Toward an Economic Theory of Leadership: Leading by Example

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This paper explores leadership within organizations. Leadership is distinct from authority because following a leader is a voluntary, rather than coerced, activity of the followers. This paper considers how a leader induces rational agents to follow her in situations when the leader has incentives to mislead them. (*JEL* D21, L29, D29)

"Example is leadership."
—Attributed to Albert Schweitzer.

Leadership, although long studied by political theorists and social scientists (see Robert J. House and Mary L. Baetz [1979] or Gary A. Yukl [1989] for surveys), has generally been neglected by economists.[1] Economic analyses of organizations have, instead, focused on formal or contractual relationships. Indeed, the players in organizations who would commonly be called leaders, such as managers, are typically modeled as *agents* of other players who are *not* commonly seen as leaders (e.g., shareholders). Such analyses, despite their great insights, shed no light on leadership. In particular, they miss what I see as the defining feature of leadership: A leader is someone with followers. Following is inherently a *voluntary* activity.[2] Hence, a central question in understanding leadership is how does a leader induce others to follow her.[3]

Even when the leader has *formal authority*—the power to coerce (directly or indirectly)—such authority is rarely absolute. Certainly sociology, political theory, and organizational behavior still see a need for leaders to encourage and motivate a following.[4] Moreover, the

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[1] A notable exception is Julio J. Rotemberg and Garth Saloner (1993), which studies the question of "leadership style." Leadership style is modeled as the degree to which the leader empathizes with followers (formally, the weight the leader's utility function assigns the followers' utility). In an incomplete contracting environment, empathy can serve as a commitment device and, therefore, be valuable. Because Rotemberg and Saloner assume that a leader's empathy is common knowledge, the issues of trust-building explored in this paper do not arise. On the other hand, empathic or "participatory" leadership is an aspect of leadership not considered in this paper.

[2] According to House and Baetz (1979 p. 343) over 70 definitions of leadership have been advanced in just the twentieth century. Similarly, Yukl (1989 pp. 3–4) offers 13 definitions. The definition offered here, that a leader is someone who induces a voluntary following, is consistent with House and Baetz's synthesis definition (p. 344): "behavior. . . intended to influence others and the degree to which such influence attempts are viewed as acceptable to the person who is the target of the influence attempt."

[3] Charles Handy (1993 p. 97) describes answering this question as one of the "Holy Grails" of organization theory.

[4] See, e.g., Part III, Ch. 9, of Max Weber's (1921) *Wirtschaft und Gesellshaft* (H. H. Gerth and C. Wright Mills, 1946); Part II of Douglas McGregor (1966); Ch. 19, "On Avoiding Being Despised and Hated," and Ch. 21, "How a Prince Should Act to Acquire Esteem," of Niccolò Machiavelli's (1532) *The Prince* (Peter Bondanella and Mark Musa, 1979); Ch. 4 of Handy (1993); and Ch. 2 of Dennis H. Wrong (1995). Motivating a following is also

people in an organization with authority are not always, or solely, the leaders. Consider, for instance, that in many academic departments the true leaders are often *not* the department chairs. Leadership is, thus, distinct from formal authority; it is, instead, an example of informal authority.[5,6] As Weber (Gerth and Mills, 1946 pp. 248–49) puts it: "The [leader] does not deduce his authority from codes and statutes, as is the case with the jurisdiction of office; nor does he deduce his authority from traditional custom or feudal vows of faith, as in the case with patrimonial power."

As an economist, I presume that followers follow because it is in their interest to do so. What could make it in their interest to follow? One answer is that they believe the leader has better information about what they should do than they have. Leadership is thus, in part, about transmitting information to followers. But this cannot be all there is to leadership: A leader must also convince followers that she is transmitting the correct information; that is, she must convince them that she is not *misleading* them.[7] Consider, for example, the owner of a small firm. Because her profits are increasing in her employees' efforts, she has an incentive to tell the employees that *all* activities deserve their fullest efforts. Rationally, the employees realize that she has this incentive and are, thus, predisposed to *dis*regard her calls to action. She must, therefore, devise a way to convince the employees to put in more effort for those activities that are truly the most important.

There are two ways that we might see her do that. One is *leader sacrifice:* The leader offers gifts to the followers (e.g., free coffee or pizza for working into the evening). The followers respond *not* because they want the gifts themselves—indeed, the gifts could be public goods that they can enjoy regardless of whether they respond—but because the leader's sacrifice convinces them that she must truly consider this to be a worthwhile activity. The other way to convince followers is *leading by example:* The leader herself puts in long hours on the activity, thereby convincing followers that she indeed considers it worthwhile. Historical instances of leading by example include Dr. Martin Luther King, Jr. marching at the *head* of civil rights marches or Joseph Stalin's decision to stay in, rather than flee, Moscow to encourage Russian resistance in the battle for Moscow (a variant of the common military situation in which a unit leader must show that he is willing to put himself in harm's way in order to induce the unit to put itself in harm's way).

To formalize this intuition and to explore its implications further, I model the leader and her followers as members of a team. The teams model, as formulated by Bengt Holmstrom (1982), is well suited to studying leadership. First, because the leader shares in the team's output, she has an incentive to exaggerate the value of effort devoted to the common activity. Second, because the information structure limits the leader's ability to coerce followers, she must induce their voluntary compliance with her wishes.

In the next section, I review the teams problem under symmetric information about the return to effort allocated to a common endeavor. Each team member—*worker*—decides how much effort to invest in the common endeavor. Under mild conditions, the optimal solution

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a large part of "how-to" analyses, such as James M. Kouzes and Barry Z. Posner (1987), Warren G. Bennis (1989), Ronald A. Heifetz (1994), and John P. Kotter (1996).

[5] Within economics, Philippe Aghion and Jean Tirole (1997) also consider the distinction between formal and less formal authority, but in the context of delegating control. In particular, their distinction between formal authority and "real" authority is that the former is "the right to decide," while the latter is "the effective control over decisions."

[6] Some authors, rather than distinguish leadership from (formal) authority, use leadership to refer to both formal authority (e.g., deriving from an office) as well as the less formal idea of leadership that I pursue. Typical means of referring to this distinction are, respectively, "formal leadership" and "emergent leadership" (House and Baetz, 1979) or "de jure leadership" and "de facto leadership" (R. S. Peters, 1967). My use of "leadership" is perhaps most closely related to Weber's idea of *charismatic* leadership (Gerth and Mills, 1946).

[7] For example, Kotter (1996), an expert on business leadership, writes (p. 9): ". . . people will not make sacrifices . . . unless they think the potential benefits of change are attractive and unless they really believe that a transformation is possible. Without *credible* communication, and a lot of it, employees' hearts and minds are never captured" [emphasis added]. Or consider Weber (Gerth and Mills, 1946 p. 249): "The charismatic leader gains and maintains authority solely by *proving* his strength in life" [emphasis added].

is for the team to share the product of its common endeavor equally. However, as Holmstrom showed, this solution is only second-best optimal: Because each worker gets only a fraction of the overall (social) return to his effort, he expends less than the first-best level of effort on the common endeavor—he fails to internalize the positive externality his effort has for the team. This *teams problem* is, thus, simply an example of the free-riding problem endemic to the allocation of public goods.

In Section II, I assume that only the leader has information about the return to effort allocated to the common endeavor. Given asymmetric information, the question is how can the leader credibly communicate this information, to the rest of the team. I consider two possible solutions. In Section II, subsection A, I view the problem as a mechanism-design problem. I show that a mechanism that makes *side payments* among team members a function of the leader's announcement about her information can duplicate the symmetric-information, second-best outcome. The leader's side payment to other workers is increasing in her estimate of the return to effort—she "sacrifices" more the better the state is.

In Section II, subsection B, I allow the leader to "lead by example"; that is, she expends effort before the other workers. Based on the leader's effort, the other workers form beliefs about the leader's information. I first show, under fairly general assumptions, that leading by example yields an outcome that is *superior* to the symmetric-information outcome. The reason for this surprising conclusion is that the hidden-information problem "counteracts" the teams problem (free-riding): The need to convince the other workers increases the leader's incentives, so she works harder. In fact, her share can be reduced but still leave her stronger incentives than under symmetric information and, thereby, increase the shares of the other workers so that they too work harder. I then proceed to derive what the optimal contract (shares) should be when the leader leads by example. I find that in a small team, she has the smallest share, but in a large team, she has the largest share. Under certain conditions, leading by example dominates symmetric information even when attention is restricted to equal shares.

