COMMENTARY ON WEITZMAN'S PAPER

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EXECUTIVE SUMMARY

A recent paper by Harvard economist Martin Weitzman (2009) analyzed potential responses to global warming in statistical/decision theoretic language. We believe the basic points he is making are already well known, and they do not add anything concrete to the climate change debate.

Weitzman's basic argument comes down to the juxtaposition of two assertions: (1) that a critical climate change parameter known climate sensitivity has a fat-tailed distribution (i.e., extreme events are more likely than previously assumed), (2) therefore, risk functions that depend exponentially on climate sensitivity are infinite. (In other words, there is no limit to the potential damage that climate change might cause.) This essential dichotomy has been studied for at least 80 years in contexts such as insurance and the risk of hydrological extremes. His review of the literature on climate sensitivity is extremely cursory, and only barely acknowledges the extensive debate that has already taken place on the plausibility of assigning probabilities to large sensitivities. His approach to Bayesian statistics is also very simplistic, ignoring the large literature that already exists on translating output from climate models into probabilistic projections of climate outcomes. Finally, by translating uncertainty about climate sensitivity into a single ill-defined "consumption" variable, he ignores the enormous literature on the impacts of climate change.

Weitzman’s main argument hinges on a decision quantity defined as “the amount of present consumption the agent would be willing to give up in the present period to obtain one extra sure unit of consumption in the future period". He points out that this depends on climate sensitivity (defined as the increase in global mean temperature to be expected, under equilibrium conditions, as a result of a doubling of atmospheric carbon dioxide from pre-industrial levels). After reviewing the literature, he concludes that current knowledge of climate sensitivity is highly uncertain, and therefore it would be reasonable to assume it has a “fat-tailed” distribution. When combined with certain risk-analysis assumptions, this leads to the conclusion that the above defined decision quantity is infinite. Literally interpreted, that would mean there is no action society could take at the present time that would adequately protect against the possibility of catastrophic climate change in the future.

Weitzman recognizes the difficulty of working with infinity and discusses various ways to avoid it. The main tool is to impose a very large but finite “value of statistical life” (VSL). However, the decision quantity still tends to infinity as the VSL tends to infinity (this is Weitzman’s “Dismal Theorem”). Other attempts to remove the infinity, such as truncating the probability distribution of climate sensitivity, are dismissed as artificial and arbitrary. Based on this, Weitzman advocates a “generalized precautionary principle” that implies very careful attention to the probability of extreme changes in climate.

In summary, we believe the literature on responses to climate change is already well ahead of Weitzman in analyzing uncertainties in the projections of climate models and translating them into specific societal consequences. We acknowledge the role that economists must play in incorporating these uncertainties into societal responses, but we do not find that Weitzman’s analysis provides any useful contribution to that role.

COMMENTARY

To put our remarks in contexts, we begin with a overview of Weitzman (2009). The overview is not intended to be comprehensive, but rather focuses on aspects most relevant to our comments.

Outline of Weitzman (2009)

Weitzman defines C be “consumption”. We found his explanation of this variable somewhat elusive. Our interpretation is that it is a measure of human welfare, normalized so that $C = C_0 = 1$ at the present day. The primary variable of interest is some future C, say 200 years hence. If C is very low, conditions for human life will be intolerable.

Associated with C is a utility function, $U(C) = C^{1-\eta}/(1-\eta)$, leading to “marginal utility” $U'(C) = C^{-\eta}$. This is the assumption of “constant relative risk aversion” (CRRA). It appears that the range of the parameter η that is of most interest is $\eta > 1$; for specific calculations, Weitzman sets $\eta = 2$.

Next, Weitzman defines $Y = \ln(C)$ and assumes $Y = F(\Delta T) = G - \gamma\,\Delta T$ where ΔT is the change in global mean temperature (in degrees Celsius) and G and γ are positive constants. Weitzman does not explain completely this choice of F (i.e., being linear in ΔT), though he suggests that the choice does not matter for his purposes, e.g. assuming $F(\Delta T) = G - \gamma\,\Delta T^2$ would give essentially the same answers.

Weitzman then defines $M = \beta \exp(-\eta Y)$ where β is a “time preference parameter” ($0 < \beta \leq 1$), leading to the interpretation that the expected value of M, $E\{M\} = \beta E\{\exp(-\eta Y)\}$, as “the amount of present consumption the agent would be willing to give up in the present period to obtain one extra sure unit of consumption in the future period”. Mathematically, E{M} is obtained by integrating the product of $\beta E\{\exp(-\eta y)\}$ and f(y) over the range of possible values of the random variable Y=y. Here, f is the pdf of Y. Weitzman notes that the critical calculation depends on the density f through what is known as its moment generating function (MGF). A “fat-tailed” distribution, in Weitzman's terminology, corresponds to any pdf for which the MGF is infinite. Weitzman points out that if Y has a normal distribution with mean μ and standard deviation s, then

$$E\{M\} = \beta \exp(-\eta\mu + 0.50(\eta s)^2)$$

is finite, but if Y has a t-distribution, E{M} is not finite. This distinction is critical in Weitzman's thesis.

