Problem Set 2, Introduction to Game Theory (Part B), Winter Term 2017
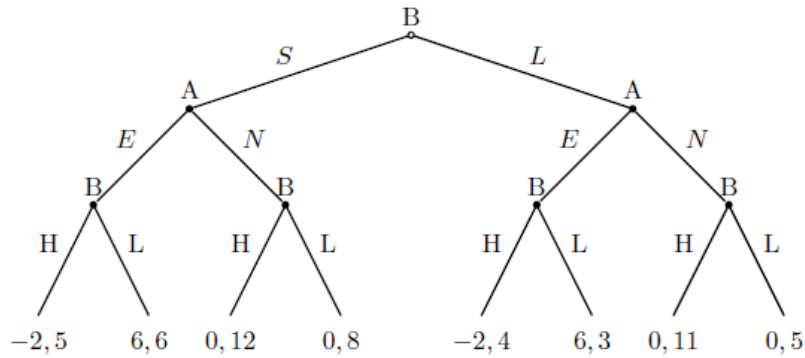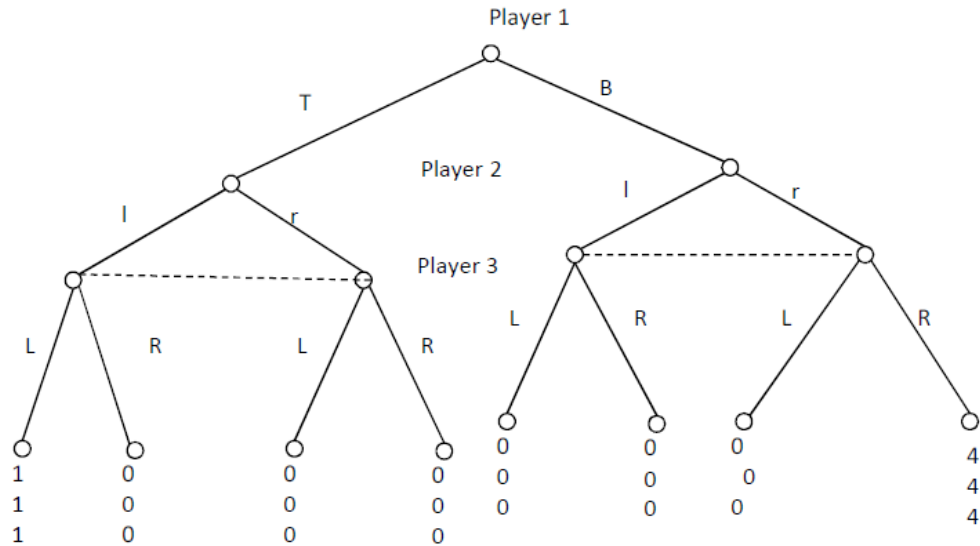
1. Now consider the following three stage game: First B decides whether to build a small or large factory, then A decides whether or not to enter and then B decides whether to produce high or low output. What is the subgame perfect Nash equilibrium of this game?
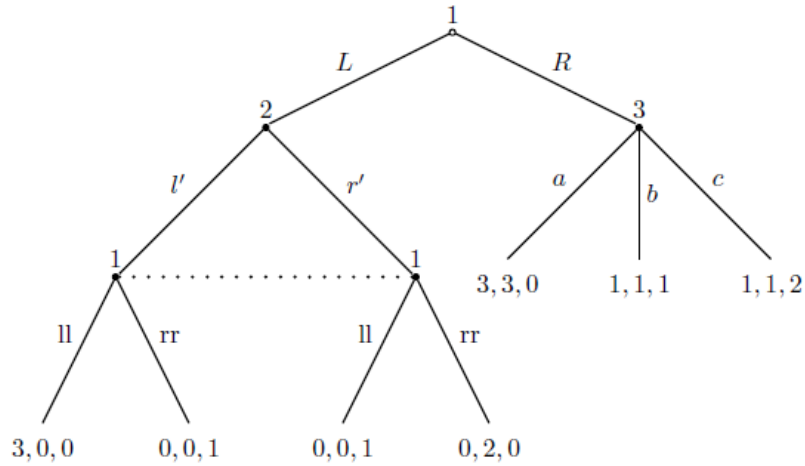

