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MULTIPLE REFERRALS AND MULTIDIMENSIONAL CHEAP TALK

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In previous work on cheap talk, uncertainty has almost always been modeled using a single-dimensional state variable. In this paper we prove that the dimensionality of the uncertain variable has an important qualitative impact on results and yields interesting insights into the "mechanics" of information transmission. Contrary to the unidimensional case, if there is more than one sender, full revelation of information in all states of nature is generically possible, even when the conflict of interest is arbitrarily large. What really matters in transmission of information is the local behavior of senders' indifference curves at the ideal point of the receiver, not the proximity of players' ideal point.

Keywords: Cheap talk, information transmission, experts.

1. introduction

Since the seminal work by Crawford and Sobel (1982), cheap talk models have been the object of extensive study. On the theoretical front, the idea that agents may transmit information sending signals that do not directly affect their utility has been scrutinized in many different environments: when there is one sender and more than one receiver;[2] when there is more than one sender but one receiver;[3] and in a repeated game.[4] On the applied front, cheap talk has proved to be very useful to understand problems in many different fields: political science, finance, and macroeconomics among many others.[5] All the models in the literature, however, have studied an environment in which the policy decision and the underlying asymmetric information are unidimensional. Typically, the relevant decision to be made is modelled as a point in the real line. In this paper we analyze a model of cheap-talk in a multidimensional setting. We show that results in this environment are qualitatively different than the conventional results in one dimension. One important insight of the existing literature is that

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[2] See, for example, Farrell and Gibbons (1986).

[3] See Gilligan and Krehbiel (1989), Austen-Smith (1993), and Krishna and Morgan (1999, 2000).

[4] See Sobel (1985) and Morris (2001).

[5] Some other examples are Matthews (1989) and Austen-Smith (1990) in political science and Stein (1989) in macroeconomics.

informativeness of the equilibrium is necessarily related to the conflict of interest between senders and the receiver; this is not true in a multidimensional environment. Indeed, in our model, contrary to the unidimensional case, if there is more than one sender, full revelation of information is generically possible, even when the conflict of interest is arbitrarily large.

It is not difficult to imagine situations in which uncertainty and the policy space are multidimensional; in fact, the decision of a policy-maker is often the solution of a trade-off between different dimensions of a problem. Consider for instance the case of the members of a committee in the Congress who report to the Floor on a complex issue like an environmental bill. An important aspect in this problem is certainly the direct impact of the bill on, say, the level of ozone; but clearly the bill may have an impact on other relevant dimensions as well: for example, firms' profits or employment. The assumption that uncertainty and the policy space are unidimensional, therefore, is often unrealistic. The simplification might still be appropriate as a 'first order' approximation if it does not have a qualitative impact on the results; if this were the case, these models might be seen as 'reduced forms' of a more complex environment. However, the results of this paper suggest that the implications of these works become questionable if the dimensionality of the problem is an issue. The debate on the informational role of committees in the Congress is a relevant example in this sense. The fact that existing theories predict that little information is transmitted if the conflict of interest is large has been used as an argument to dispute an informational role of committees since empirically committees are composed by preference outliers.[6] However, this conclusion would not be valid in a multidimensional world.

To understand the intuition that drives the analysis, consider a policy-maker who chooses a two-dimensional policy $y = (y_1, y_2)$. The policy outcome is represented by the vector $x = y + \theta$, where $\theta \in \Re^2$ is the state of the world that is unknown to the policy-maker but observed by two experts who may each send a signal. The agents have quasi-concave utilities over outcomes and we normalize the ideal point of the policy-maker at the origin:[7] In Figure 1 we represent the indifference curves of the experts at the ideal point of the policy-maker.[8] The policy-maker may listen to the experts' advice, but then takes the optimal policy given his posterior beliefs. For example, if he is sure that the state is μ, then the optimal policy is $y = -\mu$. The key idea on which we build the analysis is that, when preferences are defined over a multidimensional policy space, agents with different ideal points still have common preferences over lower dimensional subsets. For example, a sender and a receiver may have very different preferred points on a plane; however the induced ideal point might be the same if they

[6] See Londregan and Snyder (1994), Krehbiel (1991), and Diermeier and Feddersen (1998), and Section 6 of this work for a more detailed discussion.

[7] When the outcome space is $\Re^d$, and utilities are strictly concave, an ideal point is a well-defined vector in $\Re^d$.

[8] In Figure 1, for simplicity, we have not drawn the policy-maker's indifference curves. The policy-maker wishes to minimize the distance between the outcome of the policy x and his ideal point, the origin in our normalization.

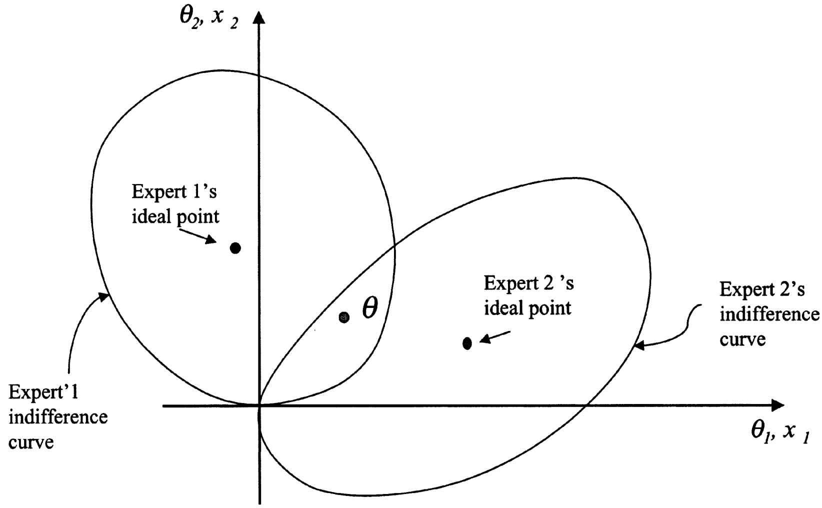

FIGURE 1.—An example.

are restricted to choose on a line in this plane. If the influence that the sender has on the receiver's action is restricted to these subsets, therefore, the sender would have no incentives to lie. The main result in our model, indeed, is that generically there exists an equilibrium in which it is possible to make each sender influential only on the dimension of common interest with the receiver, and these dimensions are sufficient to identify the state of the world.

This intuition can be easily seen in the particular example described by Figure 1. In spite of the fact that the ideal points (and so the conflict with the receiver) may be arbitrarily large, full revelation is possible in equilibrium for any state of nature θ. Consider these strategies: the experts truthfully reveal the state of the world; the policy-maker believes that the horizontal coordinate is equal to the horizontal coordinate of expert l's declaration and the vertical is equal to the vertical coordinate of expert 2's declaration. Consider now expert l's incentive to lie. Given that expert 2 follows the equilibrium strategy, expert 1, with a deviation $\hat{s}_1$ can induce only an outcome of the form:[9]

$$(1) \qquad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} - \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} = \begin{pmatrix} \theta_1 - \hat{s}_1 \\ \theta_2 - s_2(\theta) \end{pmatrix} = \begin{pmatrix} \theta_1 - \hat{s}_1 \\ 0 \end{pmatrix}$$

where the second equality follows from the definition of policy-maker's beliefs and the third follows from the definition of expert 2's equilibrium strategy $s_2(\theta)$. From Figure 1 it is immediately apparent that any deviation from truthful revelation is strictly worse for expert 1. Equation 1, in fact, implies that for any state θ,

[9] In this example, the variables θ_1 and θ_2 are, respectively, the horizontal and vertical coordinates of θ.

a deviation by expert 1 induces a point that necessarily lies on the horizontal axis: in this example, expert l's indifference curve is tangent with this coordinate at the origin and truthful revelation is strictly optimal. The same holds for expert 2; the strategy is optimal for the policy-maker who obtains for any θ his ideal point (the origin, after the normalization), and his posterior beliefs are correct in equilibrium.

Clearly this is a very special example, because in Figure 1 we have imposed the condition that indifference curves are tangent to the coordinate axes. However we show that the same intuition holds for generic quasi-concave utilities, even when each expert has an arbitrarily large conflict of interest with the policy-maker in all the dimensions of the problem and there are more than two dimensions. We also discuss how this intuition on the structure of the dimensionality of the environment can be applied to construct more informative equilibria in the case of a single sender. Finally, we discuss robustness of the equilibrium to collusion and other variations of the model.