My focus in this model is on what the leader does to induce a following. I do not consider the questions of how the leader is chosen or why people want to be leaders. In the conclusion, Section III, I examine these questions at a speculative level and with an eye toward future research. I also relate my analysis to the theoretical and empirical findings of political theory and other social sciences.

The idea that one set of players will base their actions on a first-mover because they believe she has information bears some relation to the "herd behavior" or "informational cascades" literature (David S. Scharfstein and Jeremy C. Stein, 1990; Abhijit V. Banerjee, 1992; Sushil Bikhchandani et al., 1992). Unlike that literature, there is no issue here of the followers ignoring their private signals—they have none. More importantly, unlike that literature, the first-mover here has an incentive to induce a following. Finally, unlike that literature, the players here can sign contracts among themselves to affect the transmission of information. Contracting also distinguishes this paper from the Stackelberg-signaling literature in industrial organization (see, e.g., Esther Gal-Or [1987] or George J. Mailath [1993]). Moreover, whereas the leader wishes to convince followers that the state is bad in that literature, here the leader wants to convince them that the state is good. This paper is also related to the growing literature on information transmission within the firm (see, e.g., Canice Prendergast [1993] or Steven D. Levitt and Christopher M. Snyder [1996]). That literature has tended, however, to view the firm in terms of formal authority and incentive contracts, whereas I am looking at less formal leadership and more voluntary cooperation. A further difference is that literature is about "a fundamental trade-off between inducing workers to tell the truth and inducing them to exert effort" (Prendergast, p. 769). My point, in contrast, might be described as the need to convincingly tell the truth can lead to more effort being exerted. In spirit, the paper closest to mine is by Shira B. Lewin (1997). She models how franchisers might seek to influence franchises to adopt a new innovation (e.g., a new kind of hamburger in a fast-food restaurant). By adopting it themselves in their company-owned outlets, franchisers hope to influence franchises to follow suit. Lewin's focus, however, is solely on franchising, which means a different contracting environment than considered here. Consequently, her analysis would not readily carry over to leadership in organizations.

All proofs can be found in the Appendix.

I. Preliminaries

Consider a team with N identical workers indexed by n. Each worker supplies effort e_n. The value to the team of its members' efforts is $V = \theta \sum_{n=1}^{N} e_n$, where $\theta \in [0, 1]$ is a stochastic productivity factor realized *after* efforts have been supplied.[8] A worker's utility is $w - d(e)$, where w is his wage and $d(\cdot)$ is an increasing, convex, and thrice-differentiable function. Assume $d(0) = 0$, $d'(0) = 0$, and that the third derivative is bounded:

$$(1) \quad 2d''(e) > d'''(e) > -[d''(e)]^2 \quad \text{for all } e > 0.$$

Condition (1) is sufficient to ensure both that production will be done by the full team rather than a subset and that information about the state is valuable (see footnote 12 *infra*). An example of a disutility function satisfying all these assumptions is $d(e) = \frac{1}{2}e^2$.

The disutility function $d(\cdot)$ can be seen as representing the forgone utility of leisure. Alternatively, it can be seen as the worker's forgone profit from reducing his efforts spent on his *private* projects. The latter interpretation is relevant if the role of leadership is seen, in part, to facilitate coordination on *common* projects or objectives.

[8] Restricting θ to $[0, 1]$ is with little loss of generality. In particular, the upper bound can be readily changed. The main effect of increasing the lower bound is to alter the initial conditions for the differential equations that define the leader's strategy in Section II, subsection B. Consequently, the leader's equilibrium strategies would be somewhat different than those presented there. Analysis of the positive-lower-bound case—available from the author—shows that the results are nonetheless quantitatively similar to those presented in Section II, subsection B.

Assume, keeping with Holmstrom, that although contracts can be written contingent on V, they cannot be written contingent on the team members' efforts. Assume, too, that contracts cannot be contingent on θ *directly* (although they can be contingent on *ex ante announcements* about θ). Finally, assume that ownership of the common endeavor cannot be traded among the team members.[9] Given these restrictions, a contract is a set of contingent wages $\{w_n(V, \hat{\theta})\}_{n=1}^{N}$, where $w_n(V, \hat{\theta})$ is worker n's wage when total value is V and the *announced* value of θ is $\hat{\theta}$. Assume that the workers cannot commit to *ex post* inefficient contracts that would have them forgo some of the value [i.e., they cannot use contracts such that $\sum_{n=1}^{N} w_n(V, \hat{\theta}) < V$]. There is no external source of funds, so $\sum_{n=1}^{N} w_n(V, \hat{\theta}) \not> V$. Hence, attention can be limited to contracts in which $\sum_{n=1}^{N} w_n(V, \hat{\theta}) = V$ for all V and $\hat{\theta}$.

The following proposition greatly enhances tractability.

PROPOSITION 1: *Assume that each worker holds the same beliefs about θ conditional on hearing $\hat{\theta}$. For any contract $\{w_n(V, \hat{\theta})\}_{n=1}^{N}$, there exists an affine-shares contract $\{s_n(\hat{\theta}), t_n(\hat{\theta})\}_{n=1}^{N}$ with the following properties:*

- *worker n is paid $s_n(\hat{\theta}) \cdot V + t_n(\hat{\theta})$ when V is realized;*
- $\sum_{n=1}^{N} s_n(\hat{\theta}) = 1$;
- $\sum_{n=1}^{N} t_n(\hat{\theta}) = 0$;

[9] This assumption is without loss of generality when team members expend efforts simultaneously. When, however, they can expend effort sequentially, then this restriction could matter. In particular, by using a series of "buy-back" contracts (see Joel S. Demski and David E. M. Sappington, 1991), in which each worker is sole owner of the endeavor when it is his turn to work on it, but then sells it to the next worker who will work on it, the first-best outcome is attainable. Depending on whether there is renegotiation and how it is modeled, this conclusion could depend on the assumption that the production process is linear in effort (see Aaron S. Edlin and Benjamin E. Hermalin, 1997, or Georg Nöldeke and Klaus M. Schmidt, 1997). Although potentially powerful at a theoretical level, the practical application of buy-back mechanisms in this context could be limited: In many team situations it is difficult to see how a team could actually utilize this scheme. How, for instance, could a business school be sequentially "sold" among its faculty?

- *this contract induces the same effort from each worker in equilibrium as would* $\{w_n(V, \hat{\theta})\}_{n=1}^N$; *and*
- *this contract yields each worker the same expected utility in equilibrium as would* $\{w_n(V, \hat{\theta})\}_{n=1}^N$.

In light of Proposition 1, there is no loss of generality in restricting attention to affine-shares contracts.

Observe that nothing so far rules out negative shares (i.e., $s_n < 0$). I feel, however, that negative shares are unrealistic, since I am unaware of any real-world use. Moreover, because they would essentially be a way of using outside funds to alleviate the balanced-budget requirement, negative shares seem inconsistent with the spirit of the teams problem. Furthermore, they could be problematic if the team members are protected by limited liability and have limited sources of outside funds. They would also be problematic if workers could engage in negative effort ("sabotage"), since they would invite workers to pursue activities detrimental to the common endeavor. Even if negative effort were impossible, negative shares could still lead to sabotage: For instance if

$$-1 \times s_m > s_n > 0,$$

then m and n would find it profitable to enter into a coalition in which n agrees to reduce his equilibrium effort to zero. For these reasons, I am comfortable ruling out negative shares:[10]

ASSUMPTION: *In an affine-shares contract,* $s_n(\hat{\theta}) \geq 0$ *for all workers n and all announcements* $\hat{\theta}$.

Under an affine-shares contract each worker's expected utility is

$$(2) \qquad \mathcal{U}_n \equiv \tilde{\theta} s_n(\hat{\theta}) \left(\sum_{m \neq n} e_m + e_n \right) + t_n(\hat{\theta}) - d(e_n),$$

where $\tilde{\theta} = \mathbb{E}_\theta\{\theta | \hat{\theta}\}$. Since increasing $s_n \geq 0$ raises the marginal benefit of effort, we have

LEMMA 1: *A worker's effort is increasing in his share (i.e.,* $\partial e_n / \partial s_n > 0$*).*

I can now solve for the optimal contract when the workers are symmetrically informed.