Weitzman writes $Y = \mu + s Z$ where Z is proportional to the change in the logarithm of C corresponding to the change in temperature ΔT (i.e., $Z = \Delta \ln(C)/(\ln(2)$ and Z has standard deviation equal to one). The standard deviation s is "loosely conceptualized as a highly stylized abstract generalization" of something like the "climate sensitivity parameter". He assumes that the pdf of Z, say $\phi(z)$, is known. Hence, for given values of μ and s, the pdf of Y is

$$f(y) = (1/s)\ \phi((y-\mu)/s).$$

Since s is unknown, he suggests a Bayesian analysis for s, combining a collection of n observations of y, denoted by **y**, and a prior pdf $\pi_0(s)$ for s. Probability theory provides a recipe (Bayes' Theorem) for finding the posterior pdf $\pi(s|\mathbf{y})$, and then the implied posterior predictive pdf $f(y|\mathbf{y})$ of Y conditional on the observed data **y**. As an illustration, if $\pi_0(s) \propto s^{-k}$, for some $k \geq 0$, and ϕ is the pdf of a standard normal random variable, then $f(y|\mathbf{y})$ is the pdf of a t-distribution with n+k degrees of freedom, and hence, tail probabilities proportional to $|y|^{-n-k}$ as |y| tends to infinity. Combining the polynomially decreasing tails of $f(y|\mathbf{y})$ with the definition of E{M}, he reaches the conclusion that E{M} is infinity, implying that this somehow has profound implications for how we should respond to climate change.

Weitzman recognizes that one would not generally work with an unbounded utility function. This leads him to introduce, in Section 3, the "VSL-like parameter". Here, VSL is the "value of statistical life"; i.e. the notion often used in life insurance calculations that one can place a specific financial cost on a human life; here this notion is extended to the life of the entire human race.

With this in mind, Weitzman assumes there is a lower bound $D(\lambda)$ (denoted "death") on consumption C, and depending on a VSL λ. He devotes substantial, though somewhat arbitrary, discussion to the development of D and λ. For our commentary, the critical point, however, is that for $\eta > 1$, $D(\lambda)$ tends to 0 as l tends to infinity.

Now Weitzman comes to what he calls "The Dismal Theorem". He redefines $M = M(C) = \beta \exp\{- \eta\ Y\}$ where $C > D$ or $Y > \ln(D)$. He then states:

Theorem 1. For fixed n and k, $E\{M|\lambda\}$ tends to infinity as λ tends to infinity.

There follows a rough, hand-waving proof, the assumptions of which are never precisely spelled out.

The rest of the paper is taken up with somewhat discursive commentary on how the Dismal Theorem is relevant to the problem of climate change. On the finding of an infinite limit, Weitzman comments “it is easy to put arbitrary bounds on utility functions, to truncate probability distributions arbitrarily, or to introduce *ad hoc* priors that arbitrarily cut off or otherwise severely dampen high values of S or low values of C”. Any of these changes lead to “an arbitrarily large but finite number” His point seems to be the arbitrariness of such a truncation. The implication is a “`generalized precautionary principle' for situations of potentially unlimited downside exposure”. He cautions over the use of Monte Carlo simulation, essentially making the well-known point that it can be difficult to detect when a Monte Carlo simulation is attempting to evaluate an infinite expectation. In the last section he considers the implications for “integrated assessment models” (these are the sort of models that are likely to be introduced in any actual climate change legislation), essentially concluding “the analysis is much more frustrating and much more subjective [than the analysis of a thin-tailed case]...because it requires some form of speculation (masquerading as an ‘assessment’) about the extreme bad-fat-tail probabilities and utilities”.

Discussion

We fail to see how the argument of Weitzman's paper is any more than the confluence of the following two assertions:

> 1. Environmental variables involved in climate change (specifically S, the “climate sensitivity” parameter) have (posterior or subjective) probability distributions that are more reasonably taken as polynomially-tailed rather than exponentially-tailed.
>
> 2. For a loss function that increases exponentially in the environmental variable S, the expectation or risk function is infinite.

The fact that this kind of dichotomy exists is both obvious and has underlain mathematical modeling of risk for many years, stretching back at least as far as Cramer's work on insurance risk or Gumbel's work on hydrological extremes, both of which originated in the 1930s.

Climate Science. What is missing from Weitzman's analysis is any sense of the detail of modern climate research. Most critically, it is debatable whether Weitzman has correctly dealt with the climate sensitivity parameter, S. The first issue is how it is defined. Weitzman defines a first climate sensitivity parameter S_1 as the proportionality constant in the formula $\Delta T = (S_1 / \ln 2) \ln (\Delta C)$. A few lines later he quotes from the Summary for Policymakers of IPCC (2007):

> The equilibrium climate sensitivity is a measure of the climate system response to sustained radiative forcing. It is not a projection but is defined as the global average surface warming following a doubling of carbon dioxide concentrations.

