Delhi School of Economics

M.A. ECONOMICS SUMMER SEMESTER
COURSE 002. INTRODUCTORY MATHEMATICAL ECONOMICS
Midterm 1
30th August 2010

Instructions. *Time: 70 minutes. Maximum Marks 15. Closed book closed notes exam. Attempt all the three questions. Five marks each. Some internal options are available.*

1. Prove the followings:

   (a) Suppose V and W are finite dimensional and that U is a subspace of V. Prove that $\exists$ a linear transformation $T : V \to W$ such that $\ker T = U$ iff $\dim U \geq \dim V - \dim W$.

   (b) $f(x)$ is a convex function iff $-f(x)$ is a concave function for any $f : R \to R$.

   OR,

   (a) Define a linear transformation $T : F^2 \to F^2$ by $T(w, z) = (z, w)$ [note the order]. Find all eigen values and eigen vectors of T. [Hint: start from the definition of eigen value.]

   (b) Matrix A is called nilpotent if $A^k = 0$ for some k. Prove that if A is nilpotent then 0 is the only eigenvalue of A.

2. Let V be the subspace of R^3 consisting of vectors $[x_1 \;\; x_2 \;\; x_3]^T$ satisfying

$$x_1 + 2x_2 - 3x_3 = 0$$
$$x_2 - 2x_3 = 0.$$

   (a) Find all vectors (a subspace) that are perpendicular (orthogonal) to V.

   (b) Find a matrix A whose column space is V, i.e., $Col(A) = V$.

   (c) What is the projection matrix P projecting onto V?

   (d) Is the projection matrix P in (c) positive definite or semi-positive definite? State your reasoning.

   (e) What is the closest vector in the orthogonal complement $V^\perp$ to $[1 \;\; 0 \;\; 0]^T$

3. Consider the following quadratic form $x^T A x$, where A is a 2 by 2 symmetric matrix. It is known that the eigenvalues of A are $\lambda_1 = 2$, $\lambda_2 = 1$, and the corresponding eigenvectors are $x_1 = [1 \;\; 2]^T$ and $x_2 = [-2 \;\; 1]^T$.

   (a) Determine A.

   (b) Find the invertible matrix S required to diagonlize A using completion of squares method.

   (c) Find the unit vector x(i.e. $||x|| = 1$) at which $x^T A x$ is maximized. What is the maximum value of $x^T A x$?

   OR,

   Consider the following bases for R^2. The v-basis is given by $v_1 = [2 \;\; -1]^T$, $v_2 = [0 \;\; 1]^T$. The w-basis is given by $w_1 = [-1 \;\; 1]^T$, $w_2 = [1 \;\; 1]^T$

   (a) Find the coordinate vector of $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ with respect to w-basis.

   (b) Suppose the matrix M maps the coordinate vector in v-basis to the coordinate vector in w-basis, i.e., $[\mathrm{x}]_w = M[\mathrm{x}]_v$. Find M.

   (c) Suppose A is the matrix for a linear transformation $T : R^2 \to R^2$ in v-basis, and B is the corresponding matrix for the same transformation in w-basis. What is the relationship between A and B? Describe it with one equation.

   (d) In part(c), are you sure that A and B have the same set of eigenvalues? State your reasoning.