Repayment incentives and the distribution of gains from
group lending*

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Abstract

Group loans with joint liability are a distinguishing feature of many microfinance pro-
grams. While such lending benefits millions of borrowers, major lending institutions ac-
knowledge its limited impact among the very poor and have shifted towards individual
loans. This paper attempts to explain this trend and to understand how borrower wealth
influences the benefits from group lending. In our model, individuals of heterogeneous
wealth face a given investment opportunity and access to credit is limited by strategic
default. We show that, in the absence of social sanctions, the poorest investors are offered
individual loans while richer borrowers opt for group lending. We characterize the con-
ditions under which social sanctions within groups improve credit market outreach and
repayment rates. These sanctions improve welfare because they substitute for bank sanc-
tions but are never imposed in equilibrium. Finally, we explore the welfare effects of group
size and show that these are, in general, ambiguous.

Keywords: microcredit, joint-liability, group lending, repayment incentives, social
sanctions. JEL codes: I38, G21, O12, O16

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1 Introduction

The ideology and practice of poverty alleviation has been deeply influenced by the idea that access to credit can empower the poor. Microfinance programs around the world cover millions of borrowers and are provided under a variety of different institutional arrangements. The overall gains from such lending are widely acknowledged but there is growing concern about their capacity to reach those at the bottom of the income distribution. This has resulted in a lively debate, but little consensus on how credit contracts can be better designed to improve credit market outreach and the welfare of poor borrowers. A recurring question within this debate is whether group loans with joint liability can effectively achieve these objectives.

Group loans were first popularized by the Grameen Bank of Bangladesh in the 1970s. It was believed that joint liability would generate social pressure on borrowers to repay loans and create a financially sustainable model of lending. This approach was questioned in the late nineties when natural disasters triggered widespread default and borrowers protested against rigidities in the lending program. The Grameen Bank responded by introducing Grameen II which made all members individually liable for their loans (Yunus, 2004; Kalpana, 2006). Within a couple of years, membership doubled, suggesting that individual loans catered to a previously unmet demand for credit.\(^1\) A similar switch to individual contracts was made by Banco Sol of Bolivia, another pioneer in group lending.

This trend towards individual contracts is far from universal. A majority of the 663 institutions reporting to the Microfinance Information Exchange (MIX) in 2009 relied on some form of joint liability and many large microfinance institutions offer a combination of group and individual loans.\(^2\) An interesting contrast to the Grameen case is provided by the microfinance sector in India which is dominated by village-based groups that strictly adhere to joint liability.\(^3\)

In this paper, we study the relationship between borrower wealth and the benefits from group

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\(^1\) Wright et al. (2006) examine membership trends until 2005 and report that “Grameen took 27 years to reach 2.5 million members and then doubled that in the full establishment of Grameen II”.

\(^2\) De Quidt et al. (2012) contains a table classifying institutions reporting to MIX and Ghate (2008) lists 129 recognized microfinance institutions in India, many of which offer both types of contracts. The Bank for Agriculture and Agricultural Cooperatives (BAAC) in Thailand also allows its members to choose between group and individual loans (Ahlin and Townsend, 2007).

\(^3\) On March 31, 2007, there were 39.9 million members in these Self-Help Groups and they constituted about three-quarters of all borrowers in the microfinance sector (Srinivasan (2009), p 5, Table 1.2).
lending contracts. The surplus from group loans is determined by two opposing effects. Joint liability generates risk-pooling within groups because successful projects subsidize unsuccessful ones. It also increases the repayment burden when multiple projects fail. For small loan sizes, the first effect dominates because each successful project can cover a large fraction of the total loan. Group lending is therefore viable and valuable for moderately poor households and yet inaccessible to the very poor.\(^4\) We find that social sanctions within groups are always welfare improving because they augment the enforcement capacity of banks without ever being implemented in equilibrium. Sanctions do not however always improve outreach and, under certain conditions, even arbitrarily large social sanctions do not provide credit access to those denied individual loans.

Our work contributes to a well-established theoretical literature on the mechanisms through which joint liability can affect investment decisions and borrower welfare.\(^5\) It is most closely related to Besley and Coate (1995) who show that joint liability has ambiguous effects on repayment rates through a mechanism similar to the one described above.\(^6\) We build on the Besley-Coate idea using a more general framework in which wealth varying across internally homogeneous groups of arbitrary size. Our analysis reveals that the two-person groups that have dominated the theoretical literature on joint liability are particular because the fraction of members contributing to repayment must be at least one-half. This fraction varies more smoothly in larger groups and it is precisely this variation that results in a positive correlation between borrower benefits and wealth. A discussion of the higher cost of borrowing for the poor has been largely absent from the joint liability literature even though it has been central to the study of income dynamics in the presence of credit constraints (Galor and Zeira, 1993; Banerjee and Newman, 1993; Matsuyama, 2000).\(^7\) Since lending organizations vary widely both in the

\(^4\)The limited prevalence of group loans among the poor could be for reasons unrelated to the incentive structure of credit contracts and we abstract from these here. For instance, such families may not have the characteristics or networks required for entrepreneurial success or transactions costs of lending to them may be high.

\(^5\)Comprehensive surveys of this literature can be found in Ghatak and Guinnane (1999) and Armendariz de Aghion and Morduch (2005).

\(^6\)Joint liability may also improve borrower selection (Ghatak, 1999) or lead to effort and project choices through peer monitoring (Stiglitz, 1990; Banerjee et al., 1994; Armendariz de Aghion, 1999). We ignore these effects of contractual choice in our model to better focus on the role of repayment incentives.

\(^7\)A different type of generalization of the Besley-Coate model considers more complicated contracts. In Rai and Sjostrom (2004) borrowers with unsuccessful projects can send messages to the bank about successful borrowers who have not repaid their loan. Bhole and Ogden (2010) allow banks to recover only part of the contracted amount.
wealth of their members and the sizes of groups they serve, we believe these results provide insight into the design of efficient lending contracts.