PROPOSITION 2: *Assume the workers have symmetric information,* $\hat{\theta}$*, when they choose their efforts. Then an optimal (second-best efficient)*[11] *contract is an affine-shares contract that has* $s_n(\hat{\theta}) = 1/N$ *for all n.*

Intuitively, the efficient way to implement *total* effort $\Sigma_{n=1}^N e_n$ is to divide the effort equally among the workers: Since the marginal disutility of effort is increasing [i.e., $d''(\cdot) > 0$], Jensen's inequality implies that even division of effort minimizes aggregate disutility of effort. A potential complication arises, however, because the teams problem means less than first-best effort. There is, thus, the possibility that team does better to "shut down" some members of the team in order to increase the incentives given the productive members of the team. As the proof of Proposition 2 makes clear, condition (1) ensures that the efficiency gains from equal division of effort outweigh any incentive gains from unequal shares.

Given Proposition 2, each worker chooses e_n to maximize

$$\mathcal{U}_n \equiv \frac{\tilde{\theta}}{N} \left(e_n + \sum_{j \neq n} e_j \right) + t(\hat{\theta}) - d(e_n).$$

[10] As a technical matter, it is worth noting that this restriction does not invalidate the proof of Proposition 1, where case 2 allows for negative shares. If the same restriction against negative incentives were imposed on the general contract, $\{w_n(V, \hat{\theta})\}_{n=1}^N$, then case 2 would not arise: Suppose worker n's equilibrium effort is zero, then no-negative incentives and $d'(0) = 0$ imply

$$\mathbb{E}_\theta\{w_n(\tilde{V} + \theta\Delta, \hat{\theta}) - w_n(\tilde{V}, \hat{\theta})\} = 0,$$

where $\tilde{V}$ is the equilibrium V and Δ is an arbitrarily small deviation. From this, (A1) is readily derived and the proof of Proposition 1 follows case 1, in which all shares are nonnegative.

[11] There is still the teams problem (free-riding), so the first-best outcome is unattainable.

Since

$$\frac{\partial^2 \mathcal{U}_n}{\partial e_n \partial \tilde{\theta}} = \frac{1}{N} > 0,$$

a worker's effort is an increasing function of $\tilde{\theta}$: An increase in $\tilde{\theta}$ raises the marginal benefit of effort, without affecting cost, so more is supplied. To summarize:

LEMMA 2: *A worker's effort is increasing in his expectation of θ under an equal-shares contract.*

II. A Hidden-Information Model

Now suppose that one of the workers, *the leader,* receives a private signal concerning θ prior to the expenditure of effort, but *after* contracts have been fixed. Assume, for convenience, that the leader's signal is perfect; that is, she learns what θ will be (this is without loss of generality since the workers are risk neutral). I will not consider in detail how she learns θ. She could be shown θ by whomever appoints her to be leader. Alternatively, it is the expectation that she knows θ that leads to her being appointed leader. Yet another alternative is that θ is the consequence of some earlier action taken by the leader.

Consider, initially, an equal-shares contract with *non*contingent side payments (i.e., t_n is a constant with respect to $\hat{\theta}$). Let $e(\hat{\theta})$ maximize a worker's utility conditional on *believing* the leader's announcement $\hat{\theta}$; that is, let it solve

$$(3) \quad \max_{e_n} \frac{\hat{\theta}}{N}\left(e_n + \sum_{j \neq n} e_j\right) - d(e_n) + t_n.$$

From Lemma 2, $e(\hat{\theta})$ is increasing in $\hat{\theta}$.

The leader's utility is

$$(4) \quad \frac{\theta}{N}((N-1)e(\hat{\theta}) + e) - d(e) + t_L,$$

which means that she has an incentive to lie to the workers by announcing the maximum possible value for $\hat{\theta}$. For this reason, the workers will rationally *disregard* her announcement. Valuable information is not utilized, which is suboptimal relative to a situation in which the leader is induced to announce her information truthfully.[12]

I consider two alternative frameworks for avoiding this inefficient outcome. First, I consider a pure mechanism solution: The contract is contingent on the leader's announcement, $\hat{\theta}$. Second, I consider "leading by example": The leader is allowed to commit to or to choose her effort publicly before the other workers choose their effort (although contracts still cannot be contingent on the leader's effort).

[12] To see that information is valuable even in the second best, recognize that absent any information about the state, expected welfare is

$$\int_0^1 \left[\theta e_L(\theta) + \theta \sum_{n \neq L} e_n(\tilde{\theta}) - d[e_L(\theta)] - \sum_{n \neq L} d[e_n(\tilde{\theta})]\right] dF(\theta),$$

where L denotes the leader, $\tilde{\theta}$ is the expected value of θ, $F(\cdot)$ is the distribution of θ, and $e_h(\cdot)$ is the solution to

$$\max_e s_h \theta e - d(e).$$

Expected welfare with information is

$$\int_0^1 \left[\theta \sum_{n=1}^{N} e_n(\theta) - \sum_{n=1}^{N} d[e_n(\theta)]\right] dF(\theta).$$

To show that this second integral is greater than the first it is sufficient to show that

$$\int_0^1 (\theta e_n(\theta) - d[e_n(\theta)])\, dF(\theta) > \int_0^1 (\theta e_n(\tilde{\theta}) - d[e_n(\tilde{\theta})])\, dF(\theta) = \tilde{\theta} e_n(\tilde{\theta}) - d[e_n(\tilde{\theta})]$$

for each $n \neq L$. This in turn follows because

$$x e_n(x) - d[e_n(x)]$$

is a convex function in x given (1).

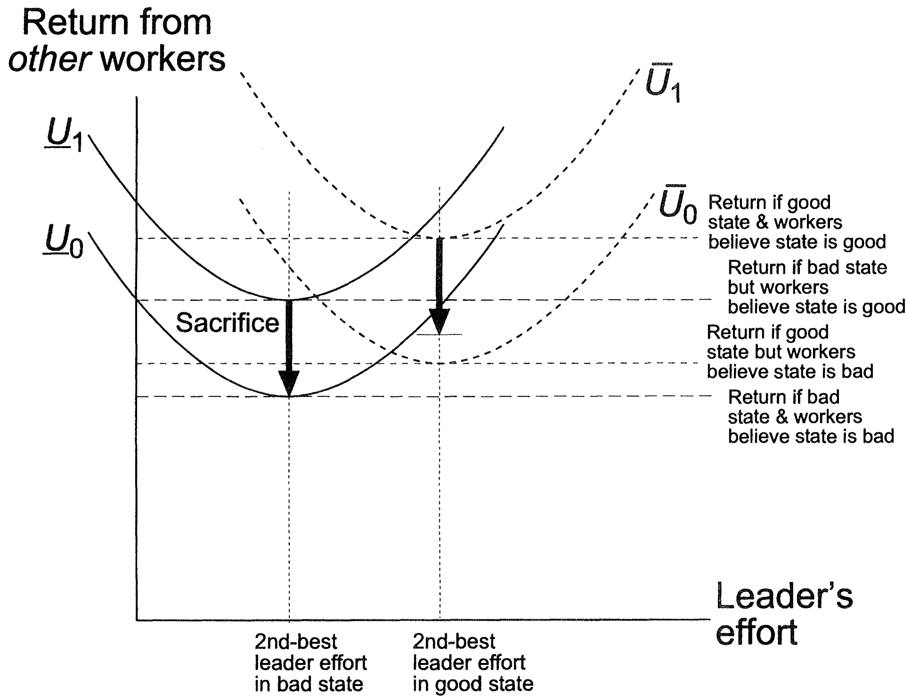

FIGURE 1. LEADER "SACRIFICE"

A. *A Mechanism-Design Solution*

By the revelation principle, I can restrict attention to mechanisms that induce the leader to tell the truth in equilibrium. Consequently, at the point when the workers choose their effort, they are all symmetrically informed in equilibrium. Proposition 2 therefore applies: If an affine-shares contract with equal shares can be found that induces truth-telling, then that contract will be optimal. I now derive such a contract.

Assign the leader index L. Consider the affine-shares contract in which

$$(5)\quad t_L(\hat{\theta}) = T - \int_0^{\hat{\theta}} z \frac{N-1}{N} e'(z)\, dz; \text{ and}$$

$$(6)\quad t_n(\hat{\theta}) = \frac{-t_L(\hat{\theta})}{N-1} \text{ for } n \neq L,$$

where T is an arbitrary constant.

