## Problem Set 3, Introduction to Game Theory (Part B), Winter Term 2017

1. Consider a two-player Bayesian game where both players are not sure whether they are playing the game $X$ or game $Y$, and they both think that the two games are equally likely. This game has a unique Bayesian Nash equilibrium, which involves only pure strategies. What is it? (Hint: start by looking for Player 2's best response to each of Player 1's actions.)

Game X
1

| 2 |  |  |  |
| :--- | :--- | :--- | :--- |
|  | L | M | R |
| T | $1,0.2$ | 1,0 | $1,0.3$ |
| B | 2,2 | 0,0 | 0,3 |

Game Y

| 1 | 2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | L | M |  |
|  | T | 1,0.2 | 1,0.3 | 1,0 |
|  | B | 2,2 | 0,3 | 0,0 |

2. Now consider a variant of this game (from Problem 1) in which Player 2 knows which game is being played (but Player 1 still does not). This game also has a unique Bayesian Nash equilibrium. What is it? (Hint: Player 2's strategy must specify what she chooses in the case that the game is X and in the case that it is Y .) Compare Player 2's payoff in the games from Problems 1 and 2 . What seems strange about this?
3. The following version of the Gift Game is a good illustration. Here, player 2 prefers the gift to be coming from a friend, but she would rather accept a gift from an enemy than to refuse the gift. Find the BNE.

4. Consider the following "Arms Race" game. Players one and two are countries. Each has the option to "Build" (B) or "Not Build" (N) arms. Player 2's payoffs are known. Player 1's payoffs are unknown. With probability p, Player 1 is of Type 1 ("Aggressive"), while with probability 1-p, Player 1 is of Type 2 ("Passive"). The payoffs are shown below. Note that player 2's payoffs depend only on the actions taken by the players, not on Player 1's type.


Player 2

Player 1 - type 2

|  | $B$ | $N$ |
| :--- | :--- | :--- |
| $B$ | 2,2 | 6,0 |
| N | 0,6 | 8,8 |

a) Explain why type 1 is called "Aggressive" and type 2 is called "Passive".
b) What are the pure strategy Bayes-Nash equilibrium outcomes of this game as a function of p ?
c) Explain in words how the nature of the incentives for player 2 change as $p$ increases and why the set of Bayes-Nash equilibria changes as a result as $p$ increases.