Taking models such as ours to the data on group lending programs is a challenge. Microfinance institutions operate in many different environments and their objectives vary from pure profit-maximization to broader social missions. Besides, empirical work on group lending programs typically estimates average impacts rather than the relationship between benefits and loan size, which is our main focus. While this makes it difficult to test the finer predictions of our model, our theoretical results are broadly consistent with the patterns found in experimental and observational data.

Several studies on targeting by microfinance institutions find that the poorest households in an area are underrepresented in group lending programs. Morduch (1998) finds that eligibility criteria for membership in the Grameen Bank were frequently violated and the non-poor were admitted as members. With the introduction of Grameen II in 2002, the Grameen Bank explicitly acknowledged that the poor are often best served outside groups.8 Dewan and Somanathan (2011) and Coleman (2006) also find participation rates rising in income in very different regional contexts.9 Sebstad and Cohen (2001) and Hermes and Lensink (2011) survey the literature on access to microfinance and emphasize the trade-off between financial sustainability and outreach.

In our model, larger bank sanctions allow groups to achieve higher repayment rates. If we interpret sanctions as a rise in the future cost of credit, there is substantial evidence that these costs influence behavior. Karlan and Zinman (2009) collaborate with a South-African bank in an unusual field experiment and show that commitments of lower future interest rates improve repayment on current loans. Other experiments find similar results on the effectiveness of dynamic incentives and also emphasize the importance of social ties in repayment decisions (Abbink et al., 2006; Cassar et al., 2007). Giné and Karlan (2009) study a microfinance institution in the Philippines that randomly assigned new areas to either joint or individual liability. As in the

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8According to their website (Grameen Bank, 2009):

A destitute person does not have to belong to a group...Bringing a destitute woman to a level where she can become a regular member of a group will be considered as a great achievement of a group.

Grameen case, the introduction of individual liability attracted many more clients. Although there was no significant difference in repayment rates across the two types of contracts, it is hard to interpret this finding since repayment rates were close to 100% under both regimes.

The next section describes our model and derives optimal contracts for individual and group loans. Section 3 compares credit outreach and borrower welfare under the two contracts in the absence of social sanctions. Section 4 characterizes the conditions under which social sanctions can improve access to credit and shows that they always improve the borrower welfare under group lending. Section 5 presents some results on group size and borrower welfare. Section 6 concludes.

2 The model

Our principal unit of analysis is a set of risk neutral households, each of whom can choose to invest in a project. The project requires one unit of capital and no other inputs. It yields a return $\rho$ with probability $\pi$ and zero otherwise. Wealth varies continuously over the $(0, 1)$ interval and households must therefore borrow if they choose to invest.

The banking system is competitive and banks offer depositors a gross return $r$, which is the opportunity cost of bank funds. They lend both to individuals and to groups of size $n$ under joint liability. To allow us to compare repayment incentives from individual and group lending contracts, we assume that all members within a group have the same level of wealth. Interest rates charged to both groups and individuals vary with repayment rates to equate the expected return from all contracts to $r$. Project returns are never observed by the bank. Under group-lending they are observed by members of the group.

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10While we have in mind the self-employment projects financed by many microfinance organizations, we do explicitly model effort decisions. The returns to the project in our model could of course be interpreted as net of effort costs.

11Assuming groups of homogenous wealth allows us to abstract from questions of within group redistribution to better focus on the risk pooling function of joint liability. In practice, many group lending programs have tried to ensure that members within groups are similar in terms of their initial endowments. In India, central banking authorities explicitly directed non-government organizations to foster savings and credit groups among households of “homogeneous background and interest” (National Bank for Agriculture and Rural Development, 1992).
The assumption of a single indivisible investment project requires some justification. One might expect poorer borrowers to invest in projects requiring smaller loans. In the Grameen case, for example, the poorest borrowers are often vegetable vendors or basket makers while slightly richer households invest in looms or small shops. Our purpose is not to model investment choices and the distribution of wealth in an economy as a whole, but rather to illustrate that for a given project, access to and benefits from joint liability are systematically related to initial wealth through differences in required loan sizes. In practice, higher initial wealth can lead to favorable credit contracts and access to a larger range of investment opportunities.

Banks have available non-pecuniary sanctions \( K \) which they can use to induce borrowers to repay their loans. These sanctions could, for example, represent harassment by bank officials, unfavorable credit contracts in the future or a loss of reputation. A commonly observed method of varying sanctions across customers is through changes in the terms of future loan contracts. Timely repayment often leads to lower interest rates on future loans, lower levels of monitoring and larger loan sizes. Evidence on the role of such dynamic incentives suggest that they are quantitatively important (Karlan and Zinman, 2009).

We denote the maximal sanction by \( \bar{K} \) and think of this as the enforcement capacity of a bank. All loan contracts specify a loan amount, an interest rate and the value of the sanction \( K \in (0, \bar{K}] \) that is levied if a borrower defaults. When a group loan is not repaid, all group members are sanctioned \( K \). Since bank sanctions provides no direct benefits to the bank and are costly to the borrower, competition ensures that the value of \( K \) chosen from the set of feasible contracts maximizes borrower welfare.

We begin by deriving the optimal contract for an individual borrower with wealth \( w \). A household with wealth \( w \) requires a loan \( L = 1 - w \). Those with unsuccessful projects necessarily default on their loans. If banks use sanctions to ensure all successful borrowers repay their loans, the competitive gross interest rate is \( \frac{r}{\pi} \) and the optimal sanction is

\[
K = \frac{r}{\pi} L. \tag{1}
\]

The bank will only lend as long as \( K \leq \bar{K} \). The largest available loan is therefore

\[
L = \frac{\pi \bar{K}}{r}. \tag{2}
\]
$ar{L}$ is a measure of credit outreach for individual loan contracts. All households prefer investing in the project to depositing their wealth in a bank if their expected return $\pi \rho$ is greater than their cost of credit, including sanctions. This requires the return $\rho$ to be greater than

$$\rho = \frac{1 - \pi}{\pi} \bar{K} + \frac{r}{\pi}.$$  \hspace{1cm} (3)

If $\rho \geq \bar{\rho}$ and $\bar{L} < 1$, there an unmet demand for credit among the poor. We assume both of these conditions.