I can now show the following proposition.

PROPOSITION 3: *The affine-shares contract that sets $s_n(\hat{\theta}) = 1/N$ for all n and defines $t_n(\hat{\theta})$ by* (5) *and* (6) *is optimal* (*second-best efficient*).

Figure 1 illustrates Proposition 3 in a two-state model. The space is the leader's effort, e_L, and the return from the other workers, $\theta\, \Sigma_{n \neq L}\, e_n$. The curves shown are the indifference curves for the leader in different states. They take a "parabolic" shape because the leader's utility is first increasing in e_L [when $\theta/N > d'(e_L)$], then hits a maximum (the bottom of an indifference curve), and finally is decreasing in e_L [when $\theta/N < d'(e_L)$].

Consider a leader in the bad state, $\underline{\theta}$. If she tells the truth, her utility is $\underline{U}_0$. Her incentive to lie is clear: Were her lie believed, she would gain $\underline{U}_1 - \underline{U}_0$ in utility. To make truth-telling credible, the leader in the good state, $\overline{\theta}$, is obligated to "*sacrifice*" enough that a leader in the bad state would have no incentive to mimic. That sacrifice, the black arrow, is $\underline{U}_1 - \underline{U}_0$. Although, the leader in the good state must sacrifice, she is willing to do so because her equilibrium utility, $\bar{U}_1 - (\underline{U}_1 - \underline{U}_0)$, exceeds her utility if she did not sacrifice, $\bar{U}_0$.[13]

Proposition 3 extends this intuition to a continuum of states. Observe too, from (5), that the leader sacrifices more the better the state is.

Leader sacrifice corresponds to real-world phenomena in which the leader promises a big party or more vacation time at the end of a big project. Alternatively, a team leader could provide pizza and coffee to team members who work evenings on a big project. In short, we have leader sacrifice whenever a leader promises a group reward to convince her team that effort pays big benefits.

Although we have so far treated this mechanism as part of an explicit side-payment contract agreed to *ex ante,* this is not necessary. Specifically, set $T = 0$ (consistent with no *prior* agreement on side payments). Suppose, in fact, that the only prior agreement is even shares (i.e., $s_n = 1/N$ for all n). Assume, however, that the leader *can* make side payments (sacrifices) if she wishes. Knowing that she can make sacrifices, the workers will interpret the sum of her sacrifices as a signal of θ. In particular, suppose the workers hold beliefs that the state is θ if the sum of the sacrifices is $-t_L(\theta)$. The workers perceive no sacrifice to mean $\theta = 0$. But since $t_L(0) = 0$ (recall $T = 0$), Proposition 3 proves it is incentive compatible for the leader to sacrifice $t_L(\theta)$ in state θ. We thus have a signaling equilibrium exhibiting leader sacrifice that does *not* rely on an explicit side-payment contract.

Even though Proposition 3 requires leader sacrifice, it does not require that the leader get less than the other workers. If T, the portion of the side payment not contingent on the announcement, is large enough, then the leader gets more than the other workers. On the other hand, under the signaling interpretation just considered, the leader is worse off than her followers. This raises the question of why she would choose or agree to be the leader in such situations. I take up this question in the conclusion.

B. *Leading by Example*

Suppose, now, that the leader can expend effort before the other workers or she can credibly commit to a level of effort.[14] Assume the other workers can observe this, but that contracts cannot be written contingent on the leader's effort. The other workers can, however, make inferences about θ based on the leader's effort.

To begin, consider an affine-shares contract satisfying

$$\text{(C1)} \qquad s_L = \frac{1}{1 + N(N-1)} \text{ and}$$

$$s_n = \frac{N}{1 + N(N-1)} \text{ for } n \neq L.$$

Note that the shares are not contingent on any announcement by the leader. Assume, too, that neither are the side payments [i.e., $t_n(\hat{\theta}) = t_n$ for all $\hat{\theta}$ and all n]. We then have the following proposition.

PROPOSITION 4: *Under contract* (C1) *there exists a separating perfect Bayesian equilibrium in which the workers mimic the leader's effort. Moreover, aggregate welfare* [*i.e.,* $V - \sum_{n=1}^{N} d(e_n)$] *is greater in this equilibrium than in the mechanism-design equilibrium of Proposition* 3.[15]

[13] This follows since the difference between U_1 and U_0,

$$\frac{\theta}{N} \sum_{n \neq L} [e_n(\bar{\theta}) - e_n(\underline{\theta})],$$

is increasing in θ [where $e_n(\hat{\theta})$ is worker n's best response to his belief $\hat{\theta}$].

[14] As a technical matter, there is the question of whether the leader can expend effort both before the others and contemporaneously with them. Whereas this is an important question in the Stackelberg-signaling literature (see, e.g., Mailath, 1993)—in particular, does zero "effort" before guarantee no "effort" with—this is not an issue here because the leader wants to convince the followers that the state is good. Hence, as will become clear, her marginal return to her effort is greater when she expends effort before rather than with the followers. That is, she will expend all her effort before rather than with.

[15] Except when $\theta = 0$. For the sake of brevity, I will not repeat this caveat later.

Since the followers make inferences about θ based on the leader's effort, the leader's effort serves as a signal of the state. In particular, the harder the leader works, the harder the followers work. This gives the leader an additional incentive beyond that generated by her share of the value created. In fact, her incentives are sufficiently increased that the team can reduce her share, thereby increasing the shares (incentives) for the other workers. Consequently, each member of the team works harder than in the pure mechanism environment of Proposition 3. Because the free-riding endemic to teams means too little effort to begin with, inducing harder work is welfare improving. Indeed, because Proposition 3 shows that a pure mechanism does as well as symmetric information, Corollary 1 follows.

COROLLARY 1: *Under contract* (C1) *aggregate welfare* [*i.e.*, $V - \Sigma_{n=1}^{N} d(e_n)$] *is greater with leading by example than under symmetric information.*

Figure 2 illustrates Proposition 4 and its corollary for a two-state model. The equilibrium is that the bad-state, $\underline{\theta}$, leader takes effort $\underline{e}^{SB}$, where "SB" stands for the second-best *under symmetric information,* and the good-state, $\bar{\theta}$, leader takes effort $\bar{e}^{*}$. The workers believe that leader effort less than $\bar{e}^{*}$ means it is the bad state, while effort greater or equal to $\bar{e}^{*}$ means it is the good state. Observe that $\bar{e}^{*} > \bar{e}^{SB}$; the good-state leader must expend more effort than under symmetric information. If she expended less, say her symmetric-information, second-best level, $\bar{e}^{SB}$, then it is no longer credible that the state is good—the bad-state leader would also choose $\bar{e}^{SB}$ because point A lies *above* her equilibrium indifference curve, $\underline{U}_0$. Indeed, to be credible, the good-state leader's effort must be such that the bad-state leader has no incentive to mimic; that is, it cannot be to the left of point B. The best the good-state leader can do is, thus, point C. Since $\bar{e}^{*}$ is closer to first-best effort, $\bar{e}^{FB}$, than is $\bar{e}^{SB}$, welfare is improved.

Proposition 4 and its corollary constitute an example of two organizational problems—free-riding and impacted information—combining to "cancel each other out." That is, the team does better when it can make the transmission of information costly. This is in the spirit of Bernard Caillaud and Hermalin (1993), where introducing a hidden-information agency problem into a signaling problem can be welfare improving because the agency costs raise the *potential* costs of signaling, thereby reducing the production distortion due to signaling.[16]

In addition to adding signaling, Proposition 4 also changes the sequence of moves in the game vis-à-vis the games considered in Propositions 2 and 3. It is signaling, however, that is solely responsible for Proposition 4 and its corollary: From (2), each worker's choice of effort is a *dominant* strategy for him or her and, hence, cannot be influenced by observing his or her co-workers' efforts.[17]

Proposition 4 does *not* establish that contract (C1) is optimal: It only establishes that when the leader leads by example, the team is better off than when it is limited to pure-announcement mechanisms as in Proposition 3. To derive the optimal contract, let s_L be the leader's share and s_W be a worker's share (from Proposition 2, it is optimal to treat the workers equally). Let $\tilde{e}(\theta)$ be the leader's equilibrium strategy (contingent on the contract in place). Since I am interested in the transmission of the leader's information, I will consider separating equilibria only; hence, $\tilde{e}(\cdot)$ is invertible. Conditional on the leader's strategy and effort, e_L, each worker chooses e_n to maximize

$$(7) \quad s_W \tilde{e}^{-1}(e_L)\left[e_n + \sum_{j \neq n} e_j\right] - d(e_n).$$

Let $\hat{e}[s_W \tilde{e}^{-1}(e_L)]$ be the solution. From (7), it is clear that $\hat{e}(\cdot)$ is increasing and differentiable. Anticipating the reaction of the workers, the leader chooses e_L to maximize

$$\theta s_L(e_L + (N-1)\hat{e}[s_W \tilde{e}^{-1}(e_L)]) - d(e_L).$$

[16] Of course, both of these are examples of the theory of the second best (e.g., monopolization of a *polluting* industry can be welfare improving) applied to organizations.

