Problem Set 1, Introduction to Game Theory (Part B), Winter Term 2017

1. Consider the infinitely repeated game with discount rate δ, where the strategic form below is the stage game. Show that for sufficiently patient players, it is a subgame perfect Nash equilibrium to play (U, L) in every period. What is the minimum δ that achieves cooperation?

Player 2; rows: Player 1

	L	R
U	1,1	-2,5
D	2,0	0,0

2. Consider an infinitely repeated Cournot duopoly game with linear demand function, $Q = \alpha - P$ and constant marginal cost, $c > 0$. Is it possible to sustain the monopoly outcome (say, each producing half of the monopoly quantity) in SPNE?

3. Consider Bertrand's model of duopoly in the case that each firm's unit cost is constant, equal to c. Let $\pi^m(p) = (p - c)D(p)$ for any price p, and assume that π is continuous and is uniquely maximized at the price p^m (the "monopoly price"). Let s be the strategy for the infinitely repeated game that charges p^m in the first period and subsequently as long as both firms continue to charge p^m, and punishes any deviation from p^m by either firm by choosing the price c for k periods, then reverting to p^m. Given any value of δ, for what values of k is the strategy pair (s, s) a subgame perfect equilibrium of the infinitely repeated game?

4. Consider the infinitely repeated Prisoner's Dilemma in which the payoffs of the component game are those given in the figure below.

Player 2; rows: Player 1

	C	D
C	x, x	0, x +1
D	x +1,0	y, y

Consider the strategy following tit-for-tat strategy:

$$s_i(h^t) = \begin{cases} C & \text{if } t = 0 \text{ or } h_j^t = C \\ D & \text{if } t > 0 \text{ and } h_j^t = D \end{cases}$$

Under which conditions on the parameters $x > y > 0$, and $0 < \delta < 1$ is the strategy profile (tit-for-tat, tit-for-tat) a subgame perfect equilibrium?