A loan contract is then given by $(L, \bar{\pi}, K)$, where $K$ depends on $L$ according to (1). Expected gains from investment depend on project returns, expected sanctions and the opportunity cost of the borrower’s own funds and are given by

$$U_i = \pi \left( \rho - \frac{r}{\pi} L \right) - (1 - \pi) K - rw.$$  \hspace{1cm} (4)

On substituting for $K$ and $w$, this expression simplifies to

$$U_i = \pi \rho - r - \frac{(1 - \pi)}{\pi} r L$$

Those with smaller loans face lower bank sanctions and therefore enjoy higher benefits.

We now turn to optimal group contracts. Advocates for group lending have stressed the importance of both joint liability and social sanctions in increasing repayment rates. We begin by determining the gains from joint liability by loan size, assuming that groups have no additional enforcement capacity in the form of social sanctions. Section 4 extends the model to incorporate such sanctions.

A group of size $n$ in which all members have the same initial wealth $w$ requires a group loan of size $n(1 - w) = nL$. If the loan is repaid whenever there are $j$ or more successful projects within the group, the repayment rate is given by the binomial probability of $j$ or more successes in $n$ trials. We denote this by $B_j$ when there is no ambiguity about the $n$ and $\pi$ under consideration.\footnote{We use the notation $B(n, j, \pi)$ when required.} Banks break even when charging $\frac{r}{B_j}$. A group lending contract with per member loan size $L$ is
feasible if there exists an integer \( j \leq n \) for which:

\[
\frac{n}{j} \frac{r}{B_j} L \leq K
\]  

\((5)\)

The left hand side of the above inequality is the contribution per successful project when exactly \( j \) successes are realized within the group. Each successful member is willing to contribute only if this is less than the maximal bank sanction. Notice that the largest loan size for which this inequality is satisfied corresponds to the value of \( j \) for which the function \( jB_j \) is maximized. The following lemma characterizes this function and is useful in deriving many of our subsequent results:

**Lemma 1.** The function \( jB_j \) is single-peaked. It starts at zero, takes the value \( n\pi^n \) at \( j = n \) and attains a maximum at \( j^* \leq \lceil n\pi \rceil \), the smallest integer above \( n\pi \). If \( \pi < \frac{1}{n} \), it is decreasing for \( j \geq 1 \) and, if \( \pi > \frac{n(n-1)}{1+n(n-1)} \), it is increasing throughout.

**Proof.** See appendix

This implies that our search for an optimal group contract need only be over values of \( j \leq j^* \) since higher values of \( j \) correspond to bigger sanctions and lower repayment rates. Suppose however, that for a given loan size \( L \), there are multiple values of \( j \leq j^* \) for which \( (5) \) holds. Corresponding to each of these values of \( j \), we have a contract of the form \((L, \frac{r}{B_j}, K_j)\), where \( K_j \) is the sanction required for each successful member to pay their share of the loan when exactly \( j \) successes occur. Analogous to \((1)\) for individual loans, we can define the minimum sanction for which repayment by each of the \( j \) successful members is incentive compatible:

\[
K_j = \frac{n}{j} \frac{r}{B_j} L
\]

\((6)\)

Since \( jB_j \) typically increases in \( j \), groups require higher sanctions to raise their repayment rates. Our first result states that the benefit from higher repayment rates more than offsets the costs of higher sanctions and groups therefore choose the contract with the highest rate of repayment.

**Proposition 1.** Consider the set of feasible group contracts \((L, \frac{r}{B_j}, K_j)\) for a per-member loan of size \( L \). Groups prefer the contract with the highest repayment rate.

**Proof.** See Appendix
Groups voluntarily choose contracts with higher sanctions in order to increase the degree of cross-subsidization within the group. A competitive credit market is important for this result because it ensures that any surplus generated by the contract is kept by the group and not the lender.\textsuperscript{13}

The welfare of each borrower under a group contract \((L, \frac{r}{B_j}, K_j)\) depends on the realized number of successful projects in the group. For fewer than \(j\) successes, each member is sanctioned \(K_j\) and the successful members keep the return from their projects. With \(j\) or more successes, equal contributions by all successful members are used to repay the loan. The opportunity cost of capital is always \(rw\). Denote by \(\pi_k\) the probability of exactly \(k\) successes in \(n\) trials. The expected per-member benefits from the group loan can then be written as

\[
U_g^j = \sum_{k=0}^{j-1} \pi_k \left( \frac{k}{n} \rho - K_j \right) + \sum_{k=j}^{n} \pi_k \left[ \frac{k}{n} (\rho - \frac{n r L}{k B_j}) \right] - rw \\
= \frac{\rho}{n} n \sum_{k=0}^{n} k \pi_k - r(L + w) - (1 - B_j) K_j \\
= \pi \rho - r - (1 - B_j) \frac{n r L}{j B_j}
\]

Having described optimal individual and group contracts, we now use these to derive the equilibrium relationship between borrower wealth and the gains from investment.

3 Equilibrium credit contracts

Equilibrium in our model consists of two wealth intervals corresponding to the households that are offered individual and group loans respectively. Each household, if offered both contracts, chooses the one with higher benefits. We start by showing that there is always a set of households who are offered individual loans but have no access to group loans. In other words, joint liability per se cannot extend credit outreach.

\textsuperscript{13}De Quidt et al. (2012) is a recent paper in which a lender monopolist extracts some of the surplus from group loans.
Proposition 2. In the absence of social sanctions the largest loan available in a group lending contract is strictly smaller than in an individual contract.