The paper is structured as follows. In Section 2 we present the model in detail. In Section 3 we discuss the problem of full revelation in the unidimensional case: we first characterize a necessary and sufficient condition for the existence of a fully revealing equilibrium in this environment when no restriction on out-of-equilibrium beliefs is imposed; then we show that even when this condition holds, the equilibria rely on the construction of implausible out-of-equilibrium beliefs. In Sections 4 and 5 we study the multidimensional case, present the result described above, and discuss robustness to collusion and other extensions. We conclude in Section 6 with the discussion of how this theory may provide new insights into the role of committees in the legislative process.

2. THE MODEL

As in the example presented in the introduction, we consider the case of a policy-maker who has to take a decision given the advice of informed experts. Let $Y \equiv \Re^d$ denote the set of alternatives for the policy-maker. Following Austen-Smith and Riker (1987), we distinguish between the policy space and the outcome space. For any policy $y \in Y$, the outcome is $x = y + \theta$ where θ is a d-dimensional vector in $\Theta \equiv \Re^d$. Nature chooses θ according to some continuous distribution function $F(\theta)$ with density $f(\theta)$, support Θ and zero expected value. The policy-maker chooses y without knowledge of θ; the experts instead observe the realization of nature.

There are three players and each of them has von Neumann Morgenstern utility function u_i: $X \to \Re$, so all the agents do not care about the policy choice y, but about the policy outcome x. We assume that the utilities $u_i(x)$ are continuous, quasi-concave and differentiable. The first two agents are called experts (the set of experts is E); each expert has an ideal point x_i. The policy-maker has ideal point x_p that we normalize to be at the origin. For simplicity we will assume quadratic utilities, but this assumption is not essential for the main results and will be relaxed later: $u_i(x) = -\sum_{j=1}^{d}(x_i^j - x^j)^2$ where x_i^j and x^j are the jth coordinate of, respectively, i's ideal point and the outcome x. Utility functions (and therefore ideal points) are common knowledge.

The timing of the interaction is as follows: (a) at time 0 nature chooses θ according to $F(\theta)$ and each expert observes the true θ; (b) at time 1 the experts are asked to report simultaneously or privately the state of nature θ to the policy-maker; (c) the policy-maker decides y and the outcome that is realized is $x = y + \theta$. A strategy for the policy-maker is a function y: $\Theta \times \Theta \to Y$ that associates each couple of declarations of the experts to a policy in Y. A belief function for the policy-maker is a function μ: $\Theta \times \Theta \to P(\Theta)$ that for each pair of proposals of the experts assesses a posterior belief P over Θ. A strategy for the ith expert is a function s_i: $\Theta \to \Theta$: for each realization of nature the expert reports a state of the world in Θ. The equilibrium concept is the Perfect Bayesian Equilibrium. Since we are interested in the equilibrium in which, for any state of the world, information is perfectly transmitted, it is useful to give the following definition.

DEFINITION 1: A *fully revealing equilibrium* is an equilibrium in which, for each true state θ, $\mu^*(s_1^*(\theta), s_2^*(\theta))(\theta) = 1$.

Note that it is difficult to achieve a fully revealing equilibrium because the policy-maker's action must be sequentially rational. If the policy-maker could commit to a policy response to the declarations of the experts, then it would be much easier to implement a fully revealing equilibrium with strategies of a type that implies that if the experts' reports disagree, the policy-maker will choose a policy that is bad for everyone. This is generally possible but would not be sequentially rational.

It is useful to introduce a further definition and a simple lemma. In a fully revealing equilibrium, as defined in the previous paragraph, the true state is always revealed to the policy-maker. However this does not imply that in equilibrium experts report the truth; any function of the true state will do as well if the policy-maker 'understands' it. This multiplicity of equilibria is a well known characteristic of cheap talk games. We define a *truthful fully revealing equilibrium* as a fully revealing equilibrium in which experts report what they observe truthfully. Note also that it is not, in general, necessarily true that after the declarations, beliefs of the receiver are degenerate (i.e. beliefs that assign probability to only one state of nature). However, there is no loss of generality restricting the analysis to this case.

LEMMA 1: *If there exists a fully revealing equilibrium, then there exists a truthful fully revealing equilibrium. If the truthful fully revealing equilibrium has nondegenerate out-of-equilibrium beliefs, then there exists a truthful fully revealing equilibrium with degenerate out-of-equilibrium beliefs.*

This simple Lemma is useful in proving the nonexistence of a fully revealing equilibrium: If we prove that no truthful fully revealing equilibrium exists, then the lemma implies that no fully revealing equilibrium exists.[10] The first part is similar to the revelation principle. In any equilibrium the strategies map states of the world to messages and the belief function maps these into a posterior. Therefore we can always define a belief function that maps directly from states to the posterior as the composite function: this belief and the related reaction function are, by definition, such that no deviation is profitable. The second part follows from the fact that for any nondegenerate posterior out-of-equilibrium belief, we can always construct a degenerate belief that puts all the mass on the point that would be optimal given the original belief. Therefore, without loss of generality, we may focus on an equilibrium in which the receiver's beliefs assign probability one to one state. With a slight abuse of notation we define this point $\mu(s_1, s_2)$; given messages s_1, s_2 the receiver believes that the state is $\mu(s_1, s_2)$ with probability one.

3. FULLY REVEALING EQUILIBRIA IN ONE DIMENSION

In this section we study the problem of fully extracting information from experts in a one-dimensional setting. The goal of this section is to find conditions for the existence of fully revealing equilibria in order to compare the results with the case of higher-dimensional policy spaces.

Gilligan and Krehbiel (1989) first proposed a cheap talk model with multiple referrals, heterogeneous preferences, and asymmetric information along one dimension. Krishna and Morgan (1999) have shown that when the biases of both the experts are not too large there is a fully revealing equilibrium. However, they have not completely characterized the conditions in which a fully revealing equilibrium exists.[11] In this section we make two points. In Proposition 1 we find a necessary and sufficient condition for the existence of a fully revealing equilibrium when experts report simultaneously and have opposed biases. In Proposition 2, however, we show that the fully revealing equilibrium requires an implausible ad hoc choice of out-of-equilibrium beliefs.

The key to the existence of a fully revealing equilibrium in one dimension is Θ, the support of the state of the world. The first result of this section is that when Θ is large enough, the policy-maker can achieve a fully revealing equilibrium even if the agents have opposing biases. Define in this one-dimensional setting $\Theta = [-W, W]$.[12] The intuition behind the fact that fully revealing equilibria may exist is the following. Nothing prevents the policy-maker from having

[10] The proof of the lemma is omitted because the intuition is straightforward. See Battaglini (1999) for further details.

[11] Krishna and Morgan (1999) explicitly consider only the symmetric case $|x_1| = |x_2|$, and show that something equivalent to $|x_1| < (W/4)$ is a sufficient condition. They also say that "it is routine to verify that [full revelation] continues to be an equilibrium for all values of x_{c1} and x_{c2} such that $0 < x_{c1} < \frac{1}{4}$ and $-\frac{1}{4} < x_{c2} < 0$" (p. 14). Proposition 1 extends this sufficient condition and adds a complementary necessity result.

[12] For simplicity we assume that the support is symmetric, but the result is clearly not driven by this assumption.

out-of-equilibrium beliefs that are conditional on the observed messages. Notice that if this is the case, then deviations from a fully revealing equilibrium become more difficult because declarations reveal some information about the true state of the world. When an expert contemplates a deviation from a fully revealing equilibrium, in fact, he must assume that the other expert and the policy-maker follow the equilibrium strategies: therefore the expert knows that some information is revealed to the policy-maker by the other expert even if he deviates. The larger W is, the more freedom we have to find the function $\mu(s_1, s_2)$, and so the larger is the set of equilibria. In the following Proposition we focus on the case of simultaneous reports and opposed biases. Without loss of generality we assume that $x_1 < 0$, $x_2 > 0$.

PROPOSITION 1: *If $d = 1$ and the experts' ideal points (x_1, x_2) are on opposite sides of the policy-maker's ideal point, then $|x_1| + |x_2| > W$ is a necessary and sufficient condition for the nonexistence of a fully revealing equilibrium.*

In proving Proposition 1 in the Appendix, we show that if $W \geq |x_1| + |x_2|$, there is an entire class of equilibria that would be fully revealing. The following example is one of them. Senders report the truth in equilibrium ($s_i(\theta) = \theta$ for $i \in \{1, 2\}$). The policy-maker believes in the declaration if they are consistent ($\mu(s_1, s_2) = s$ if $s = s_1 = s_2$); if $s_1 \leq s_2$ he believes that the state is $(s_1 + s_2)/2$; and if $s_1 > s_2$ he believes that the state is W when $s_1 < 2x_2 - W$ and $-W$ when $s_1 \geq 2x_2 - W$. The policy that is implemented is optimal given these beliefs: $y(s_1, s_2) = -\mu(s_1, s_2)$.

