Delhi School of Economics

M.A. ECONOMICS SUMMER SEMESTER
COURSE 002. INTRODUCTORY MATHEMATICAL ECONOMICS
Midterm 1
12th September 2011

Instructions. *Time: 70 minutes. Maximum Marks 15. Closed book closed notes exam. Attempt all the three questions. Five marks each. Some internal options are available.*

1. Do the followings:

   (a) Find the matrix **P** that projects every vector $\overrightarrow{b}$ in R^3 onto the line in the direction of $\overrightarrow{a} = (2, 1, 3)$:

   (b) What are the column space and nullspace of **P** ? Describe them geometrically and also give a basis for each space.

   (c) What are all the eigenvectors of **P** and their corresponding eigenvalues? (You can use the geometry of projections, not a messy calculation.) The diagonal entries of **P** add up to _______.

2. (a) Let **S** be the standard basis of R^2 , and **B** be the basis {(1,4), (2,9)}. Let T: $R^2 \to R^2$ be the linear map defined by $T(x, y) = (2x - y, x - 2y)$, for all $x, y \in R$.
   (i) Find the coordinates of each of the standard basis vectors in basis **B**.
   (ii) Find the matrix $[T]_{\mathbf{B}}$ for T relative to basis **B**.

   (b) True or False (Explain your reasoning):
   (i) If {u_1, u_2} is linearly independent and {v_1, v_2} is linearly independent, then {u_1, u_2, v_1, v_2} is linearly independent.
   (ii) If {u_1, u_2} is a spanning set of V and {v_1, v_2} is another spanning set of V , then {u_1, u_2, v_1, v_2} is also a spanning set of V .?

3. (a) Assume that V = U ⊕W for two subspaces U and W of V . Let {u_1, . . . , u_m} be a basis for U and let {w_1, . . . ,w_n} be a basis for W. Prove that {u_1, . . . , u_m,w_1, . . . ,w_n} is a basis for V . (Hint: what do you know about dim U ⊕W?).

   (b) Let $T : R^n \to R^k$ be a real matrix (not necessarily square). If the nullspace of T is {0}, show that the matrix T^*T is invertible and positive definite.

   OR,

   a. Let $P_3(F)$ be a vector space of polynomials with coefficients in F and degree ≤3, and let T: $P_3(F) \to P_3(F)$ be defined as $T(f(x)) = f(x + 1)$, for all $f(x) \in P_3(F)$
   For example: $T(x^2 + x) = (x + 1)^2 + (x + 1)$
   (i) T is a linear map.
   (ii) Compute $Mat(T; B)$ i.e.$[T]_B$ where B is basis $\{1, x, x^2, x^3\}$ of $P_3(F)$.

   b. Prove the statement: A set $X \subset R^n$ is convex iff it contains any convex combination of vectors $x_1, ..., x_m \in X$.