Proof. The largest individual loan $\bar{L}$ is not feasible as a group loan if, for all $0 < j \leq n$, 

$$\frac{n}{j} \bar{L} \frac{r}{B_j} > K.$$ 

Substituting for $\bar{L}$ from (2), we can rewrite this condition as 

$$n\pi > jB_j$$

The LHS is the expectation of a binomial distribution with parameters $n$ and $\pi$ and writing this as a sum, we see that the above condition always holds:

$$n\pi = \sum_{k=0}^{n} k\pi_k > \sum_{k=j}^{n} k\pi_k \geq j \sum_{k=j}^{n} \pi_k = jB_j.$$ 

This result is driven by the fact that banks extract less from groups than from individual loans of size $\bar{L}$ and must therefore charge higher interest rates. Any individual loan resulting in a successful project gets repaid, so the repayment rate on all such loans, irrespective of their size, is simply $\pi$. In contrast, under group lending, banks recover nothing from successful projects in defaulting groups. These leakages from the banking system increase in loan size and cause the inequality in (5) to fail at $\bar{L}$.

Although joint liability does not extend outreach, it can increase borrower welfare for small loan sizes by pooling risk. As seen in Proposition 1, groups achieve this by opting for maximal sanctions. Using the expressions for borrower welfare derived in (4) and (7), the difference in welfare for a given loan size under the two contracts is given by:

$$U_g - U_i = Lr \left[ \frac{1 - \pi}{\pi} - \frac{(1 - B_j) n}{j} \right].$$

If a group contract $(L, \frac{r}{B_j}, K_j)$ satisfies the condition
\[
\frac{1 - \pi}{\pi} > \frac{(1 - B_j)n}{B_jj}
\]  
(9)

then each member is better off than they would be under an individual contract. The following result identifies one interesting case for which this condition always holds.

**Proposition 3.** If a per-member loan \( L \) is small enough to require only one successful project for group repayment, borrower welfare is always higher under group lending contracts.

*Proof.* Rewriting (9) for \( j = 1 \), we get
\[
1 - (1 - \pi)^n - n\pi(1 - \pi)^{n-1} > 0.
\]
The above expression is simply the binomial probability of more than one success in \( n \) trails and is greater than zero for all \( n \geq 2 \).

This result tells us that if a group needs only one success for repayment, group loans are always preferred to individual contracts. Any increase in loan size that does not require additional successes in the group further exploits the ability of groups to pool risk and we see from (8) that the difference in welfare from the two types of contracts is locally increasing in loan size.

## 4 Social sanctions

In this section we allow social sanctions within groups to augment the enforcement capacity of the bank. We show that these sanctions always increase borrower welfare from group loans and they may also extend credit outreach beyond \( \bar{L} \), the largest loan offered as an individual contract. We model these sanctions in a manner similar to Besley and Coate (1995). We assume that the group imposes a utility cost \( \gamma \) on members who are able to contribute their fair share of the group loan but choose not to do so. Since the size of this sanction does not depend on the number of contributing members, it is best interpreted as a fall in a borrower’s status within the
community or a partial or total exclusion from group activities.\textsuperscript{14} Members with unsuccessful projects are not sanctioned since they have zero returns and cannot contribute.\textsuperscript{15}

Once we allow for social sanctions, our feasibility constraint in (5) becomes

\[
\frac{n}{j} \frac{r}{B_j} L \leq \min(\rho, \bar{K} + \gamma)
\]  \hspace{1cm} (10)

A group lending contract with per member loan size \( L \) is feasible if there exists some \( j \) at which members with successful projects are both able and willing to pay their share of the loan. This requires that each member’s contribution be less than the return \( \rho \) and that it be lower than the sum of bank and social sanctions. We can think of these two constraints as the liquidity and group incentive constraints respectively. Given a loan size \( L \), let \( \tilde{j} \) be the smallest value of \( j \) for which (10) is satisfied. If fewer than \( \tilde{j} \) successes occur, the group defaults, the bank sanctions all members and no social sanctions are used. If, on the other hand, more than \( \tilde{j} \) successes are realized, each successful member contributes their fair share \( \frac{n}{\tilde{j}} \frac{r}{B_{\tilde{j}}} L \) towards repayment.

A natural next question is the degree to which social sanctions can extend outreach. Such sanctions relax the incentive compatibility constraint and allow a larger share of project returns to be extracted from each successful member. This increases possibilities for risk pooling and allows groups to repay the bank loan in states with fewer successful projects. Notice however from (10) that higher sanctions can only improve repayment incentives as long as the total value of bank and social sanctions are below the project return \( \rho \). Beyond this point it is not repayment incentives but project returns that restrict the size of group loans. The following proposition outlines the conditions under which social sanctions allow households requiring more than \( \bar{L} \) to obtain credit through group loans.

**Proposition 4.** Let social sanctions \( \gamma \geq \bar{\rho} - \bar{K} \). Then \( \bar{L} \) is a feasible group loan if either of the following conditions hold:

1. \( n = 2 \)

2. \( \pi \leq \frac{1}{2} \) and \( n \pi \) is an integer

\textsuperscript{14}There is no distinction between the total sanction and the sanction per contributing member in the Besley-Coate model because only groups of two are permissible.

\textsuperscript{15}One might ask whether it is plausible that group members are sanctioned if all members are successful and do not repay, i.e. they default collectively. We could allow for no sanctions in this case, but this would complicate the model and for moderately sized groups, this is a low probability event.
3. \( \pi \leq \frac{1}{4} \)

In contrast, for large \( \pi \), \( \rho = \bar{\rho} \) and \( n > 2 \), there always exist loan sizes which will be offered as individual loans but not as group loans even if social sanctions are arbitrarily large.

Proof. See Appendix.