The equilibria constructed in the proof of Proposition 1 and the one displayed above are just theoretical possibilities and we do not claim that any of them is plausible. For example, notice that in the equilibrium of the example above, if $s_1 > s_2$ and both s_1 and s_2 are near zero, the policy-maker believes that the state is extreme (in the example, W or $-W$): this does not seem plausible but it is necessary in order to discourage deviations, since the policy-maker cannot determine which expert has deviated. For this reason we argue in favor of a plausible restriction on out-of-equilibrium beliefs that eliminates these equilibria. The existence of a fully revealing equilibrium in the previous proposition relies on the fact that, following an out-of-equilibrium pair of messages, we are able to construct ad hoc beliefs that support the desired outcome. The assumption that the support of θ is bounded is a radical way of restricting out-of-equilibrium beliefs: since no state is larger than W, clearly no out-of-equilibrium belief can put weight on states larger than W. The a priori assumption that the support is bounded, however, is not a good assumption, and it is not necessary for restricting out-of-equilibrium beliefs. There is not a widely accepted way to refine beliefs in games with a continuum of types. We introduce a simple restriction on out-of-equilibrium beliefs that parallels consistency in the sequential equilibrium concept and has a straightforward interpretation.

We define an *ε-perturbed* game as the game described above in which each expert i independently observes the true state of nature with probability $1 - \varepsilon_i$

and with probability ε_i observes a random state $\tilde{\theta}$: a random variable with continuous distribution $G_i(\cdot)$, density $g_i(\cdot)$, and the same support as θ. We may interpret this as a situation in which each expert may commit a mistake with probability ε_i, or with this probability he is not an expert. In a perturbed game, we may compute the policy-maker's beliefs following any pair of messages θ, θ'; for any prior $f(\theta)$, beliefs depend on $\varepsilon = (\varepsilon_1, \varepsilon_2)$, $G(\cdot) = (G_1(\cdot), G_2(\cdot))$ and the experts' strategies: $\mu(G, \varepsilon, s^*(\theta))$. An equilibrium is *robust* if there exists a pair of distributions $G_i(\cdot)$ for $i = 1, 2$ and a sequence $\varepsilon^n = (\varepsilon_1^n, \varepsilon_2^n)$ converging to zero such that out-of-equilibrium beliefs of the equilibrium are the limit as $\varepsilon^n \to 0$ of the beliefs that the equilibrium strategies would induce in an ε^n-perturbed game: $\mu(G, \varepsilon^n, s^*(\theta)) \to \mu^*(s^*(\theta))$. The idea behind this restriction on beliefs is simple. In equilibrium, after receiving a pair of messages that are inconsistent, the policy-maker believes that at least one of the experts has made a mistake. Given the distribution of the state of nature and the distribution of the wrong observation, the policy-maker will assign a posterior probability to the event where each expert has observed the wrong variable, and with this posterior he accesses a belief on the state of nature. We require that there exist a $G_i(\cdot)$, $i = 1, 2$, and a sequence ε^n such that this process may be rationalized: this requirement imposes consistency on the construction of the posterior beliefs. Note that we are not making a specific assumption regarding the distribution of the wrong observation in order to preserve as much generality as possible to the restriction in beliefs. Given this definition, we have the following proposition.

PROPOSITION 2: *If $d = 1$ and both x_1 and x_2 are large enough in absolute value, then there exists no robust fully revealing equilibrium for any $W \in (-\infty, +\infty)$.*

As we have seen above, the equilibria constructed in Proposition 1 have beliefs that are 'discontinuous': Even if the declarations reveal almost the same state of the world, out-of-equilibrium beliefs are substantially different from this state. If the senders report two different states, the event in which both experts are simultaneously wrong is irrelevant for ε small, so the messages convey information because in equilibrium typically one sender is right. If one sender deviates with a message that reveals a state that differs very little from the true state (and so is very near the state revealed by the other sender), the posterior belief cannot be arbitrarily distant from this value. When beliefs are not too much 'discontinuous' in the messages, following an argument similar to Proposition 1, we can see that at least one expert has a deviation that induces a point that is strictly preferred to the outcome with full revelation.

Before moving on to the two-dimensional case, it is useful to summarize the results of this section. First, in a more general environment than Gilligan and Krehbiel (1989) it is possible to construct fully revealing equilibria with just two experts and one dimension. We have found a simple, necessary and sufficient condition for existence of a fully revealing equilibrium. Second, even when they exist, these equilibria are not plausible because, at least for x_1 and x_2 large, they rely on an ad hoc construction of out-of-equilibrium beliefs.

In the following section we show why and how the results differ if we consider the problem in more than one dimension.

4. EQUILIBRIUM IN TWO OR MORE DIMENSIONS

In the introduction we saw an example of a two-dimensional environment in which full revelation occurs for any state of nature. That example was special since we imposed the nongeneric condition that the indifference curves of the experts are tangent at the coordinate axes. In this section we generalize the intuition showing that in a multidimensional setting we have an equilibrium with full revelation for the general case in which this condition does not hold. In the following result we consider the two-dimensional case and continue to assume quadratic utilities. The simple generalization to the case with more than two dimensions and quasi concave utilities is presented in the discussion that follows.

PROPOSITION 3: *If* $d = 2$, *then for any* x_1 *and* x_2 *such that* $x_1 \neq \alpha x_2\ \forall \alpha \in \Re$, *there exists a fully revealing, robust equilibrium.*

PROOF: First, we provide some definitions. For any $a \in \Re$, and for any $i = 1, 2$, define

$$l_i(a) \equiv \{z \in \Re^2, \nabla u_i(0,0) \cdot z = a\}. \tag{2}$$

The locus $l_i(a)$ has a simple geometric interpretation (see Figure 2): $l_i(0)$ is the tangent of the indifference curve of the ith agent at the ideal point of the

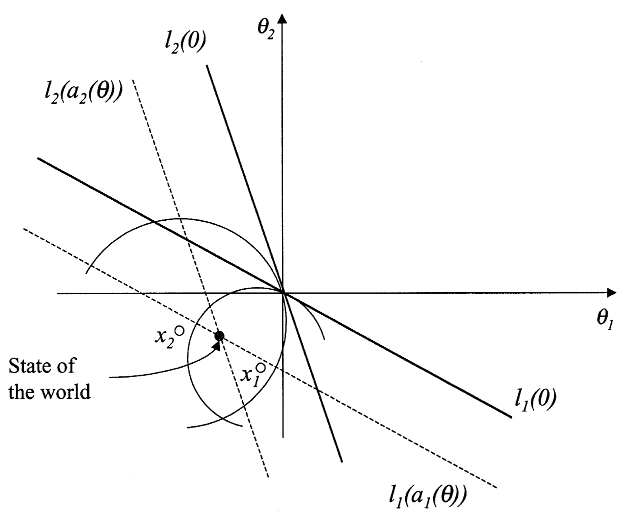

FIGURE 2.— The general case: it is possible to construct a new coordinate system to exploit experts' conflict of interest.

policy-maker; for any $a \in \Re, l_i(a)$ identifies one and only one line parallel to $l_i(0)$. Notice that, given the assumption that $\nexists \alpha \in \Re$ such that $x_1 = \alpha x_2, l_1(0)$ and $l_2(0)$ are linearly independent vector spaces. Linear independence of the ideal points, in fact, implies linear independence of the gradients of utility at the origin and therefore it implies linear independence of any $\alpha \in l_1(0), \beta \in l_2(0)$ since they are respectively orthogonal to $\nabla u_1(0,0)$ and $\nabla u_2(0,0)$. For this reason, $\forall \theta \in \Re^2$, there exists a unique vector $(a_1, a_2) \in \Re^2$ such that $\theta = l_1(a_1) \cap l_2(a_2)$. We may define the function $a(\theta)$: $\Re^2 \to \Re^2$ that, for each θ, associates the couple $a_1(\theta), a_2(\theta)$ uniquely defined by the previous equality. It is routine to verify that $\forall (a, b, c, d) \in \Re$:

$$l_i(a) \cap l_j(b) + l_i(c) \cap l_j(d) = l_i(a+c) \cap l_j(b+d). \tag{3}$$

We are now ready to prove the proposition. Each expert is required to report a number, s_i. Consider the following strategies and beliefs:

$$s_i(\theta) = a_j(\theta) \quad \forall i, j = 1, 2, \qquad i \neq j, \tag{4}$$

$$\mu(s_1, s_2) = l_1(s_2) \cap l_2(s_1), \tag{5}$$

$$y(s_1, s_2) = -\mu(s_1, s_2). \tag{6}$$

We claim that these strategies and this belief are a robust equilibrium. Given the other players' strategies, player i, choosing $\hat{s}_i$, may induce a point:

$$\begin{aligned} \theta - \mu\big(s_j(\theta), \hat{s}_i\big) &= \theta - l_i\big(s_j(\theta)\big) \cap l_j(\hat{s}_i) \quad \text{by (5)}, \\ &= \theta - l_i\big(a_i(\theta)\big) \cap l_j(\hat{s}_i) \quad \text{by (4)}, \\ &= l_i\big(a_i(\theta)\big) \cap l_j\big(a_j(\theta)\big) - l_i\big(a_i(\theta)\big) \cap l_j(\hat{s}_i) \\ &\qquad \text{by definition of } a(\theta), \\ &= l_i(0) \cap l_j\big(a_j(\theta) - \hat{s}_i\big) \quad \text{by Claim 1.} \end{aligned}$$