2. What are the subgame perfect Nash equilibria of the following games?
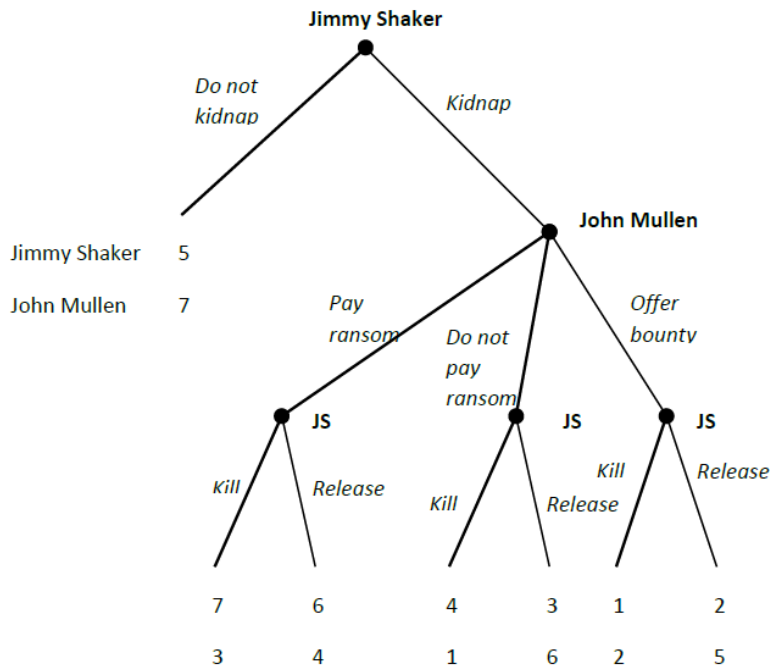
a)

b) Players, namely, 1, 2 and 3 play the game below. Compute all the subgame perfect Nash equilibria for this game.

Tree 1

Node 1 (top): branches L and R
- L → Node 2
- R → Node 3

Node 2: branches l' and r' (both leading to Node 1, connected by a dotted information set)
Node 3: branches a, b, c
- a → $3,3,0$
- b → $1,1,1$
- c → $1,1,2$

Left Node 1: branches ll and rr
- ll → $3,0,0$
- rr → $0,0,1$

Right Node 1: branches ll and rr
- ll → $0,0,1$
- rr → $0,2,0$

3. Jimmy Shaker and accomplices plan to kidnap Sean Mullen, son of a multimillionaire, John Mullen in the movie *Ransom* (1996). The decided ransom is $2,000,000. If Jimmy Shaker decides against this kidnapping then John Mullen retains his son. However if the kidnapping takes place, there is no guarantee Sean will be handed over to his parents alive. In such a case, John Mullen may take three possible actions: pay up the ransom, not pay and announce a reward of equal amount on the head of the kidnapper. Find all SPNE

Jimmy Shaker (top node): branches *Do not kidnap* and *Kidnap*
- *Do not kidnap* → Jimmy Shaker 5, John Mullen 7
- *Kidnap* → **John Mullen** node

John Mullen: branches *Pay ransom*, *Do not pay ransom*, *Offer bounty*
- *Pay ransom* → JS node: *Kill* → (7, 3); *Release* → (6, 4)
- *Do not pay ransom* → JS node: *Kill* → (4, 1); *Release* → (3, 6)
- *Offer bounty* → JS node: *Kill* → (1, 2); *Release* → (2, 5)

4. Simon is walking home late at night when suddenly he realizes that there is someone behind him. Before he has a chance to do anything, he hears, "I have a gun, so keep your mouth shut and give me your wallet, cell phone, and iPad." Simon doesn't see a gun, but does notice that the mugger has his hand in his coat pocket, and it looks like there may be a gun in there. If there is no gun, Simon thinks he could give the mugger a hard shove and make a run for it. But if there is a gun, trying to escape will result in him being shot. He would prefer to hand over his belongings than risk serious injury (utility 2). Earlier that evening, the mugger was engaging in his own decision making as he debated whether to use a gun. Because the prison sentence is longer when a crime involves a gun. Note that if he has a gun, the mugger has a choice to show it or not in the beginning. If he shoots, he gets a utility of 3 but in case of a peaceful robbery, he gets utility 4 if shows the gun, 5 if carries the gun but doesn't show and 6 if he doesn't carry a gun. He'd really like to conduct the theft without it but if Simon runs, he ends up with utility of 2. Simon is worse off if he gives up his things with utility 3 when the mugger doesn't have a gun. On the contrary, he gets 6 if he runs in such a situation. If mugger carries the gun, Simon derives a utility of 5 if he surrenders when shown a gun as compared to a situation where he is not (utility 4). Draw the game tree and the pure strategy SPNE.