When \( \gamma \geq \bar{\rho} - \bar{K} \), as is assumed in the proposition, repayment is constrained by project returns and not by repayment incentives. Recall that \( \bar{\rho} \) is the minimum value of the project return at which investment is profitable for all wealth levels with individual loan contracts. The project success rate \( \pi \) affects outreach under group lending because when projects are risky, returns in the good state are high and social sanctions allow these to be extracted from successful members. This proposition also illustrates that the two-member groups that have been exclusive focus of much of the existing literature on group lending are a special case for which high enough sanctions always extend outreach.\(^{16}\)

If the project return \( \rho \) is larger than \( \bar{K} + \gamma \), repayment incentives are binding. The bank sanction needed to achieve a group repayment rate \( B_{j} \) is now lower than in (6) and is given by:

\[
K'_{j} = K_{j} - \gamma = \frac{n}{j} \frac{r}{B_{j}} L - \gamma
\]

The optimal group contract for a loan of size \( L \) with social sanctions \( \gamma \) is now \((L, \frac{r}{\tilde{B}_{j}}, K'_{\tilde{j}})\) where \( \tilde{j} \) is the smallest of value of \( j \) for which \( K'_{\tilde{j}} \leq \bar{K} \). The benefit from a group contract with social sanctions \( \gamma \) is:

\[
U_{j}^{g} = \pi \rho - r - (1 - B_{j}) \left( \frac{n}{j} \frac{r}{B_{j}} L - \gamma \right)
\]

The following result summarizes the effects of social sanctions on borrower welfare and on the repayment rates:

**Proposition 5.** For group lending contracts that rely on some bank enforcement, higher social sanctions always increase borrower welfare. If the largest per member loan requires at least two

\(^{16}\)When \( n\pi \) is an integer, the mean, median and mode of a binomial distribution coincide. This allows us to prove the first part of this proposition under weaker conditions on \( \pi \).
or more successful projects within the group for repayment, higher social sanctions also raise repayment rates.

A formal proof of the proposition is in the appendix. Social sanctions are welfare improving because they substitute for bank sanctions but are never imposed in equilibrium. They also improve repayment incentives by increasing the total value of sanctions faced by each member.

5 Group size

The literature on group lending has focussed on two-member groups and fixed loan sizes. In this section, we explore the interaction between borrower wealth, project characteristics and group size. We show that the relationship between group size and borrower welfare is not, in general monotonic. We begin with analytical results for some special cases followed by some numerical illustrations.

**Proposition 6.** If \( \pi \geq \frac{n(n-1)}{1+n(n-1)} \), the largest loan available to a group of size \( n \) is both feasible and generates higher per-member welfare in smaller groups. In contrast, for sufficiently small loans and no social sanctions within groups, a fall in group size lowers welfare and two member groups are never optimal.

**Proof.** See appendix

The second part of the proof is intuitive. Group loans raise welfare whenever they lead to lower expected sanctions. If a loan is small enough to require only one success for repayment, large groups are favored because the probability of one or more successes, \( B(1, n, \pi) \), is increasing in \( n \). The result will hold for zero or small social sanctions. If social sanctions are large, they could substitute fully for bank sanctions and since no sanctions will then be implemented in equilibrium, larger groups would have no added benefits.

For the first part of the result, we know from Lemma 1 that when the project success probability is greater than \( \frac{n(n-1)}{1+n(n-1)} \), the largest loan offered to a group of size \( n \) requires all members to succeed. This is more likely for smaller groups. To complete the argument, we need to show that loans of this size are feasible for a smaller group. This is done in the appendix.
Moving away from these limit cases we find non-monotonic effects of group size on the benefits per borrower. Figure 1 illustrates how the optimal group contract and the gains from group lending change as a function of group size. We include all even-sized groups of between 2 and 40 members. The required loan size is $L = .25$ and the project success probability is $\pi = .5$. Bank and social sanctions are $.6$ and $.4$ respectively and the risk free interest rate is 20 per cent. Starting with a group of size 2, an increase in size initially lowers welfare because the optimal contract for sizes of 4 and 6 requires at least half of the members to succeed, and the probability of this event is decreasing in size. In a group of size 8, the optimal contract is based on 3 successful projects. The fraction of successes in the group lending contract therefore falls, resulting in higher repayment rates and borrower benefits. The gain from group loans follows a scissor-like pattern which mirrors the changes in the value of $j$ on which the optimal group contract is based. This example illustrates that general results on size effects are elusive.

6 Conclusion

This paper is motivated by the now common observation that group lending with joint liability is more successful for moderately poor households than for the very poor. We highlight one mechanism which generates this pattern of benefits by examining how credit contracts vary by
loan size. We characterize conditions under which individual contracts support larger loan sizes than group lending. We also show that among those who have access to group loans, wealthier households gain most because they can pool risk more effectively, resulting in higher repayment rates and infrequent bank sanctions.

Our paper extends the standard approach in the group lending literature in two important respects. First, we move away from two person groups to arbitrary group sizes. Second, we allow the nature of the credit contract to vary by project and borrower characteristics. We believe these extensions are significant, not least because both credit contracts and group sizes observed in the microfinance sector vary considerably. To focus on repayment incentives we ignored the effects of group lending on borrower selection and on project and effort choices. There may be interesting interactions between wealth and credit contracts along these dimensions that remain unexplored and generalizing existing models to address these aspects of group lending could be fruitful.

It is difficult to link our model tightly to existing empirical studies on group lending because these rarely focus on the relationship between loan size and impact. Our analysis suggests that this is a useful direction to pursue empirically. In terms of policy, our results suggest that effective strategies to expand rural credit require a mix of contractual arrangements, and the excessive focus on group lending that we have witnessed in recent years is perhaps misplaced.

Appendix

Proof of Lemma 1

Denote by $\lfloor n\pi \rfloor$ be largest integer below $n\pi$ and by $\lceil n\pi \rceil$ the smallest integer above it. The function $jB_j$ takes the value zero at $j = 0$ and is positive for all $j > 0$. We begin by showing that it is decreasing to the right of $\lceil n\pi \rceil$. We then establish that it is single-peaked by showing that, if for some $j^* < \lceil n\pi \rceil$, $jB_j > (j + 1)B_{j+1}$, then this relationship holds for all $j$ in the range $j^* \leq j \leq \lfloor n\pi \rfloor$. Finally, for the case where $\lceil n\pi \rceil = n$, we derive the lower bound on $\pi$ given in the lemma for which $jB_j$ is increasing throughout.