Since $\hat{s}_i$ is any number in $\Re$, agent i may choose any value for $(a_j(\theta) - \hat{s}_i)$ and so any point in $l_i(0)$. But, by construction, u_i has a unique point of tangency with $l_i(0)$: the origin, i.e. the ideal point of the policy-maker. The origin is the optimal outcome that i may induce, so the optimal strategy is to set $\hat{s}_i = a_j(\theta)$, as prescribed by the equilibrium. Therefore, there is no profitable deviation for agent i, $\forall i = 1, 2$. Clearly beliefs are consistent and the policy choice is optimal given the beliefs. The equilibrium is robust since there is no out-of-equilibrium message pair and therefore beliefs are always well defined. *Q.E.D.*

The key point in understanding the general case is that if x_1 and x_2 are linearly independent, we can construct two axes that span the policy space and exploit the conflict of interest between the two experts exactly in the same way as in the particular case described in the introduction, in which experts' indifference curves are tangent to the coordinate axes. See Figure 2 in which x_1 and x_2 are generic, linearly independent vectors in $\Re^2$. Given quadratic utilities, the tangents at $(0, 0)$ of the respective utilities are linearly independent, so they span. Note that if agent i had to choose an outcome in $l_i(0)$, he would choose $(0, 0)$, the ideal point of the policy-maker. But in equilibrium this is exactly what is going to happen. Agent j in fact will be honest on the l_j dimension so i is forced to choose in $l_i(0)$. Notice that if x_1 and x_2 are linearly dependent, then this equilibrium is not possible. In this case l_1 and l_2 would coincide and so they would not span the entire space. However, if x_1 and x_2 are 'just an ε' linearly independent (for example $x_1 = \alpha x_2 + \varepsilon, \alpha \in \Re$), then the result holds. This shows that the multidimensional analysis is qualitatively very different from the unidimensional.

As we mentioned in the introduction, the intuition of the result may be divided in two parts. On the one hand it is possible to make each sender influential only in the dimension of common interest with the receiver; on the other hand, the dimensions can be chosen to span the entire outcome space; so by combining the signals, the receiver may fully extract all the available information. To understand the connection between these two parts, it is useful to consider the case of a single sender in two dimensions. At first analysis, it might seem possible to use the same intuition to fully extract information from the expert, at least in the particular dimension that is tangent to his indifference curve. In the two senders case we only need a coordinate system in which the indifference curves of each sender is tangent to the set that the sender may induce when the other coordinate is truthfully revealed: we require only this because in equilibrium each sender i, given the other sender's message, may induce outcomes only in this set ($l_i(0)$ in the equilibrium that we have constructed). In the single-agent case we still need a tangency condition for full revelation, but this condition should hold not only in one point, the origin in our normalization, but for any parallel translation of this dimension. This would happen because, in the single-agent case, the other coordinate would not be revealed and therefore sender i would induce an outcome on a parallel to $l_i(0)$ with probability one. If the indifference curve is not tangent, we would have a conflict in this dimension too. With quadratic utilities, we can find a coordinate system[13] that satisfies this requirement. In this case, contrary to the single agent and unidimensional case studied by Crawford and Sobel (1982), in which only a countable set of informative signals can be sent in equilibrium, there would be full revelation of one dimension and the other dimension would be partially revealed through a partitional equilibrium. However, this would not be true in general. If we only assume quasi-concavity, the gradient of the indifference curve would generically depend on both dimensions and there is no guarantee that this is still tangent to a line parallel to $l_i(0)$, i.e. that there is no conflict of interest. This means that the two parts of the intuition described before are intimately related. It is not only necessary to have two experts because we need revelation in two dimensions; without the second expert, in fact, we would not generally have full revelation in any dimension.

This has two implications: first that the 'dimension of common interest' is an equilibrium phenomenon. It is not generally possible to identify ex ante a dimension of common interest for a sender and a receiver. This dimension needs to

[13] The tangent to the sender's indifference curve and its orthogonal.

be identified using some information on the state of the world. The concept of conflict of interest itself is actually endogenous in a multidimensional setting: As we have seen, it is not the distance in ideal points that is relevant, but the independence of the slope of the indifference curves in one point. The second implication is that the issue of what is the most informative equilibrium in a multidimensional environment with one sender is still open. The fact that, in the single-agent case, we cannot generically construct an equilibrium that is fully revealing with the mechanism described above does not mean that the intuition exploited in this paper cannot be used to characterize a more efficient equilibrium in the single agent case too. An endogenous choice of the dimensions of communication, in fact, is still likely to play a key role in the characterization of the most informative equilibrium with one sender.

Before presenting extensions and other properties of the equilibrium, it is important to note that the intuition behind the results presented in this and in the previous sections does not depend on the special assumptions that have been made for simplicity, in particular, quadratic utility and focus on a two-dimensional space. Consider first the assumption of quadratic utilities. In Proposition 3 we exploit the fact that the tangents of the indifference curves at the ideal point of the policy-maker are linearly independent and therefore span the policy space. With quadratic utilities, this is implied by linear independence of the ideal points: in this case, in fact, the gradients of the indifference curves at the origin are $\nabla u_i(0,0) = -2x_i$. However, when we assume only that utilities are quasi-concave, there is not necessarily a relationship between the ideal points and the tangents, but the intuition and the essence of the result carry through. In this case, to obtain the result, we only need that the gradients of the indifference curves at the ideal point of the receiver $(\nabla u_1(0,0), \nabla u_2(0,0))$ be linearly independent, which is a condition that is generically verified. In fact, if this condition holds, the tangents are also linearly independent and we can construct a new coordinate system exactly in the same way as in Proposition 3: since the utilities are quasi-concave, no point on the tangent is strictly better for the sender than the tangency point. As a matter of fact, we do not need differentiability of utility functions either. Indeed, more generally, when utility is quasi-concave, the upper contour set $P_i(0) = \{x \in \Re^d | u_i(x) \geq u_i(0)\}$ is convex so, by the supporting hyperplane theorem, there exists a hyperplane that has points that are never strictly preferred to the origin. If the utility function is not differentiable, this hyperplane may not be unique, but still exists.

Therefore, the main assumption that is used in the construction of this equilibrium is that utilities are quasi-concave. In the two-dimensional case, the set of outcomes preferred by the sender to the receiver's ideal point is convex and can be separated by a line. Quasi-concavity is also the relevant assumption in the unidimensional case. The necessary and sufficient condition that we have characterized in Proposition 1 ($|x_1| + |x_2| > W$) clearly depends on the assumption of quadratic utilities. In this case ideal points characterize the upper contour set; for example, for $x_i < 0, P_i(0) = [2x_i, 0]$. However, the same intuition would still hold under the more general assumption of convexity of the upper contour set. Here too, once we drop the assumption of quadratic utilities, a relationship between the ideal points and the set of preferred outcomes no longer exists. Therefore, the necessary and sufficient condition in this case cannot depend only on the ideal points; but more in general on $P_i(0)$: in this case $P_1(0) := [\tilde{x}_1, 0]$ and $P_2(0) := [0, \tilde{x}_2]$ where $\tilde{x}_i$ $i = 1, 2$ are determined by the shape of the utility function. However, the main insight of Proposition 1 would still be true. The condition would not be on the agents' biases, as measured by $\tilde{x}_1$ and $\tilde{x}_2$, independently taken, but a measure of both of them together would be relevant. However, in the case of utility functions that are symmetric around the ideal point, we have that $\tilde{x}_i = 2x_i$ and the characterization would be the same as in Proposition 1. The same considerations are also true for Proposition 2 where the relevant variable is the upper contour set $P_i(0)$; in this case, it would be necessary that the bias of the experts, as measured by $\tilde{x}_1$ and $\tilde{x}_2$, is in absolute value large enough, a condition that is implied by large ideal points if the upper contour sets are strictly convex.