[17] If, however, buy-back contracts (see footnote 9 *supra*) were feasible, then changing the sequencing of moves would, in itself, affect the set of equilibrium outcomes.

The first-order condition is

$$\theta s_L\left(1 + (N-1)s_W\hat{e}'[s_W\tilde{e}^{-1}(e_L)] \times \frac{1}{\tilde{e}'[\tilde{e}^{-1}(e_L)]}\right) - d'(e_L) = 0.$$

In equilibrium, the e_L that solves this first-order condition must equal $\tilde{e}(\theta)$. Making that substitution, we have

$$\theta s_L\left(1 + (N-1)s_W\frac{\hat{e}'(s_W\theta)}{\tilde{e}'(\theta)}\right) - d'[\tilde{e}(\theta)] = 0. \tag{8}$$

The leader's strategy, $\tilde{e}(\cdot)$, is then the solution to this differential equation. As is well known for signaling games (see, e.g., Mailath, 1987), the initial condition for this differential equation is fixed by the requirement that the "worst" type, 0, get her maximum utility conditional on being identified as the worst type. It is readily shown, therefore, that $\tilde{e}(0) = 0$.

In general, solving the differential equation (8) subject to the initial condition $\tilde{e}(0) = 0$ is wicked hard. To ensure an analytic solution, which simplifies the exposition, assume that $d(e) = \frac{1}{2}e^2$. I can, then, establish the following lemma.

LEMMA 3: *Assume* $d(e) = \frac{1}{2}e^2$. *Let the leader's share be* s_L. *Then her equilibrium strategy is* $\tilde{e}(\theta) = k(s_L)\theta$, *where*

$$k(s_L) = \frac{s_L + \sqrt{4s_L - 3s_L^2}}{2}. \tag{9}$$

I can now solve for the optimal contract. Aggregate welfare is

$$\theta\tilde{e}(\theta) + \theta(N-1)\hat{e}(s_W\theta) - \tfrac{1}{2}(\tilde{e}(\theta)^2 + (N-1)\hat{e}(s_W\theta)^2)$$
$$= \theta^2[k(s_L) + (N-1)s_W - \tfrac{1}{2}(k(s_L)^2 + (N-1)s_W^2)].$$

Note that maximizing aggregate welfare with respect to the shares is independent of the value of θ. That is, the heretofore implicit assumption that the shares are *not* functions of announcements about θ is without loss of generality. Since shares sum to one, maximizing aggregate welfare is equivalent to maximizing

$$k(s_L) + 1 - s_L - \frac{1}{2}\left(k(s_L)^2 + \frac{(1-s_L)^2}{N-1}\right) \tag{10}$$

with respect to s_L. The solution has the following properties.

PROPOSITION 5: *Assume* $d(e) = \frac{1}{2}e^2$. *Under the optimal contract, the leader works at least as hard as any worker and strictly harder if* $N \geq 3$. *The leader's share,* s_L, *is declining in N, but is bounded below by a number approximately equal to* 0.128843. *Finally, the leader's share is less than any worker's (i.e.,* $s_L < s_W$*) if* $N \leq 6$, *but greater than his if* $N \geq 7$.

Proposition 5 establishes that optimal leading by example can mean that the leader works harder than any individual worker. She works harder, in part, because that is necessary for her to signal her information. In a large team ($N \geq 7$) she also works harder because she gets a larger share of the value created.

That the leader gets a larger share than any other worker in a large team might, at first, seem inconsistent with the intuition given above for the earlier leading-by-example result, Proposition 4. There, recall, the signaling incentive made it possible to increase the incentives (shares) of the other workers without excessively diminishing the leader's overall incentives ("sum" of signaling and share incentives). This effect is still present here. But a second effect exists: The strength of the signaling incentive depends, in part, on the leader's share. If, for instance, her share were zero, then she would have no incentive to signal.[18] Consequently, the need to preserve the

[18] She would also have no incentive to misrepresent θ, but from Proposition 3 that is *not* even the most efficient pure mechanism, so it must be dominated by leading by example.

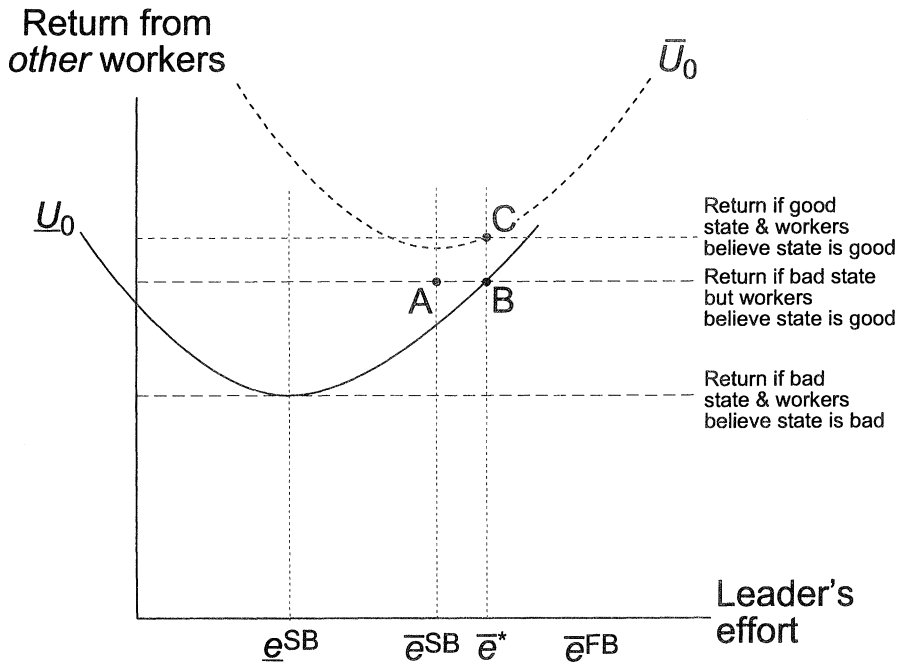

FIGURE 2. LEADING BY EXAMPLE

signaling incentive places a lower bound on the leader's share. Since the average share must tend to zero as the size of the team increases, a lower bound on the leader's share means that eventually her share will exceed that of any other worker as the team grows.

That the leader's share is ever less than the other workers' may, at first, seem counterfactual.[19] After all, in most corporations, it is typically upper management (the "leaders") who receive compensation contingent on the corporation's performance, while the production workers receive little contingent compensation. But—to the extent they are teams at all—these are large teams; so this is consistent with Proposition 5. In contrast, consider smaller teams like maîtres d' and waiters or floorwalkers and department-store salespeople. In these teams, the workers' compensation is more contingent than the leaders: Waiters typically get a larger share of the tips than the maître d' and the salespeople get a larger share of the commissions than the floorwalker.[20]

For teams in which leadership is especially informal (e.g., an academic committee of "equals"), their ability to write contracts could be even more limited than considered here. In particular, the shares are likely fixed and roughly equal. This seems particularly true when the value, V, is nonmonetary or indivisible (e.g., a public good). It is therefore worth considering leading by example when shares are fixed at $1/N$. Since the leader has the same share as the other workers, but also a signaling incentive, it follows that she works harder.

[19] Of course, using side payments (i.e., $\{t_n\}$), it could be that the leader's *overall* compensation is greater.

[20] Admittedly, it could be argued that tips and commission are paid on the basis of individual performance rather than group performance so these are not examples of teams. However, it is not uncommon for waiters to pool their tips or salespeople to pool their commissions, particularly when team work is expected.

