is indeed possible, despite Weitzman's conclusions based on his Dismal Theorem. This conclusion holds even in situations where the expected marginal rate of substitution between consumption today and the future is infinite, at least on the margin.

This result puts us back in the realm that we started in: The choice of a climate policy depends on consideration of the discount rate, control costs and effectiveness, and the effects of climate change on future incomes. None of these elements is easy to estimate or uncontroversial to apply. The question still remains of what will happen under future climate scenarios and what can and should be done. Cost-benefit analysis may be deemed the wrong framework with which to make these momentous decisions. But if so, it is not fat-tailed distributions or infinite expected marginal utilities that should motivate this abandonment.

4 References

Weitzman, M.L. (2008), On Modeling and Interpreting the Economics of Catastrophic Climate Change, Working Paper.