1. $jB_j$ is decreasing to the right of $j = \lceil n\pi \rceil$:
Consider \( n > j \geq \lceil n\pi \rceil \). The function \( jB_j \) is strictly decreasing at \( j \) if \( jB_j > (j + 1)B_{j+1} \). The LHS can be written as \( jB_{j+1} + j\pi_j \) and rearranging terms gives us the condition:

\[
j\pi_j > B_{j+1}
\]  

(13)

The binomial probabilities, \( \pi_j \) and \( \pi_{j+1} \) are respectively

\[
\pi_j = \pi^j(1 - \pi)^{n-j} \frac{n!}{j!(n-j)!}
\]

and

\[
\pi_{j+1} = \pi^{j+1}(1 - \pi)^n \frac{n!}{(j+1)!(n-j-1)!}.
\]

Writing \( \pi_j \) in terms of \( \pi_{j+1} \) and multiplying by \( j \), we get

\[
j\pi_j = \left( \frac{j}{n-j} \frac{1 - \pi}{\pi} \right) (j + 1) \pi_{j+1}
\]

But \( \frac{j}{n-j} \frac{1 - \pi}{\pi} \geq 1 \) if \( j \geq n\pi \), which implies

\[
j\pi_j \geq \pi_{j+1} + j\pi_{j+1}.
\]

In the same way, we can express \( j\pi_{j+1} \) as a function of \( \pi_{j+2} \):

\[
j\pi_{j+1} = \left( \frac{j}{n-j-1} \frac{1 - \pi}{\pi} \right) (j + 2) \pi_{j+2}.
\]

Since \( \frac{j}{n-j-1} \frac{1 - \pi}{\pi} > 1 \) for \( j \geq n\pi \),

\[
j\pi_{j+1} > \pi_{j+2} + j\pi_{j+2}
\]

Combining the expressions for \( j\pi_j \) and \( j\pi_{j+1} \) we get

\[
j\pi_j > \pi_{j+1} + \pi_{j+2} + j\pi_{j+2}.
\]

For all \( k \leq n - 1 \), we can continue expressing \( j\pi_k \) as a function of \( \pi_{k+1} \), and we obtain:

\[
j\pi_j > \sum_{k=j+1}^{n} \pi_k = B_{j+1}.
\]

As a result, \( jB_j \) is strictly decreasing to the right of \( \lceil n\pi \rceil \).

2. \( jB_j \) is single-peaked to the left of \( \lceil n\pi \rceil \):

We use the property of a Binomial distribution that the mode \( M \) of the distribution is either \( \lfloor n\pi \rfloor \) or \( \lceil n\pi \rceil \) (Kaas and Buhrman, 1980).
Let us first consider any value $j', \ 1 \leq j' \leq \lfloor n\pi \rfloor$ for which $jB_j$ is increasing at $j'$. From (13), this implies that $j'\pi_{j'} \leq B_{j'+1}$. But since $\pi_j$ is maximized at the mode, which is at least as large as $j'$, the LHS $j\pi_j$ is increasing for all $j < j'$. The upper tail probability, $B_{j+1}$ is strictly decreasing in $j$ throughout. It follows that $\forall j < j'$, $jB_j$ must be increasing at $j$.

Now suppose that $j'' \leq \lfloor n\pi \rfloor$ is the smallest value of $j$ at which $B_j$ is decreasing, or equivalently, $j\pi_j \geq B_{j+1}$. Since $j''$ is less than the mode, this inequality must hold for all $j$ between $j''$ and the mode by the same reasoning given above, i.e. $j\pi_j$ is increasing in $j$ until the mode and $B_{j+1}$ is strictly decreasing in $j$. If no such value exists, $jB_j$ is increasing throughout.

3. For $jB_j$ to be increasing throughout, we require the mode to be greater than $(n-1)$ and in addition, using (13), $(n-1)\pi_{n-1} < \pi_n$. We can rewrite this inequality as

$$n(n-1)\pi^{n-1}(1-\pi) < \pi^n$$

or

$$\pi > \frac{n(n-1)}{1+n(n-1)}$$

However, at the values of $\pi$ satisfying the above condition, $n\pi > n-1$ so that $M \geq n-1$.

**Proof of Proposition 1**

Consider a loan of size $L$ and denote by $R_j$ the net benefit to the group from a contract in which repayment occurs whenever there are $j$ or more successes. The interest rate charged is $\frac{r}{B_j}$ and the sanction required for incentive compatibility is $K_j = \frac{r}{j B_j} L$. Recall that such a loan is feasible if $K_j \leq \bar{K}$. We will show that $R_j > R_{j+1}$ whenever both contracts are feasible.

We can restrict our consideration to contracts with $j+1 \leq \lfloor n\pi \rfloor$ (the smallest integer greater than or equal to $n\pi$) since we know by Lemma 1 that contracts with higher values of $j$ are never optimal. We can write the difference in benefits from $R_j$ and $R_{j+1}$ as

$$R_j - R_{j+1} = \Delta_{j-1} + \Delta_j + \Delta_{j+1}.$$ 

$\Delta_{j-1}$ is the difference in group returns under the two contracts for states with fewer than $j$ successes and $\Delta_j$ and $\Delta_{j+1}$ are the corresponding differences when exactly $j$ successes occur and more than $j$ successes occur. In the first case, the group is sanctioned under both contracts,
in the third case, it is not sanctioned under either contract, and when exactly \( j \) successes occur, it is sanctioned under \( R_{j+1} \) but not under \( R_j \). The difference in group returns under the two contracts for each of these states is therefore

\[
\Delta_{j-1} = \sum_{l=0}^{j-1} \pi_l \left[ (l \rho - nK_j) - (l \rho - nK_{j+1}) \right] = n(1 - B_j)(K_{j+1} - K_j)
\]

\[
\Delta_j = \pi_j \left[ (j \rho - \frac{nLr}{B_j}) - (j \rho - nK_{j+1}) \right] = \pi_j nK_{j+1} - \pi_j \frac{nLr}{B_j}
\]

\[
\Delta_{j+1} = \sum_{l=j+1}^{n} \pi_l \left[ \frac{nLr}{B_{j+1}} - \frac{nLr}{B_j} \right] = nLr \frac{B_j - B_{j+1}}{B_j B_{j+1}} B_{j+1} = \pi_j \frac{nLr}{B_j}
\]