The second simplifying assumption is the dimensionality of the policy space. In the previous section we analyzed the case with two experts and two dimensions. However, two experts are also sufficient for full revelation in more than two dimensions. Consider the three dimensional case. Take expert i ($i = 1$ or 2) and fix the plane that is tangent to his indifference curve at the policy-maker's ideal point; call this plane T_i. For expert 1, fix one vector on T_1; call it v^1. For expert 2, fix two linearly independent vectors in T_2; call them v_1^2, v_2^2. Clearly $\{v^1, v_1^2, v_2^2\}$ spans the three-dimensional space. Consider an equilibrium in which the declaration of expert 1 is interpreted as a coordinate in the v^1 dimension and the declaration of expert 2 is interpreted as the coordinate in the v_1^2, v_2^2 dimension. Given that expert 1 reports the truth, expert 2 will have to choose a point on T_2; by construction, the optimal choice in T_2 is the ideal point of the policy-maker, so expert 2 will reveal the true coordinate (in the v_1^2, v_2^2 coordinate system). Given that expert 2 is honest, expert 1 will have to choose a point in v_1 and therefore will be honest. This argument can be generalized to more than three dimensions. The key intuition, here too, is simple: with generic utilities we can always find a set of vectors in the two hyperplanes tangent to the indifference surfaces at the receiver's ideal point that span the entire outcome space.

5. EXTENSIONS AND OPEN QUESTIONS

A few characteristics of the equilibrium in Proposition 3 seem important: robustness to the senders' collusion, a different order of reports, and noise in the observations. We present these issues in the remainder of this section. We also discuss questions that are still open for further research.

Collusion Proofness. Robustness of the equilibrium to collusion seems a very important property. Consider the example of two informed lobbies and a policy-maker. The possibility of secret agreements between lobbies is more than plausible and it is not desirable to assume it away. Under some conditions the equilibrium constructed above is robust with respect to this problem.

To study this issue it is useful to formalize the concept of collusion proofness of an equilibrium. For any equilibrium $\{y^*(\cdot,\cdot), s_1^*(\cdot), s_2^*(\cdot), \mu^*(\cdot,\cdot)\}$ we may define the induced game $\Gamma(y^*)$ as a game where the players are the two experts, strategies are the same as before, and utilities are defined: $\tilde{u}_i(s_1, s_2) = u_i(\theta + y^*(s_1, s_2))$. Clearly the original equilibrium is an equilibrium of this game, but there may be other equilibria. Assume that there exists an equilibrium that Pareto dominates $s_1^*(\theta), s_2^*(\theta)$.[14] Especially if we assume that agents may collude, then the original game would be at least suspect: Agents would coordinate on the Pareto superior equilibrium. Therefore we make the following definition:

DEFINITION 2: An equilibrium of the original expertise game is *collusion-proof* if the induced game $\Gamma(y^*)$ has no Pareto superior equilibria.

This definition does not coincide with definitions used in other work on collusion.[15] In this literature, collusion is ruled out if players cannot coordinate on an action that is strictly Pareto superior. In order to pursue this approach, however, it is necessary to assume that players can sign binding agreements at the colluding stage; this seems a strong assumption. The approach followed in this paper, on the contrary, is in line with the *coalition proofness* concept introduced by Bernheim, Peleg, and Whinston (1987). The structure of the model and the role of the players allow us to restrict attention to the coalition formed only by experts. What is important is that the coalition of experts cannot commit to a joint strategy. This is consistent with the structure of the model because it preserves its noncooperative nature.

Using Definition 2, we have the following result.

PROPOSITION 4: *With quadratic utilities and* $x_1 \neq \alpha x_2 \ \forall \alpha \in \Re$, *the equilibrium constructed in Proposition* 3 *is collusion-proof.*

PROOF: Assume it is not collusion-proof. Then there exists a strictly Pareto superior equilibrium $s_1'(\theta), s_2'(\theta)$, which clearly must induce an outcome different from the origin. The following condition must hold, otherwise there would be a profitable deviation for one of the experts:

$$l_i(\alpha_i(\theta) - s_j'(\theta)) \cdot \nabla u_i(\theta - l_i(s_j'(\theta)) \cap l_j(s_i'(\theta))) = 0 \quad \forall i = 1, 2. \tag{7}$$

The first term of the left-hand side of (7) is the direction of allowed deviation for agent i at equilibrium, the second is the gradient of i's utility at the equilibrium outcome. Condition (7) means that for each agent, at the equilibrium outcome, the indifference curve of the agent must be tangent to the direction of allowed deviation for the agent; if this is not true, then there is a profitable deviation. Given quadratic utilities, the locus of possible outcomes with this property is the line connecting the ideal point of agent i with the origin. However, the intersection of these two loci contains only the origin since $x_i \neq \alpha x_j$. *Q.E.D.*

[14] At least one agent is strictly better off and neither is worse off.

[15] See, for instance, Tirole (1992).

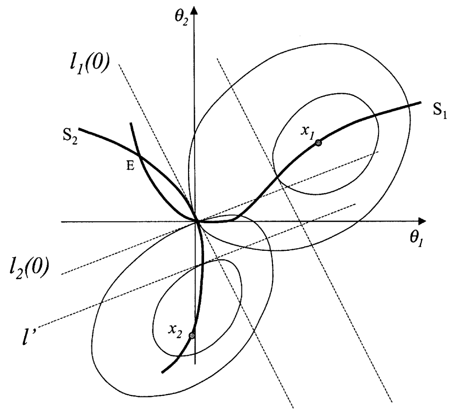

FIGURE 3.—Collusion proofness with quasi concave utilities: an example.

As we have seen, the result of Proposition 3 generalizes to quasi-concave utilities under essentially the same condition that we have with quadratic utilities (linear independence of the gradients of the utility curves at the receiver's ideal point). Instead, in the case of Proposition 4, the generalization requires a more restrictive condition, which however has an interesting interpretation. Consider Figure 3. For any declaration of sender 1, sender 2 induces a point in a parallel of $l_2(0)$, say l'. For any of these sets, we may identify sender 2's optimal choice and consider the set of these points (call it S_2); we may do the same for sender 1 and identify the set S_1. Formally, if we assume differentiable utilities, we might define

$$S_i = \left\{ x_1, x_2 \left| \frac{\partial u_i(x_1, x_2)/\partial x_1}{\partial u_i(x_1, x_2)/\partial x_2} = \frac{\partial u_i(0,0)/\partial x_1}{\partial u_i(0,0)/\partial x_2} \right. \right\}.$$

At the receiver's ideal point, the sets S_1 and S_2 intersect; this characterizes an equilibrium since it means that at this point the declaration of each sender is optimal given the declaration of the other. With quadratic utilities there is only one point of intersection because these sets are straight lines. However, in general, depending on the level curves, we may have different cases. For collusion proofness we do not require that these lines have a unique intersection, as it happens in the quadratic case, but, as in Figure 3, only that they do not intersect at a point that is preferred by both senders to the receiver's ideal point. In the case of the figure, we have another equilibrium of $\Gamma(y^*)$ at E, but this equilibrium is not Pareto superior for the experts with respect to the origin.

More formally, given the upper contour sets $P_i(0,0) = \{x_1, x_2 | u_i(x_1, x_2) \geq u_i(0,0)\}$, if we define the set $P = P_1(0,0) \cap P_2(0,0)$, this condition holds whenever $S_1 \cap S_2 \notin P$.[16] It is not difficult to show examples where this condition is not

[16] When the upper contour sets are strictly convex, $S_1 \cap S_2$ is a point. When the set is not strictly convex, clearly we require that no point in $S_1 \cap S_2$ belongs to P.

satisfied. The condition, however, is interesting because it shows the relationship between the possibility of full extraction of information and the degree of the senders' conflict of interest. As we have seen in Proposition 3, full revelation is possible in equilibrium even in extreme cases in which the experts have interests that are very much in line, i.e. even if $\nabla u_1(0,0) = \alpha \cdot \nabla u_2(0,0) + \varepsilon$ with $\alpha > 0$ and ε is a negligible vector. Once we take into consideration the possibility of collusion, the situation is different. If $\alpha > 0$, the more ε is negligible, the larger is the set of points that both experts prefer to the receiver's ideal point. Therefore it is more likely that there exists a Pareto superior equilibrium, and full revelation is not collusion-proof. However, when senders have opposed preferences ($\alpha < 0$), as ε converges to zero, the area that is Pareto superior for the senders (P) converges to the empty set since indifference curves are almost tangent, and collusion proofness is almost surely satisfied.