PROPOSITION 6: *Assume an equal-shares contract. Then the leader works harder in a separating equilibrium than any individual worker and the value of the common endeavor, V, is greater. Moreover, if* $d(e) = {}^1/_2 e^2$, *then leading by example yields greater aggregate welfare than the pure mechanism of Proposition 3 or under symmetric information.*

By redefining the variables, this proposition can be applied to the situation in which citizens—the "team"—make private donations to a public good.[21] Proposition 6 suggests that when some donors are better informed about the worthiness of a public good (e.g., charity) than others, it could be welfare improving to have them donate before the less well-informed donors. This could explain, for instance, why solicitations for donations often list the amounts given by earlier donors. It is also a different conclusion than reached in the case where donors are symmetrically informed: Under symmetric information, simultaneous donations increase welfare vis-à-vis sequential donations (Hal R. Varian, 1994).

III. Conclusion

Leadership, as distinct from authority, is an important phenomenon in organizations—a point made abundantly clear by the literature in other social sciences. This literature has not, however, adequately explained a defining feature of leadership: How do leaders induce a following among rational agents? Clearly—and this is a point made by this literature—followers must conclude that they do better to follow than not; but why do they conclude this? And how does the leader generate this conclusion? This article has argued that followers follow because they become convinced: (i) that the leader has superior information, and (ii) that the leader, despite incentives to the contrary, is not misleading them; that she is informing them honestly. This second task is achieved by the leader convincingly signaling her information either by sacrificing or by setting an example.

In the analysis presented here, I treated these two methods of signaling information as alternatives, with leading by example dominating leading by sacrifice. This raises two questions: First, if leading by example dominates leading by sacrifice, why do we ever see the latter? Second, would some hybrid of the two outperform leading by example? An obvious answer to the first question is that it may be impossible to sequence activities so the leader goes first. Alternatively, the leader is incapable of doing the same task as the workers (although she may pursue other leadership actions—see below). The answer to the second question is no: As discussed, both leading by sacrifice and leading by example are forms of signaling. In the former, the signal is a transfer, so it has no *direct* impact on welfare. In the latter, it is a productive action, so it *directly* increases welfare. For this reason, signaling by example is welfare superior to signaling by sacrifice. A hybrid scheme, which would reveal some information in a nonproductive manner, cannot, therefore, increase incentives to reveal information in a productive manner, although it could decrease them.

As noted, I have so far assumed that the leader's action is qualitatively the same as the workers'. Instead, she could do "leader" activities like planning. Although this changes the mechanics of my model, it does not alter its underlying logic: Workers will still look to the leader's actions for signals about the return to their own efforts.[22] The same is possible if we left the teams setup and assumed individual productivity measures were available *for the workers.* Their willingness to accept their individual incentive contracts or their performance under them (or both) could depend on what they inferred about θ from the leader's actions.[23]

Although generating a following is a critical aspect of leadership, there are admittedly other

[21] An earlier working paper explored this in greater detail. Contact the author for the relevant section of that working paper.

[22] An earlier working paper explored this in greater detail. Contact the author for a copy of the relevant section.

[23] This analysis assumes that the value of θ could not be verified from the individual performance measures; i.e., there would need to be other stochastic factors.

aspects of leadership to investigate: What else do leaders do? Which traits make a good leader? To what extent is leadership defined or limited by its cultural context? How are leaders chosen? And why do people want to be leaders?

Beginning with this last question, my analysis admittedly said little beyond pointing out that the leader's participation constraint could be met by increasing her side payment. In some cases, side payments are a sufficient explanation for why individuals are willing or seek to become leaders. A dean, for instance, might induce someone to become department chair by promising her reduced teaching, higher pay, or more faculty slots.

Certainly it seems in many cases that leadership is sought for reasons other than remuneration. For instance, it could be that the leader sees leadership as a way of winning divine or historical approval (consider, e.g., biblical prophets or Joan of Arc). A related view is that the leader derives utility directly from having a following. To the extent that leadership is the result of a desire to have a following, the leader could come to be ''led'' by the followers: For instance, suppose each follower's utility is

$$\theta e - d(e),$$

but followers are ignorant of *their* marginal value, θ, for e. Then followers follow the leader only so long as she ''proves'' herself in the Weberian sense (footnote 7 *supra*) by signaling the correct θ. To gain a following, the leader must, therefore, determine what the followers prefer and match her example to what *they* most prefer—she must be ''in touch'' with her followers or ''feel their pain.''

When leaders want to be leaders, we would often expect competition to be leader. How such competition plays itself out depends in large part on the dimensions over which would-be leaders compete. There are many possible modeling strategies: There could be competition to learn θ first or best; there could be competing common endeavors with different marginal returns, θ_1 and θ_2, each championed by a different would-be leader; there could be just one winner (leader) to this competition, or multiple winners each leading a bloc or sect drawn from the original team.

In other situations, however, people become leaders reluctantly. For instance, suppose that it was common knowledge within the team that member L knew θ or could learn it costlessly and secretly. Then L would have no choice but to serve as leader:[24] Knowing she knows θ, her teammates would rationally wait to see what she did before deciding how much effort to expend themselves.[25] But knowing this, L would take this into account when choosing her own actions, which is what leadership—as modeled here—is about. That is, by virtue of their better information (or the belief that they have better information), individuals could find leadership thrust upon them. Moreover, even if they would prefer not to be leader, they are forced to act as leader once they have gained the mantle of leadership in this way. Hence, if leadership is underrewarded, ignorance could be bliss, which could create undesirable consequences for organizations: Incentives to learn valuable information would be blunted or eliminated.

On the other hand, organizations may not want *everyone* seeking information when leading by example dominates symmetric information. This suggests that organizations may be structured to keep information in the hands of the leaders and prevent its premature dissemination to the masses.[26] This could help to

[24] This is not an unreasonable supposition. Empirical work (see House and Baetz [1979] for a survey) finds that individuals exhibiting energy (low cost of learning θ), intelligence (high likelihood of knowing θ, low cost of learning), education (ditto), self-confidence (suggests knowing θ), and task-relevant knowledge (knowing θ) are more likely to be made leaders than those not exhibiting these traits.

[25] If L starts off not knowing θ, but can learn it costlessly and secretly, then the analysis is more involved, but reaches the same conclusion. Suppose there were an equilibrium in which L learned θ with probability $\zeta \in [0, 1)$. Given that her teammates cannot see whether she has, in fact, learned θ, L cannot do worse by learning θ; in fact, she does better, because this information allows her to choose a better action from the perspective of her own utility. Hence, $\zeta \not< 1$ in equilibrium.

[26] Although empirical work (see, e.g., the survey by House and Baetz) suggests that a leader who is overly secretive may alienate followers. What is unclear from this literature is whether followers are alienated because they

explain why planning and information gathering are actions often associated with leaders. Indeed, these are typically viewed as part of leadership.

Political theory and other social sciences have focussed much of their research attention in leadership on the question of establishing legitimacy. Within political theory, this question has been raised primarily with respect to formal authority (de jure leadership), and so is orthogonal to the less formal notion of leadership (de facto leadership) considered here.[27] Other social sciences have taken up the issue in the context of less formal leadership. There the focus has been on the traits and behaviors critical for establishing legitimacy. Traits can be both acquired and intrinsic. Many of the traits that have been found to be important empirically (see, e.g., House and Baetz), such as education and task-specific knowledge, would seem correlated with knowing θ and so in keeping with our analysis (see also footnote 24 *supra*). Intrinsic traits, such as height, sex, race, and class, are more difficult to embed in a model of fully rational actors, although they are perhaps consistent with a model of semi-rational actors. For instance, potential leaders from a marginalized group could lack legitimacy because of a stereotype that they do not know θ (so their example is seen not to convey information) or because there is a social cost—stigma—associated with being perceived to have followed the lead of someone in the marginalized group. Either could explain, for example, why a group of men might not follow the lead of a woman in repairing a car.

Behavior that establishes legitimacy is behavior that shows that the would-be leader's goals and objectives are consistent with those of her putative followers; that is, the leader must show that she is within the team's culture rather than outside it (House and Baetz).[28] These could represent actions to show that her preferences accord with her followers or that it is reasonable to believe she knows θ. Alternatively, these actions can be seen as demonstrating empathy so as to build trust, either for the reasons put forth in Rotemberg and Saloner (1993) or to allow the leader greater flexibility (e.g., to permit followers to go along with actions that are costly in the short run because they trust they will pay off in the long run).