Summing these three terms gives us

\[
R_j - R_{j+1} = n(1 - B_j)(K_{j+1} - K_j) + \pi_j nK_{j+1}
\]

and substituting for \( K_j \) and \( K_{j+1} \) we have \( R_j - R_{j+1} > 0 \) as long as

\[
n^2 Lr \left[ \frac{\pi_j}{(j+1)B_{j+1}} + (1 - B_j) \frac{jB_j - (j+1)B_{j+1}}{jB_j(j+1)B_{j+1}} \right] > 0
\]

or

\[
\pi_j + (1 - B_j) \frac{jB_j - (j+1)B_{j+1}}{jB_j} > 0
\]

Using \( B_{j+1} = B_j - \pi_j \) we need

\[
(j + 1)\pi_j > B_j \left[ 1 - B_j + \pi_j \right]
\]

or

\[
(j + 1)\pi_j > B_j \left[ \pi_0 + \pi_1 + \cdots + \pi_j \right].
\]

The sum of the \((j + 1)\) terms on the RHS of the above inequality is always smaller if \( \pi_j \) is increasing in \( j \). Since any contract under consideration must have \((j + 1) < \lceil n\pi \rceil\), we always have \( j \leq \lfloor n\pi \rfloor \) and we know that \( \pi_j \) is increasing in this range. Therefore, for all feasible contracts, we have \( R_j > R_{j+1} \)

**Proof of Proposition 4**

We start with the first part of the proposition and show that individual contracts can be implemented as group loans under the stated conditions on \( n \) and \( \pi \). When social sanctions are
higher than $\bar{\rho} - \bar{K}$, we show that even if returns take their minimum value of $\bar{\rho}$, repayment is feasible for low values of $\pi$.

A group of loan of size $\bar{L}$ can be repaid if there exists $j'$ such that

$$\frac{n}{j'} \frac{r}{B_{j'}} \bar{L} \leq \bar{\rho}$$

Substituting for $\bar{L}$ from Equation (2) and for $\bar{\rho}$ from Equation (3), this condition can be rewritten as

$$\frac{n}{j'} \frac{\pi}{B_{j'}} \bar{K} \leq \frac{1 - \pi}{\pi} \bar{K} + \frac{r}{\pi}$$

Now, since $L_i = \frac{\pi K}{r} < 1$, $\frac{r}{\pi} > \bar{K}$. It is therefore enough to show that

$$\frac{n}{j'} \frac{\pi}{B_{j'}} \bar{K} \leq \frac{1 - \pi}{\pi} \bar{K} + \bar{K},$$

or equivalently

$$\frac{n}{j'} \frac{\pi}{B_{j'}} \leq \frac{1}{\pi}. \quad (14)$$

The rest of the proof uses two well-known results on the binomial distribution from Kaas and Buhrman (1980). The first is that for integer values of the mean $n\pi$, the median, $m$, is equal to the mean. The second is that for non-integer values of the mean, the median is either $\lceil n\pi \rceil$, the smallest integer above the mean or $\lfloor n\pi \rfloor$, the largest integer below the mean.

We will first prove the result for integer values of $n\pi$, with $\pi \leq \frac{1}{2}$. We will then consider non-integer values of $n\pi$ and finally show that $\bar{L}$ is always a feasible group loan when $n = 2$.

If $n\pi$ is an integer, set $j' = n\pi = m$. The inequality in (14) is now

$$\pi \leq B_m$$

But since $m$ is the median, $B_m \geq \frac{1}{2}$ so with $\pi \leq \frac{1}{2}$ this is always true.

Now consider non-integer values of the mean. The median in this case must either be $\lceil n\pi \rceil$ or $\lfloor n\pi \rfloor$. If the median $m = \lceil n\pi \rceil$ then set $j' = \lceil n\pi \rceil$ in (14). Since $n\pi < \lceil n\pi \rceil$, the LHS is of (14) is smaller than when $n\pi$ is an integer and the result goes through a fortiori.

If instead $m = \lfloor n\pi \rfloor$, consider first $j' = \lfloor n\pi \rfloor = 1$. In this case $n\pi$ is strictly less than 1 and (14) holds if $B_1 > \pi$. But $B_1 = 1 - (1 - \pi)^n$ so this is always true. If $\lceil n\pi \rceil > 1$, set $j' = \lceil n\pi \rceil = m$. 

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We can now rewrite (14) as
\[ \frac{n}{m} \frac{\pi}{B_m} \leq \frac{1}{\pi}. \]

But since \( n\pi < \lceil n\pi \rceil \), it is enough to show that
\[ \pi \leq \frac{m}{m+1} B_m \]

The ratio \( \frac{m}{m+1} \) is increasing in \( m \) and takes its minimum value of \( \frac{1}{2} \) when \( j = 2 \). The minimum value taken by \( B_m \) is also \( \frac{1}{2} \), so the above inequality holds whenever \( \pi \leq \frac{1}{4} \).

For the case where \( n = 2 \), set \( j' = 2 \) and (14) can now be expressed as \( \frac{2}{2} \frac{\pi^2}{B_2} \leq 1 \). But \( B_2 = \pi^2 \), so this holds with equality.

We finally show that for large \( \pi \) and \( n > 2 \), there always exist loan sizes that will be implemented as individual but not group contracts.