Sequential Referrals. When experts report sequentially, the conditions identified in the previous section are not sufficient to guarantee the existence of a fully revealing equilibrium. To see what may go wrong in the argument in this case, consider Figure 4. Given the reaction function of expert 2, we may find the points that expert 1 may induce, i.e. the points that, given the message of expert 1, expert 2 would choose. This is the set S_2 defined above, which, in the example represented in Figure 4, is a straight line connecting sender 2's and the receiver's ideal points. This means that if sender 1 is believed in the l_1 dimension, he may induce any point in this set; as can be seen from Figure 4, this

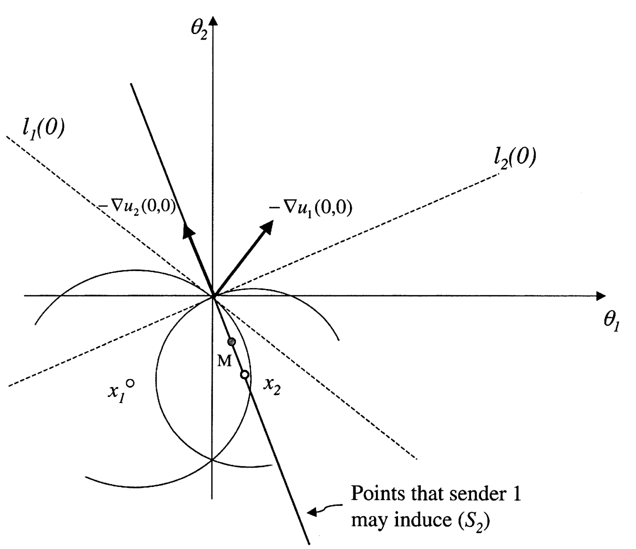

FIGURE 4.— Illustration of the sufficient condition for existence of a fully revealing equilibrium with sequential referrals.

implies that there might be a deviation that induces a point (M in the figure) that is strictly preferred to the origin by sender 1.

With quadratic utility functions the mechanism of Section 4 gives full revelation in the sequential game if and only if the experts' biases are exactly orthogonal. However the result is relevant for two reasons. First, the condition is so restrictive only if we limit the analysis to quadratic utilities. Consider Figure 3 and assume that sender 1 reports first. Sender 1 may induce any point in S_2 because, for any declaration, 2 would choose one of these points. However, in this example, the set S_2 never intersects the points that sender 1 strictly prefers to the receiver's ideal point. In this case, the condition for full revelation in the sequential case is $S_2 \cap P_1 = \varnothing$. The second reason why this is interesting is that we have not proven that there is not a fully revealing equilibrium. We have only shown that the idea developed in the previous sections cannot always be immediately applied.

The case of sequential reports in a multidimensional environment is interesting because it confirms how results in one dimension fail to be true in a multidimensional world. The characterization of the most informative equilibrium with sequential referrals when $S_2 \cap P_1 \neq \varnothing$, and so full revelation is not possible, is an open question. Krishna and Morgan (2000) study this problem in a unidimensional environment. However, even in this framework, it is possible to show that equilibria are partitional only under restrictive conditions; therefore the frontier of informativeness of equilibria can be drawn only for subclasses of equilibria.[17] In the multidimensional case, the complete analysis will be considerably harder since a much larger set of strategies is possible. In this situation too, the idea of making a sender influential in a subset of the policy dimensions is likely to be important in the characterization of the most informative equilibrium.

Noise and Residual Uncertainty. In general, in the learning process of a decision-maker who hears experts, we may identify two main components. On the one hand, we have the strategic interaction studied above, due to the conflict of interests and the incentives to lie. On the other hand, even if we ignore the issue of incentive compatibility for truthful revelation, we still have the problem of statistical aggregation of informative signals that are only imperfectly correlated to the true state of the world. Of course, both components are typically present in a sender-receiver relationship. In this paper, however, we have ignored the statistical aggregation problem and focused on the strategic relationship and the incentives to report truthfully. In the equilibrium constructed in Proposition 3, in fact, the receiver believes one and only one expert in some endogenously determined dimension. This depends on the fact that both experts know the true state of the world: if one of the two reports truthfully on the assigned dimension, hearing the second sender does not add any information. If both experts receive only an imperfectly informative signal, then it might be useful to combine the knowledge of the two senders in order to reduce noise. Actually we would have a trade-off; it might be optimal to sacrifice incentives for truthful revelation, but use

[17] Krishna and Morgan (2000) consider the class of partitional (or in their terminology, 'monotonic') equilibria. Unlike the case analyzed by Crawford and Sobel (1982), in fact, with more than one sender, Perfect Bayesian Equilibria are not necessarily equivalent to partitional equilibria.

all the available signals. In order to focus on the intuition of how information can be extracted, we have not analyzed this issue, which would have added an extra dimension to the problem. In doing this we have followed the approach of almost all the previous work since Gilligan and Krehbiel (1989) and Krishna and Morgan (1999, 2000), where this issue is ignored. Only the work by David Austen-Smith attempted to combine the two issues in a single analysis. However, although this research effort yields very insightful results in one dimension (Austen-Smith (1993)), in more than one dimension it becomes exceedingly difficult to analyze and has yielded only some exploratory examples (Austen-Smith (1991)).

The characterization of the most informative equilibrium in a multidimensional environment when this issue is added is still an open question. The reason why the study of this issue is complicated is precisely the existence of a trade-off between incentives for full revelation, which, as we have seen above, tend to imply a restriction of the set of outcomes where the sender is influential, and the reduction of noise, which implies that the more signals are used in equilibrium for the updating of beliefs, the better it is. The most informative equilibrium on the true state of the world, may actually be such that the incentive compatibility constraints are relaxed. However, the intuition that underpins the analysis in the previous sections is important to understand these situations in which senders' observations are noisy as well. In effect, with quadratic utilities, even if the agents observe a noisy signal, for example $\tilde{\theta} = (\theta_1 + \varepsilon_i^1, \theta_2 + \varepsilon_i^2)$, where for each sender $i = 1, 2$ the $\varepsilon_i^1, \varepsilon_i^2$ have mean zero and are independently distributed, we would be able to have full revelation. If agent 2 reveals the truth in his dimension as in Proposition 3, the dimension l_1 in which sender 1 is influential would be a parallel to $l_1(0)$ because of the noise in sender 2's observation. However, on average the observation of sender 2 is correct, and with quadratic utilities the variance is not relevant in the decision. Therefore sender 1 incentives would be preserved.[18] This is not generally true for a quasi-concave utility function since in this case even if the other sender is correct on average, the variance of the observation matters. However, if the variance of the residual noise ε_i is very small, only the local behavior of the utility functions would be relevant and a quadratic approximation of the utility function would be a good fit. As a result, an optimal choice of the dimension of information transmission would reduce substantially the conflict of interest and, although in general we would not have full revelation in all states, it would yield a more informative equilibrium.

[18] To see this intuition, consider the example of the two experts with ideal points on the orthogonal axis and when θ_1, θ_2 are independent. Sender i's utility is $u_i = -E[\sum_{t=x,y}(\theta_t - m_i^t - x_i^t)^2]$ where x_i^t is the bias of i on the t dimension; in this example, $x_1^1 = 0$ and $x_2^2 = 0$. Since in this example preferences of sender 1 and the receiver are aligned on coordinate 1, if sender 2 is truthful on the y dimension, the utility for sender 1 is $u_1 = -(E(\theta_x \mid \tilde{\theta}_i) - m_1^x)^2 + K$, where m_i^t is the message of sender i and K is a constant that depends on the variance of noise in the observations. So $m_1^x = E(\theta_x \mid \tilde{\theta}_i)$ is optimal and full revelation of the available information on the x dimension is still the best strategy for sender 1. Clearly, the same holds for sender 2. This argument can be extended to the general case in which ideal points are linearly independent. A complete proof is available from the author. See also Battaglini (2002).

6. CONCLUSIONS

In this paper we have shown that there are insights to be gained from the analysis of multidimensional cheap talk. Multidimensionality is not just a technical change in the model, it implies results that are qualitatively different. Contrary to the one-dimensional case, full revelation of information is typically possible in two or more dimensions, and under some conditions, the result is robust to collusion and other perturbations. Besides full revelation, we believe the analysis provides a new intuition on how information can be transmitted in equilibrium between senders and a receiver with conflicts of interest.