Although, as this discussion makes clear, there is more to leadership than the analysis presented here, this analysis is both consistent with the analyses in other literatures and offers explanations for many of them. Moreover, it is intended to suggest that the framework put forth here can be extended to explain a wider set of questions dealing with leadership (although in many instances *how* it should be extended is a question for future research).

Appendix

PROOF OF PROPOSITION 1:

Let $\tilde{\theta} = \mathbb{E}_\theta\{\theta \mid \hat{\theta}\}$. Let $\tilde{e}_n$ be worker n's equilibrium level of effort given $\{w_n(\cdot, \hat{\theta})\}_{n=1}^N$ and let $\tilde{V}$ be the corresponding equilibrium value (observe that $\tilde{e}_n$ is a function of $\hat{\theta}$ and $\tilde{V}$ is a function of θ and $\hat{\theta}$). We need to consider two cases: (1) $\tilde{e}_n > 0$ (interior solution) for all n, and (2) $\tilde{e}_n = 0$ (corner solution) for some n. We begin with case 1. There exists a $\bar{\Delta} > 0$, such that $\tilde{e}_n - \Delta \geq 0$ for all $\Delta \in [0, \bar{\Delta}]$. Observe that equilibrium requires

$$\mathbb{E}_\theta\{w_n(\tilde{V}, \hat{\theta})\} - d(\tilde{e}_n) \geq \mathbb{E}_\theta\{w_n(\tilde{V} + \theta\Delta, \hat{\theta})\} - d(\tilde{e}_n + \Delta) \text{ and}$$

$$\mathbb{E}_\theta\{w_n(\tilde{V}, \hat{\theta})\} - d(\tilde{e}_n) \geq \mathbb{E}_\theta\{w_n(\tilde{V} - \theta\Delta, \hat{\theta})\} - d(\tilde{e}_n - \Delta)$$

inherently dislike asymmetries of information or because secretive leaders do a bad job of transmitting this information through leader sacrifice or leading by example.

[27] Although political theorists do worry, to some extent, about capturing followers' (subjects') ''hearts and minds.'' A topic, e.g., in Machiavelli's *The Prince* (Bondanella and Musa, 1979) or in Wrong (1995).

[28] Although, ironically, once she has established that she is within the culture, one of her roles as leader is often to change the culture (see Edgar H. Schein [1992] for an analysis in the context of organizations or Stephen Skowronek [1997] for an analysis in the context of U.S. presidents changing the political culture).

for all $\Delta \in (0, \bar{\Delta}]$. As a preliminary step, we will show that

$$\text{(A1)} \qquad \sum_{n=1}^{N} d'(\tilde{e}_n) = \tilde{\theta}.$$

The equilibrium conditions imply

$$\text{(A2)} \quad \frac{d(\tilde{e}_n + \Delta) - d(\tilde{e}_n)}{\Delta} \geq \frac{\mathbb{E}_\theta\{w_n(\tilde{V} + \theta\Delta, \hat{\theta}) - w_n(\tilde{V}, \hat{\theta})\}}{\Delta}$$

and

$$\text{(A3)} \quad \frac{\mathbb{E}_\theta\{w_n(\tilde{V}, \hat{\theta}) - w_n(\tilde{V} - \theta\Delta, \hat{\theta})\}}{\Delta} \geq \frac{d(\tilde{e}_n) - d(\tilde{e}_n - \Delta)}{\Delta}$$

for all $\Delta \in (0, \bar{\Delta}]$. Summing (A2) and (A3) yields

$$\text{(A4)} \quad \sum_{n=1}^{N} \frac{d(\tilde{e}_n + \Delta) - d(\tilde{e}_n)}{\Delta} \geq \frac{\mathbb{E}_\theta\{\sum_{n=1}^{N} [w_n(\tilde{V} + \theta\Delta, \hat{\theta}) - w_n(\tilde{V}, \hat{\theta})]\}}{\Delta} = \tilde{\theta}$$

and

$$\text{(A5)} \quad \frac{\mathbb{E}_\theta\{\sum_{n=1}^{N} [w_n(\tilde{V}, \hat{\theta}) - w_n(\tilde{V} - \theta\Delta, \hat{\theta})]\}}{\Delta} = \tilde{\theta} \geq \sum_{n=1}^{N} \frac{d(\tilde{e}_n) - d(\tilde{e}_n - \Delta)}{\Delta}$$

for all $\Delta \in (0, \bar{\Delta}]$. Letting $\Delta \to 0$, (A4) and (A5) implies (A1). Define

$$s_n(\hat{\theta}) = \frac{1}{\tilde{\theta}} d'(\tilde{e}_n).$$

From (A1), $\Sigma_{n=1}^{N} s_n(\hat{\theta}) = 1$. Define

$$t_n(\hat{\theta}) = \mathbb{E}_\theta\{w_n(V, \hat{\theta}) | \hat{\theta}\} - s_n(\hat{\theta}) \cdot \tilde{\theta} \cdot \sum_{j=1}^{N} \tilde{e}_j.$$

Since $\Sigma_{n=1}^{N} w_n = V$ and $\Sigma_{n=1}^{N} s_n = 1$, $\Sigma_{n=1}^{N} t_n = 0$. Under this affine-shares contract, each worker chooses e_n to maximize

$$t_n(\hat{\theta}) + s_n(\hat{\theta})\tilde{\theta} \sum_{j=1}^{N} e_j - d(e_n).$$

Regardless of his beliefs about his fellow worker's efforts, it is a dominant strategy for him to choose the e_n that solves

$$s_n(\hat{\theta})\tilde{\theta} - d'(e_n) = 0.$$

Since, however, $s_n(\hat{\theta}) = d'(\tilde{e}_n)/\tilde{\theta}$ and $d(\cdot)$ is convex, this first-order condition has the unique solution $e_n = \tilde{e}_n$. So $\{\tilde{e}_n\}_{n=1}^{N}$ remains an equilibrium (in fact, it is unique) under this affine-shares contract. Finally, since it induces the same effort, it yields the same expected utility by construction.

Now consider case 2. Expressions (A2) and (A4) are still valid. Letting $\Delta \to 0$ yields

$$\text{(A6)} \qquad \sum_{n=1}^{N} d'(\tilde{e}_n) \geq \tilde{\theta}.$$

Let I be index the set of workers supplying positive effort. Then for $m \notin I$, we have $\tilde{e}_m = 0$, so $d'(\tilde{e}_m) = 0$. Expression (A6) can, thus, be rewritten as

$$\text{(A7)} \qquad \sum_{n \in I} d'(\tilde{e}_n) \geq \tilde{\theta}.$$

For $n \in I$, let $s_n(\hat{\theta}) = d'(\tilde{e}_n)/\tilde{\theta}$. As shown above, these shares will induce the $n \in I$ to supply the same effort as when $\{w_n(\cdot, \hat{\theta})\}_{n=1}^{N}$ was the contract. Observe, too, that these shares are all positive. From (A7), we have $\Sigma_{n \in I} s_n \geq 1$. Clearly, for $\{m | m \notin I\}$ one can find $s_m(\hat{\theta}) \leq 0$ such that

$$\sum_{n \in I} s_n + \sum_{m \notin I} s_m = 1.$$

Since, $s_m(\hat{\theta}) \leq 0$, it follows that it will induce m to choose $e_m = 0 = \tilde{e}_m$. Hence, these shares sum to one and induce the same efforts in equilibrium as did $\{w_n(\cdot, \hat{\theta})\}_{n=1}^N$. Finally, it is clear that one can construct $\{t_n(\hat{\theta})\}_{n=1}^N$ just as was done in case 1 so that the equilibrium utilities are the same and $\Sigma_{n=1}^N t_n = 0$.

