Problem Set 4, Introduction to Game Theory (Part B), Winter Term 2017

1. Consider the following model of an auction for an indivisible object. There are two bidders $i = 1,2$ with private i.i.d. valuations, $v_i \in \{0,1\}$. We also know that $\Pr[v_i = 1] = \Pr[v_i = 0] = 0.5$ for $i = 1,2$. Only bids of 0 or 1 are allowed.

   (a) Assume that the object is allocated using a "second price" rule. That is, the highest bidder wins and pays the second highest bid. Ties are broken assigning the object with equal probability to either bidder. Show that there is no Bayesian Nash equilibrium in which both bidders always bid 0, regardless of their valuations.

   (b) Now assume that the object is allocated using a "first price" rule. That is, the highest bidder wins and pays his own bid. Ties are broken assigning the object with equal probability to either bidder. Show that there is a Bayesian Nash equilibrium of this new game in which both bidders always bid 0, regardless of their valuations.

2. Consider the following first-price, sealed-bid auction where an indivisible good is sold. There are $n \geq 2$ buyers indexed by $i = 1, 2, 3, \ldots, n$. Simultaneously, each buyer i submits a bid $b_i \geq 0$. The agent who submits the highest bid wins. If there are $k > 1$ players submitting the highest bid, then the winner is determined randomly among these players – each has probability $1/k$ of winning. The winner i gets the object and pays his bid b_i, obtaining payoff $v_i - b_i$, while the other buyers get 0, where $v_1, \ldots\ldots, v_n$ are independently and identically distributed with probability density function f where

$$f = \begin{cases} 3x^2 & x \in [0,1] \\ 0 & \text{otherwise.} \end{cases}$$

   a) Compute the symmetric, linear Bayesian Nash equilibrium.
   b) What happens as n → ∞?

3. There are n bidders indexed by $i = 1, 2, 3, \ldots, n$, the valuations $v_1, \ldots\ldots, v_n$ are independently and identically distributed with Uniform distribution over 0 to 1.
   i. Compute the symmetric BNE for the first price auction.
   ii. Compare the revenue to the seller from the first auction and the second price auction. Does it confirm to the *Revenue Equivalence Theorem*?
   iii. Find the optimal reserve price in the second price auction.