Although a model of cheap talk with multiple senders can be used to study many different problems, the first applications were in political science, in particular to explain the role of committees and debate in the Congress (Gilligan and Krehbiel (1989), Austen-Smith (1990)). This work may directly contribute to this literature and in particular to the debate on the so-called 'informational' theory of legislative committees. According to this theory, committees in the Congress act as senders of a cheap talk game where the decision-maker (the government or the median voter) is the receiver. This theory, however, is controversial. Although it is not possible to measure informational flows, unidimensional theories imply that information transmission is effective only when the conflict of interest between the floor and the committee is not too large; Krehbiel (1991) calls this the 'outlier principle'. This prediction has been questioned on empirical grounds. As Londregan and Snyder (1994) put it: "the dominant view among congressional scholars is that many congressional committees and subcommittees are not representative of the entire chamber from which they are selected but instead have a relatively strong preference for serving particular interests" (Londregan and Snyder (1994, p. 233)).[19]

Very little work has been done to reconcile informational theory with empirical evidence. In this paper, we show that when the analysis is multi-dimensional, the evidence described above is not in conflict with an informational theory of legislative organizations since the 'outlier principle' does not necessarily hold in a multidimensional environment. This result, therefore, may help to guide future empirical work. Existing research has always used a measure of conflict that is inherently unidimensional since it positions agents on the right-left spectrum. Even the works that have attempted to measure heterogeneity of committees' members have focused only on the variance of the ideological position of their members (Krehbiel (1991), Dion and Huber (1997)). This focus is due to the fact that the reference model of existing empirical literature is the traditional

[19] Although formal empirical analyses have presented mixed results, even the empirical studies that support the informational theory show significant conflicts of interest between the floor and committees. From their empirical analysis, Londregan and Snyder (1994) conclude: "these results are inconsistent with the implications of models that emphasize the asymmetric information problems arising from committee expertise. . . " Typically, in this literature a measure of the ideological bias of the committee's members is compared with a measure of the ideology of the floor of the congress. Typically the ideological bias is measured using direct ratings provided by interest groups (Krehbiel (1991), Londregan and Snyder (1994)).

"Hotelling style" unidimensional spatial model in which the distance of ideal points is the key variable. The analysis of this work, however, shows that the dimensionality of the problem should be taken into consideration and may provide a new theoretical framework for future empirical investigations.

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APPENDIX

Proof of Proposition 1

Necessary Condition. We prove that for $W \geq |x_1| + |x_2|$ we may find a truthful fully revealing equilibrium. It is sufficient to prove that there is a fully revealing equilibrium with out-of-equilibrium beliefs that assign positive probability only at one point. With a slight abuse of notation call this point $\mu(\theta', \theta)$: i.e. given θ', θ, the policy-maker will believe that the true state is $\mu(\theta', \theta)$ with probability one. For any couple θ', θ, incentive compatibility (IC) for full revelation implies that $u_1(\theta - \mu(\theta', \theta)) \leq u_1(0)$ and $u_2(\theta' - \mu(\theta', \theta)) \leq u_2(0)$. The first IC condition implies:

(a1) $\theta - \mu(\theta', \theta) \geq 0 \Rightarrow \mu(\theta', \theta) \leq \theta$ or

(a2) $\theta - \mu(\theta', \theta) \leq 2x_1 \Rightarrow \mu(\theta', \theta) \geq \theta - 2x_1$.

The second IC condition implies:

(b1) $\mu(\theta', \theta) \geq \theta'$ or

(b2) $\mu(\theta', \theta) \leq \theta' - 2x_2$.

An equilibrium exists if for all the out-of-equilibrium couples we can find a $\mu(\theta', \theta)$ such that one inequality of the first group and one of the second are simultaneously satisfied and $\mu(\theta', \theta) \in [-W, W]$: there is no incentive to deviate and the belief is in the support of θ. We consider the possible cases and we show that the set of beliefs that satisfy the required conditions is nonempty for any couple (θ', θ). If $\theta' \leq \theta$, then we may satisfy (a1) and (b1) choosing $\mu(\theta', \theta) \in [\theta', \theta]$. So from now on we assume that $\theta' > \theta$. Assume that $\theta \geq 0$ and consider (b2) and (a1), so $\mu \leq \min(\theta' - 2x_2, \theta)$. If $\theta' - 2x_2 \geq -W$ just take $\mu(\theta', \theta) \in [-W, \min(\theta' - 2x_2, \theta)]$; it is possible since $[-W, \min(\theta' - 2x_2, \theta)]$ would be nonempty. If $\theta' - 2x_2 < -W$, consider conditions (a2) and (b1) instead. Inequality (a2) requires $\mu(\theta', \theta) \geq \theta - 2x_1$, so (b1) and (a2) are implied by $\mu(\theta', \theta) \geq \theta' - 2x_1$ since $\theta' > \theta$, which, again, is implied by $\mu(\theta', \theta) \geq 2x_2 - W - 2x_1$. By $W \geq |x_1| + |x_2|$ we have $2x_2 - W - 2x_1 \leq W$ so the set $[2x_2 - W - 2x_1, W]$ is not empty and it is just sufficient to take $\mu \in [2x_2 - W - 2x_1, W]$. Assume now that $\theta < 0$. Consider first the case in which $\theta' \geq 0$. By (a1) and (b2) it is sufficient that $u \leq \min(-W, \theta' - 2x_2)$. Therefore, if $\theta' - 2x_2 \geq -W$, we may just choose $\mu(\theta', \theta) = -W$. If, instead, $\theta' - 2x_2 < -W$, then consider (a2) and (b1), which are satisfied if $\mu \geq \max(W, \theta - 2x_1)$; but $\theta < 0$ and, in this case, $|2x_1| < W$ so we may choose $\mu = W$. Now consider the case in which $\theta' < 0$. Inequalities (b2) and (a1) are satisfied if $\mu \leq \min(-W, \theta' - 2x_2)$; if $\theta' - 2x_2 \geq -W$ we can still choose $\mu(\theta', \theta) = -W$. Assume $\theta' - 2x_2 < -W$ and consider (a2) and (b1), which, as can easily be verified, are implied by $\mu(\theta', \theta) \geq 2x_2 - W - 2x_1$. By $W \geq |x_1| + |x_2|$ we have $2x_2 - W - 2x_1 \leq W$ so the set $[2x_2 - W - 2x_1, W]$ is not empty, and again it is sufficient to take $\mu \in [2x_2 - W - 2x_1, W]$. Therefore, we may construct a well defined belief function for any couple of declarations so that the IC constraints are satisfied and there is full revelation.

Sufficient Condition. We prove that if $W < |x_1| + |x_2|$, then there cannot exist a fully revealing equilibrium. By Lemma 1 it suffices to show that no truthful fully revealing equilibrium with degenerate out-of-equilibrium beliefs exists. For this we need only prove that there exist a θ and a θ' such that no couple of inequality a and b can be satisfied. Consider $\theta' = \min\{2x_2 - W - \varepsilon, W - \varepsilon\}$ and $\theta = \theta' - \varepsilon$ for $\varepsilon > 0$, arbitrarily small. It is easy to verify that no couple of inequality chosen, respectively, one from (a.1) or (a.2) and the other from (b.1) or (b.2) can be verified, so full revelation is impossible.

PROOF OF PROPOSITION 2

Once we introduce the restrictions on out-of-equilibrium beliefs described in Section 3, Lemma 1 does not necessarily hold. In the proof of Lemma 1, in fact we have exploited the indeterminacy of out-of-equilibrium beliefs, but now we are restricting the set of feasible beliefs. To see that, at least theoretically, we may have a fully revealing equilibrium that is not truthful, consider this case: Each expert is pooling, and the declaration of each expert reveals that the state is in a set (say A_i for agent i); however, the intersection of the two sets is a singleton, and so information is fully revealed. The restriction introduced in Section 3 is such that out-of-equilibrium beliefs depend on the equilibrium strategies (i.e. on the sets A_i). In particular, it may be possible that, through the choice of the sets A_i, we may construct out-of-equilibrium beliefs that support full revelation; such an equilibrium would clearly have beliefs that are different from the case of truthful strategies. Therefore, in order to prove that there exists no robust fully revealing equilibrium, it is not enough to prove that there exists no robust truthful fully revealing equilibrium. For this reason, in the following proof we do not invoke Lemma 1.

Assume that there exists a fully revealing equilibrium. For any message s_i sent in equilibrium by agent i, define a set: $A_i(s_i) := \{\tilde{\theta} \in \Theta | s_i(\tilde{\theta}) = s_i\}$. If there exists a fully revealing equilibrium, it must be that for any θ, $A_1(s_1(\theta)) \cap A_2(s_2(\theta)) = \{\theta\}$. Two conditions must be satisfied. First, if there exists a fully revealing equilibrium and one agent pools, then the other agent will be able to choose any point in the pooling set: otherwise some states would never be revealed. In particular, for any $\tilde{\theta}$ in $A_i(s_i)$, there must be a message s_j such that $A_1(s_i) \cap A_2(s_j) = \{\tilde{\theta}\}$. Second, to have incentive compatibility, it is necessary that agent j does not strictly prefer any point θ' in $A_i(s_i(\theta))$ to θ. For this reason, it must be that for any $t_{l+1}, t_l \in A_1(s_1)$ such that $t_{l+1} > t_l$, then $t_{l+1} \geq t_l + 2x_2$. Assume not: if $t_{l+1} < t_l + 2x_2$, then in state t_{l+1} agent 2 may report that the state is t_l so the outcome would be $0 < t_{l+1} - t_l < 2x_2$; this would be a profitable deviation for agent 2. In the same way, for $v_{n+1}, v_n \in A_2(\theta)$ such that $v_{n+1} < v_n$, it must be $v_{n+1} \leq v_n + 2x_1$.