PROOF OF PROPOSITION 2:

Let $\tilde{\theta} = \mathbb{E}\{\theta|\hat{\theta}\}$. Collectively, the workers seek to

$$\text{(A8)} \quad \max_{\{s_n\}} \sum_{n=1}^{N} (\tilde{\theta}e(s_n) - d[e(s_n)])$$

$$\text{(A9)} \quad \text{subject to } \sum_{n=1}^{N} s_n = 1,$$

where $e(\cdot)$ is defined by the individual worker's maximization problem

$$\max_e \tilde{\theta}s_n e - d(e) + \tilde{\theta}s_n \sum_{m \neq n} e_m.$$

Since $s_n \geq 0$, $d'(0) = 0$, $e \geq 0$, and the maximand is differentiable everywhere, all solutions must be *interior* solutions. Moreover, the problem is globally concave, so the first-order condition is also sufficient. Hence, $e(\cdot)$ is defined to be the unique solution to

$$\text{(A10)} \quad \tilde{\theta}s_n - d'(e) = 0.$$

The first-order condition for (A8) is

$$\text{(A11)} \quad \tilde{\theta}e'(s_n) - d'[e(s_n)] \cdot e'(s_n) - \lambda = 0 \quad \text{for all } n,$$

where λ is the Lagrange multiplier on (A9). Equal shares (i.e., $s_n = 1/N$ for all n) satisfy this first-order condition. We need to show, however, that this represents a global maximum. This follows if $\tilde{\theta}e(s_n) - d[e(s_n)]$ is concave in s_n: The second derivative is

$$\tilde{\theta}e''(s_n) - d''[e(s_n)] \cdot [e'(s_n)]^2 - d'[e(s_n)]e''(s_n).$$

Using (A10) this becomes

$$-\tilde{\theta}(1 - s_n) \frac{\tilde{\theta}d'''(e)}{[d''(e)]^2} e'(s_n) - \tilde{\theta}e'(s_n).$$

If positive for any s_n, it would be positive for $s_n \to 0$. But then

$$0 > \tilde{\theta}d'''(e) + [d''(e)]^2$$

$$> d'''(e) + [d''(e)]^2$$

(recall $\tilde{\theta} \leq 1$), which contradicts (1).

PROOF OF PROPOSITION 3:

Given Proposition 2, all we need show is that this contract induces truth-telling. The leader announces $\hat{\theta}$ to maximize

$$\frac{\theta}{N}(e + (N-1)e(\hat{\theta})) + t_L(\hat{\theta}) - d(e).$$

The first-order condition is

$$\text{(A12)} \quad \theta \frac{N-1}{N} e'(\hat{\theta}) - \hat{\theta} \frac{N-1}{N} e'(\hat{\theta}) = 0.$$

Truth-telling is a solution. Moreover, the left-hand side of (A12) is positive for $\hat{\theta} < \theta$ and negative for $\hat{\theta} > \theta$, so the first-order condition is sufficient as well as necessary.

PROOF OF PROPOSITION 4:

Let $e^*(\theta)$ maximize a worker's utility given that he has inferred θ; hence,

$$\text{(A13)} \quad \theta \frac{N}{1 + N(N-1)} - d'[e^*(\theta)] = 0.$$

Suppose $e^*(\theta)$ is also the leader's strategy under (C1). Then it follows from (A13) that the other workers' best response is to mimic the leader's effort; i.e., to choose $e = e^*(\theta)$. It remains to check whether $e^*(\theta)$ is the leader's best response to a mimic strategy. Given a mimic strategy, the leader maximizes

$$\text{(A14)} \quad \theta \frac{1}{1 + N(N-1)} \times (e + (N-1)e) - d(e).$$

The first-order condition is identical to (A13); hence, conditional on choosing an e in the range of $e^*(\cdot)$, choosing $e^*(\theta)$ is best. What about out-of-equilibrium effort not in the range of $e^*(\cdot)$? Since $e^*(0) = 0$ and $e \geq 0$, the only possible out-of-equilibrium e is $e > e^*(1)$. But given (A13), $e > e^*(1)$ would not be optimal for the leader even if it engendered belief $\theta = 1$. So $e^*(\theta)$ is indeed the leader's best response.

Now contrast it with the equilibrium of Proposition 3. Aggregate welfare is

$$\sum_{n=1}^{N} (\theta e_n - d(e_n)), \tag{A15}$$

which is strictly concave. Let $e^{FB}(\theta)$ maximize (A15). As Holmstrom showed, $e^{FB}(\theta)$ would be the solution if *each* worker were given a 100-percent share.[29] Since equilibrium effort is an increasing function of a worker's share (recall Lemma 1), it is sufficient to show that

$$\frac{1}{N} < \frac{N}{1 + N(N-1)} < 1, \tag{A16}$$

in order to conclude that $e(\theta) < e^*(\theta) < e^{FB}(\theta)$ and, thus, that leading by example yields greater aggregate welfare than the mechanism from Proposition 3. Simple algebra confirms that (A16) holds for all $N > 1$.

PROOF OF LEMMA 3:

From (7), it follows that $\hat{e}(\cdot)$ is the identity function, so $\hat{e}'(\cdot) \equiv 1$. Shares sum to one, hence $(N-1)s_W = 1 - s_L$. Rewrite (8) as

$$\theta s_L\left(1 + (1 - s_L)\frac{1}{\tilde{e}'(\theta)}\right) - \tilde{e}(\theta) = 0.$$

It is readily checked that $\tilde{e}(\theta) = k(s_L)\theta$ solves this differential equation when $k(\cdot)$ is defined by (9).

PROOF OF PROPOSITION 5:

As preliminaries, two claims are established.

Claim 1: If $k(s_L) > s_W$ ($k(s_L) \geq s_W$), then the leader works harder (as hard) as any individual worker.

Proof: $\tilde{e}(\theta) = k(s_L)\theta$, while $\hat{e}(\theta) = s_W\theta$.

Claim 2: $k(s_L) > s_L$.

Proof: Since $s_L \in (0, 1)$,

$$s_L^2 < 4s_L - 3s_L^2.$$

Hence,

$$s_L < \frac{s_L + \sqrt{4s_L - 3s_L^2}}{2} = k(s_L).$$

First, the leader's share is declining in N: The cross-partial derivative of (10) with respect to s_L and N is

$$-\frac{1 - s_L}{(N-1)^2} < 0.$$

Hence, s_L is declining in N.

The first-order condition for (10) is

$$[1 - k(s_L)]k'(s_L) - 1 + \frac{1 - s_L}{N - 1} = 0. \tag{A17}$$

Let $N \to \infty$ and solve (A17). This yields a lower bound for s_L of

$$\frac{10}{9} - \frac{19 \times 2^{1/3}}{9(187 + 9\sqrt{93})^{1/3}} - \frac{(187 + 9\sqrt{93})^{1/3}}{9 \times 2^{1/3}},$$

which is approximately 0.128843. Since

$$s_W = \frac{1 - s_L}{N - 1},$$

it follows from the lower bound that $s_L > s_W$ for $N \geq 8$.

We need only consider $N \leq 7$ to complete the proof. Table A1 summarizes the relevant data.

PROOF OF PROPOSITION 6:

Since each worker gets $1/N$ and knows θ when choosing effort, his effort is the same as

[29] Of course, such a contract is infeasible since the *sum* of shares must equal 100 percent.

TABLE A1—DATA ON OPTIMAL CONTRACTS AND EFFORTS

N	s_L	s_W	$k(s_L)$
2	0.3333	0.6667	0.6667
3	0.2149	0.3926	0.5320
4	0.1818	0.2727	0.4871
5	0.1668	0.2083	0.4655
6	0.1584	0.1683	0.4529
7	0.1531	0.1412	0.4446

in Proposition 3 or under symmetric information (Proposition 2). To show a greater value of V, we need only verify that the leader's equilibrium effort, $\tilde{e}(\theta)$, exceeds $e(\theta)$ (her symmetric-information best response to a share of $1/N$): Equation (8) yields

$$\frac{\theta}{N}\left(1+\frac{N-1}{N}\frac{\hat{e}'\left(\frac{\theta}{N}\right)}{\tilde{e}'(\theta)}\right)=d'[\tilde{e}(\theta)].$$

Since $\hat{e}'(\theta/N)$ and $\tilde{e}'(\theta)$ are positive, the left-hand side exceeds θ/N, which, since $d(\cdot)$ is convex, establishes that $\tilde{e}(\theta) > e(\theta)$. Given this, aggregate welfare is greater if $\tilde{e}(\theta) < e^{FB}(\theta)$. When $d(e) = {}^1\!/_2 e^2$, we have $e(\theta) = \theta/N$ and $e^{FB}(\theta) = \theta$. Using Lemma 3,

$$k(1/N)=\frac{1+\sqrt{(4N-3)}}{2N},$$

which is less than one for all $N \geq 2$, so $\tilde{e}(\theta) = k(1/N)\theta < \theta = e^{FB}(\theta)$.

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