It seems that Weitzman's S_1 includes the assertion that the response to any other level (than doubling) of carbon dioxide concentrations may be derived through a simple scaling relationship. We are unable to find any support for this assertion in any major work on climate change.

Based on an analysis of 22 climate studies that are cited in Chapter 9 of IPCC (2007), Weitzman concludes $P\{ S_1 > 10\,^{o}C \}$ is approximately 0.01. The IPCC report makes many qualifications about the dangers of extrapolating to values of S much above 4^{o}C. To some extent this supports Weitzman's suggestion to use fat-tailed, conservative distributions. However, other authors have questioned whether there is any scientific basis for assigning probabilities to such large values of the climate sensitivity, in particular a comment by Allen and Frame (2006).

However, Weitzman is not content to use previously published statements about S_1 that are supported by the combined authority of IPCC. Instead, he argues that this is the wrong definition, because it does not allow for feedbacks (eg., the idea that increased temperatures themselves may result in increased carbon emissions and therefore enhance the global warming.) He defines a second climate sensitivity parameter, S_2, that includes this feedback mechanism. His authority for this is Scheffer *et al*. (2006) and Torn and Harte (2006). We note that neither of these papers is referenced anywhere in the main chapters of IPCC (2007) dealing with detection/attribution of climate change and the projections of future climate change (i.e., Chapters 9--11 of the Working Group 1 Report). This suggests that, while the two cited papers may be perfectly sound, their ideas have yet to enter the mainstream climate literature.

Bayesian Analysis. It is unclear what exactly Weitzman means by his Bayesian analysis. What are the "observations" **y**? He does not discuss this at all. In fact there has been some discussion of the idea of thinking of the set of all available climate models as a sample from a hypothetically infinite set of possible climate models, with a view to constructing formal inference statements; for example, Smith *et al*. (2009) contains some discussion along these lines. Also, there are many other papers that have discussed the use of Bayesian methods for climate change analysis and climate forecasting, in situations far more concrete than that considered by Weitzman (e.g., Berliner and Kim 2008). At best, Weitzman's interpretation of Bayesian statistics is extremely simplistic, and ignores much work that has already been done along these lines.

Utility Functions. Weitzman's reliance on the exponential cost function is questionable and may be the source of his "dismal" conclusions. As noted by Weitzman, utility functions are traditionally bounded. Indeed, this is a primary motivation for his analysis in Section 3. By using an exponential cost M, the analyst suggests that there are important, exponentially different impacts due to a temperature rise of 20^{o}C versus one of 18^{o}C. More conventional suggestions consistent with utility theory are that M should level off rather than explode exponentially. Weitzman's treatment of the issue is to bound the cost in a fashion that bounds its logarithm (Y) from below. This leads to a finite E{M} for each bound, but as then bounds grow, the exponential form of the utility leads to an infinite limit. This is a mathematical property of the integral (i.e., expectation) of the exponential function.

Indeed, Weitzman's "Dismal Theorem" looses its impact when one recognizes that even sharp tailed distributions can lead to expectations that tend to infinity. For example, in Weitzman's Eq. 5, as μ tends to negative infinity (we are concerned with negative y), E(M) tends to infinity.

The "Dismal Theorem" relies on pdf's with nonzero tails and exponential cost functions. Regarding the use of "nonzero" tailed pdf's, Weitzman dismisses the truncation of the pdf as arbitrary. However, avoiding this admittedly difficult step leads to a circumstance in which our conclusions are determined by the combined behaviors of the cost M and the pdf at values of temperature changes, etc., that are implausible. To illustrate, Weitzman (p. 8) suggests (for example) that $P(? T>10^oC) = 0.05$ and $P(? T>20^oC) =0.01$. What about $P(? T>40^oC)$, $P(? T>100^oC)$? These values decay at a polynomial rate. That combined with an exponential growth in loss leads to expectations that grow to infinity. That is, the results of the analysis accrue from assumptions regarding implausible circumstances.

Simplicity and Generality. We think it is extremely unsatisfactory that Weitzman uses such a simplistic framework for translating the climate sensitivity parameter S (however defined) into a "consumption" variable C. A huge part of the climate science literature is concerned with impacts of climate change: without even more than scratching the surface, one could mention the literature on extreme weather events and their consequences (e.g floods), on hurricanes, and on sea level change. Although in each of these cases the literature is both incomplete and controversial, data and models are available to start making detailed calculations of specific consequences and their probabilities.

Weitzman's paper reinforces a few general points that are essentially well known, but does not get into the details of climate change analysis at all. Potential indicators of the lack of relevance to climate change analysis and policy making are the suggestions of Weitzman that many specific selections he makes (e.g., the utility function, the prior on S, the likelihood function of the data **y**) do not matter in his final conclusions. In some contexts such generality is a positive aspect of mathematical analysis. However, in this case, we believe that the lack of dependence on crucial features of climate science, data collection, etc., indicate that the simplicity of Weitzman's analysis robs it of relevance.

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