We now consider the beliefs that may follow after a pair of out-of equilibrium signals $s_1(\theta'), s_2(\theta'')$ if the restriction on beliefs is satisfied; we then show that given these beliefs we always have a profitable deviation for at least one sender. For each $l \in A_i(\theta)$, we may define the posterior probability that the state is l given the true state is in $A_i(\theta)$ as

$$p(l \mid A_i(\theta)) = \frac{f(l)}{\sum_{k \in A_i(\theta)} f(k)}.$$ [20]

Consider two states of nature $\theta', \theta'' \in [-B, B]$ for $B < \min\{|x_1|, |x_2|\}$ and $\theta' \neq \theta''$. For sender 1, consider the set $A_1(s_1(\theta'))$ so that, by the choice of B, $A_1(s_1(\theta')) \setminus \{\theta'\} \notin [-B, B]$; in the same way, for sender 2, consider the set $A_2(s_2(\theta''))$; as before, $A_2(s_2(\theta'')) \setminus \{\theta''\} \notin [-B, B]$. We can write:

$$E\big(\theta \mid A_1(s_1(\theta'))\big) = \frac{\sum_{k \in A_1} kf(k)}{\sum_{k \in A_1} f(k)} = \frac{\theta' f(\theta') + \sum_{k \in A_1,\, k \notin [-B,B]} kf(k)}{\sum_{k \in A_1} f(k)}$$
$$= \frac{\theta' f(\theta') + \sum_{k \in A_1,\, k \geq \theta' + 2x_2,\, k \leq \theta' - 2x_2} kf(k)}{f(\theta') + \sum_{k \in A_1,\, k \notin [-B,B]} f(k)}.$$

[20] As a heuristic justification for this, consider $A_2(\theta, \delta) = \{[l_1, l_1 + \delta], [l_2, l_2 + \delta], \dots\}$ so, by Bayes' rule,

$$p([l_1, l_1 + \delta] | A_2(\theta, \delta)) = \frac{F(l_1 + \delta) - F(l_1)}{\sum_{k \in A_2(\theta, \delta)} F(l_k + \delta) - F(l_k)};$$

dividing by δ both the numerator and the denominator, we have

$$\frac{\delta^{-1}(F(l_1 + \delta) - F(l_1))}{\sum_{k \in A_2(\theta, \delta)} \delta^{-1}(F(l_k + \delta) - F(l_k))} \longrightarrow \frac{f(l)}{\sum_{k \in A_i(\theta)} f(k)} \quad \text{as} \quad \delta \longrightarrow 0.$$

For details consult Kolmogorov (1950, par. 3, p. 51).

Since the second term of the numerator of the last expression converges to zero as x_2 increases, it must be that, for x_2 large enough, $E(\theta \mid A_1(s_1(\theta'))) \in [-\widetilde{B}, \widetilde{B}]$ where $\widetilde{B} \in (B, \min\{|x_1|, |x_2|\})$. In the same way we can prove that $E(\theta \mid A_2(s_2(\theta''))) \in [-\widetilde{B}, \widetilde{B}]$ for x_1 large enough in absolute value.

Given the equilibrium strategies, in any ε^n-perturbed game the event "observation of the couple $s_1(\theta'), s_2(\theta'')$ by the policy-maker" is the union of three disjoined events: the event in which agent 1 is right and agent 2 observes the wrong state; the event in which agent 2 is right and agent 1 observe the wrong state; the event in which both agents observe the wrong state. So in any ε^n-perturbed game and for any $G(\cdot)$, $E(\theta \mid A_1, A_2)$ is a convex combination of $E(\theta \mid A_1(\theta))$, $E(\theta \mid A_2(\theta))$ and $E(\theta)$ and therefore $E(\theta \mid A_1, A_2) \in [-\widetilde{B}, \widetilde{B}]$ for $\min\{|x_1|, |x_2|\}$ large enough. It follows that, after a pair $s_1(\theta'), s_2(\theta'')$, beliefs must be in $[-\widetilde{B}, \widetilde{B}]$ as $\varepsilon^n \to 0$: but then, by Proposition 1, since $\widetilde{B} < |x_1| + |x_2|$, we may find θ', θ'' in $[-\widetilde{B}, \widetilde{B}]$ such that either sender 1 has a profitable deviation in state θ'' or sender 2 has a profitable deviation in state θ'. Since this holds for any distribution of the wrong signal for the experts $G(\cdot)$ and any converging sequence ε_n, it follows that no out-of-equilibrium belief that satisfies the restriction supports a fully revealing equilibrium.

REFERENCES

AUSTEN-SMITH, D. (1990): "Information Transmission in Debate," *American Journal of Political Science*, 34, 124–152.

——— (1991): "Information Acquisition and Orthogonal Argument," in *Political Economy: Institutions Competition and Representation*, Proceedings of the Seventh International Symposium in Economic Theory and Econometrics, ed. by W. A. Barnet, H. J. Melvin, and N. Schofield. Cambridge, New York, and Melbourne: Cambridge University Press, pp. 407–436.

——— (1993): "Interested Experts and Policy Advice: Multiple Referrals under Open Rule," *Games and Economic Behavior*, 5, 3–44.

AUSTEN-SMITH, D., AND W. RIKER (1987): "Asymmetric Information and the Coherence of Legislation," *American Political Science Review*, 81, 897–918.

BATTAGLINI, M. (1999): "Multiple Referrals and Multidimensional Cheap Talk," The Center for Mathematical Studies in Economics and Management Science, Discussion Paper No. 1295.

——— (2002): "Legislative Organization with Imperfectly Informed Experts," Mimeo, Princeton University.

BERNHEIM, B. D., B. PELEG, AND M. WHINSTON (1987): "Coalition-Proof Nash Equilibria I: Concepts," *Journal of Economic Theory*, 42, 1–12.

CRAWFORD, P. V., AND J. SOBEL (1982): "Strategic Information Transmission," *Econometrica*, 50, 1431–1451.

DIERMEIER, D., AND T. J. FEDDERSEN (1998): "Information and Congressional Hearings," The Center for Mathematical Studies in Economics and Management Science, Discussion Paper No. 1236.

DION, D., AND J. D. HUBER (1997): "Sense and Sensibility: the Role of Rules," *American Journal of Political Science*, 41, 945–957.

FARRELL, J., AND R. GIBBONS (1986): "Cheap Talk with Two Audiences," *American Economic Review*, 79, 1214–1223.

GILLIGAN, T. W., AND K. KREHBIEL (1987): "Collective Decision-Making and Standing Committees: An Informational Rational for Restrictive Amendments Procedures," *Journal of Law, Economics & Organization*, 3, 287–335.

——— (1989): "Asymmetric Information and Legislative Rules with a Heterogeneous Committee," *American Journal of Political Science*, 33, 459–490.

KOLMOGOROV, A. N. (1950): *Foundations of the Theory of Probability*. New York: Chelsea Publishing Co.

KREHBIEL, K. (1991): *Information and Legislative Organization.* Ann Arbor: University of Michigan Press.

KRISHNA, V., AND J. MORGAN (1999): "Asymmetric Information and Legislative Rules: Some Amendments," Mimeo, Pennsylvania State University and Princeton University.

——— (2000): "A Model of Expertise," *The Quarterly Journal of Economics*, 116, 747–775.

LONDREGAN, J., AND J. SNYDER (1994): "Comparing Committee and Floor Preferences," *Legislative Studies Quarterly*, 19, 233–267.

MATTHEWS, S. (1989): "Veto Threats: Rhetoric in a Bargaining Game," *Quarterly Journal of Economics*, 104, 347–369.

MORRIS, S. (2001): "Political Correctness," *Journal of Political Economy*, 109, 231–265.

SOBEL, J. (1985): "A Theory of Credibility," *Review of Economic Studies,* 52, 557–573.

STEIN, C. J. (1989): "Cheap Talk and the Fed: A Theory of Imprecise Policy Announcements," *American Econmomic Review*, 79, 32–42.

TIROLE, J. (1992): "Collusion and the Theory of Organization," in *Advances in Economic Theory: Sixth World Congress*, Volume 2, Econometric Society Monographs, No. 21, ed. by J. Laffont. Cambridge: Cambridge University Press, pp. 151–220.