By Lemma 1, the function \( jB_j \) is maximized at \( j^* = n \) when \( \pi > \frac{n(n-1)}{1+n(n-1)} \) and we know that \( B_n = \pi^n \). If the group loan is equal to \( \bar{L} \), each successful member in the group is required to contribute \( \frac{r}{\pi} \bar{L} \), while the minimal return to the project \( \bar{\rho} \) is
\[ \bar{\rho} = \frac{(1-\pi)}{\pi} \bar{K} + \frac{r}{\pi} = \frac{(1-\pi)}{\pi} r\bar{L} + \frac{r}{\pi} \]

Required payments are therefore higher than \( \bar{\rho} \) whenever
\[ \bar{L} > \frac{\pi^n}{\pi - \pi^{n-1} + \pi^n}. \]

If \( n > 2 \), the RHS of the above expression is smaller than one and there exist loan sizes \( \bar{L} \) that are not feasible group loans even for arbitrarily high social sanctions.

**Proof of Proposition 5**

For the first part of the proof, consider an initial level of social sanctions \( \gamma_0 \) and a group loan of size \( L \) that is offered in equilibrium and repaid whenever there are at least \( j \) projects are successful within the group. This implies that \( j \) is the smallest integer for which the following inequality holds:

\[ n \frac{r}{B_j} L \leq \bar{K} + \gamma_0 \]
For any sanctions \( \gamma > \gamma_0 \), this inequality continues to hold and so the same repayment rate \( B_j \) is feasible. The benefits to a borrower from this loan are given by (12) which is reproduced below and is clearly increasing in \( \gamma \) for a given \( j \).

\[
U^g_j = \pi \rho - r - (1 - B_j) \left( \frac{n}{j} \frac{r}{B_j} L - \gamma \right)
\]

Groups may be able to do further increase expected benefits by choosing contracts with repayment rates. If the parameters \( \rho, \bar{K} \) and \( \gamma_0 \) are such as to allow all group loans to need only one successful project to enable repayment, this is clearly not possible, since \( B_1 \equiv B(n, 1, \pi) \) is the maximum possible repayment rate. If, on the other hand, some group loans require more than one success, we show that there always exist loan sizes for which repayment rates increase.

Denote by \( L^g_1(\gamma) \) the largest per member group loan that can be repaid with only one success if social sanctions are equal to \( \gamma \). This is given by \( L^g_1(\gamma) = (\bar{K} + \gamma) \frac{B_1}{nr} \). The difference in this loan size for social sanctions \( \gamma \) and \( \gamma_0 \) is given by \( \frac{B_1}{nr} (\gamma - \gamma_0) \). All groups for which the per-member loan required lies in the interval \( (L^g_1(\gamma_0), L^g_1(\gamma)) \) have higher loan repayment rates and higher net benefits from borrowing than they would with sanctions \( \gamma_0 \). More generally, if \( j \) successes are required with social sanctions \( \gamma \),

\[
L^g_j(\gamma) = (\bar{K} + \gamma) \frac{jB_j}{nr}.
\]

Groups with loan sizes in the interval \( (L^g_j(\gamma_0), L^g_j(\gamma)) \) required at least \( (j + 1) \) success for repayment with \( \gamma = \gamma_0 \) and now require at most \( j \) successes. Since the distribution of wealth has a continuous distribution of the \((0, 1)\) interval and repayment rates increase for some loan sizes, the overall repayment under group lending must go up with an increase in social sanctions if initial repayment rates are below their maximum feasible level.

**Proof of Proposition 6**

For a group of size \( n \), we know from Lemma 1 that if \( \pi \geq \frac{n(n-1)}{1+n(n-1)} = \bar{\pi}(n) \), the largest loan requires all \( n \) members to succeed. Since \( \bar{\pi}(n) \) is increasing in \( n \), smaller groups will also require all members to succeed if \( \pi \geq \bar{\pi}(n) \). For these values of \( \pi \), we see from (5) that the largest loan available to groups with \( n \) members is

\[
\bar{L}^g_n = \bar{K} \bar{\pi}^n
\]
which is decreasing in $n$. The largest loan available to a group of size $n$ is therefore also available to a group of size $n - k$.

For a loan of size $\bar{L}^g(n)$, the difference in borrower welfare for groups with $(n - k)$ members and $n$ and members is at least

$$r\bar{L}^g(n)\left[\frac{1 - \pi^n}{\pi^n} - \frac{1 - \pi^{n-k}}{\pi^{n-k}}\right] > 0.$$ 

This is a lower bound on the relative benefits of the smaller group since, for a loan of size $\bar{L}^g(n)$, a group with $(n - k)$ members may need less than $(n - k)$ successes for repayment and welfare is increasing in the repayment rate.

We now show that borrower welfare is increasing in group size for very small loans. Consider a loan size $L$ which is repaid by a group of size $n$ and a group of size $n - 1$ with only one success. The difference in net benefits for a borrower in a $n$ member group relative to an $(n - 1)$ member group is given by

$$U^g_n - U^g_{n-1} = (1 - B(1, n - 1, \pi))\frac{(n - 1)Lr}{B(1, n - 1, \pi)} - (1 - B(1, n, \pi))\frac{nLr}{B(1, n, \pi)}$$

$$= Lr\left[\frac{(1 - \pi)^{n-1}}{1 - (1 - \pi)^{n-1}}(n - 1) - \frac{(1 - \pi)^n}{1 - (1 - \pi)^n}\right]$$

$$= Lr\frac{(1 - \pi)^{n-1}}{(1 - (1 - \pi)^{n-1})(1 - (1 - \pi)^n)}[n\pi - 1 + (1 - \pi)^n]$$

The term in parenthesis can be written as

$$n\pi - (1 - (1 - \pi)^n) = \sum_{l=0}^{n} l\pi_l - \sum_{l=1}^{n} \pi_l$$

which is strictly positive for all $n \geq 2$. 

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References


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