Issues in Economic Systems and Institutions: Part II: Communication

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Can Cheap Talk Be Informative?

- Examples:
  - Nations in conflict make verbal threats and promises ("We will not allow a nuclear Iran.")
  - Politicians make campaign promises ("Read my lips, no more taxes.").
  - Policy makers declare their economic diagnoses ("Fundamentals of the economy are strong.")
  - Experts make recommendations ("Lehman Brothers: strong buy.")
  - Companies advertise their products and prospects ("Fevicol heals everything.")
- Difference from signaling: in cheap talk, messages have no *exogenous* cost, but may have *endogenous* costs.

Cheap Talk: Optimists and Pessimists

"Simply by making noises with our mouths, we can reliably cause precise new combinations of ideas to arise in each other's minds."

– Stephen Pinker.

"Oh what a tangled web we weave, when first we practice to deceive."

– Walter Scott.

"An oral contract is not worth the paper it is written on."

– Yogi Berra.

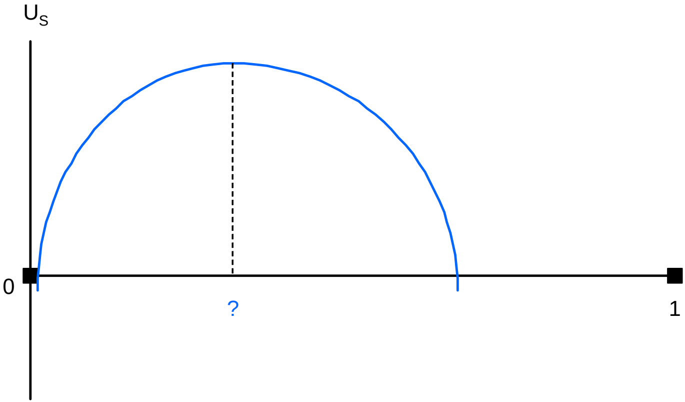
The Crawford-Sobel Model

- **Players:** Sender (S) and Receiver (R).
- **State-of-the-world:** $\theta \sim U[0,1]$, **action:** $a \in [0,1]$.
- **Information:** S knows θ, R knows only the distribution of θ.
- **Decision-making authority:** only R can choose a.
- **Preferences:** single peaked and state dependent.

$$\begin{aligned} U_R(a,\theta) &= -(a-\theta)^2 \\ U_S(a,\theta) &= -(a-b-\theta)^2 \end{aligned}$$

  Bliss points: θ for R, $\theta + b$ for S. $b > 0$ is the degree of **bias**.
- **Move sequence:** S sends message $m \in M$ to R, then R chooses a.

Payoff in Pictures



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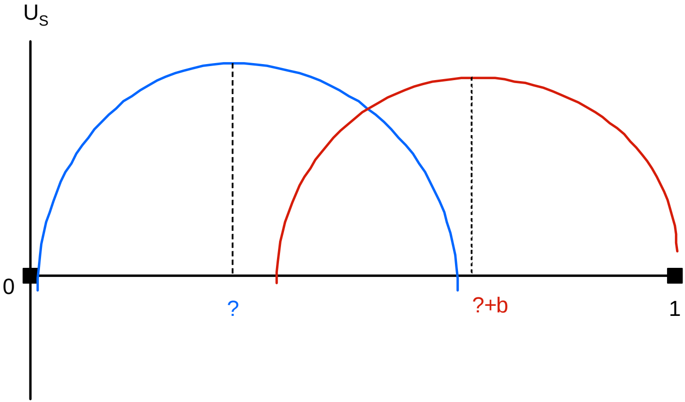

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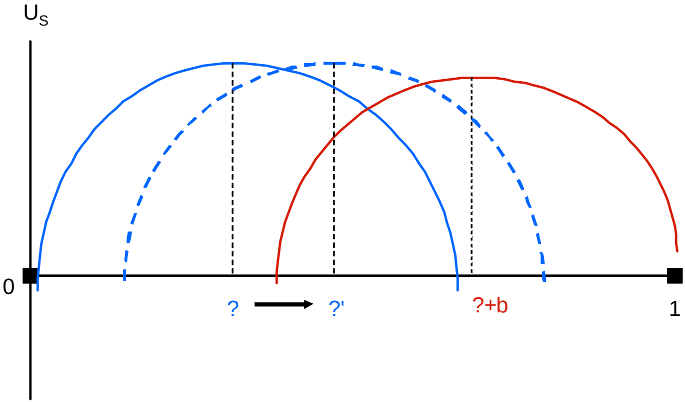

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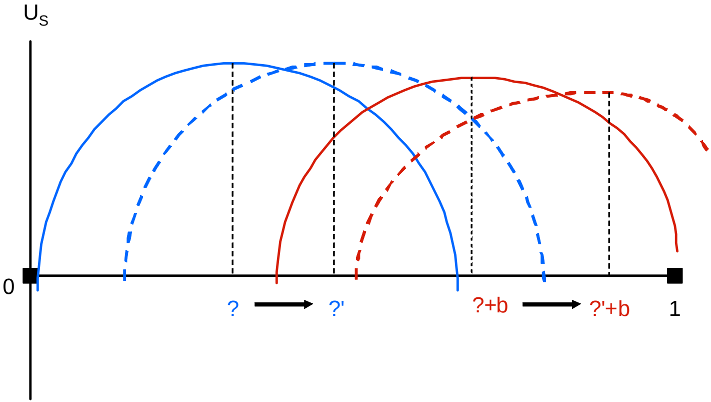


Perfect Bayesian Equilibria

- Sender's strategy: $m(\theta)$ (information-message mapping).
- Receiver's beliefs: $g(\theta|m)$ (conditional prob distributions).
- Receiver's strategy: $a(m)$ (conditional action choice).
- Sender's best response:

$$U_S(a(m(\theta)), \theta) \geq U_S(a(m), \theta) \quad \forall m \in M, \theta \in [0, 1]$$

- Bayesian beliefs:

$$g(\theta|m') = \frac{1}{\int_{m(\theta)=m'} \theta d\theta} \text{ for } m' \text{ s.t } \exists\theta \text{ s.t. } m(\theta) = m'$$

- Receiver's best response:

$$a(m) = \arg\max_a \int_0^1 U_R(a, \theta) g(\theta|m) d\theta$$

The Babbling Equilibrium

- S's strategy: $m(\theta)$ is independent of θ.
- Beliefs: $g(\theta|m) = 1$ for all θ, m (posterior same as prior).
- R's strategy: $a(m) = \frac{1}{2}$ for all m.
- Essentially, S sends a message randomly ("babbles"). R ignores the message and chooses his *ex ante* optimal action.
- Since R never listens to S, S has no incentive to talk meaningfully. Since S never talks meaningfully, R has no reason to listen to him.
- For all b, a babbling equilibrium exists.
- There may be other equilibria.

Partition Equilibria: Two Intervals

- S's strategy:

$$
\begin{aligned}
m(\theta) &= m_1 \text{ for } \theta \leq \theta_t \\
&= m_2 \text{ for } \theta > \theta_t
\end{aligned}
$$

- R's beliefs:

$$
\begin{aligned}
g(\theta|m_1) &\sim U[0, \theta_t] \\
g(\theta|m) &\sim U[\theta_t, 1] \quad \text{for all } m \neq m_1.
\end{aligned}
$$

Partition Equilibria: Two Intervals

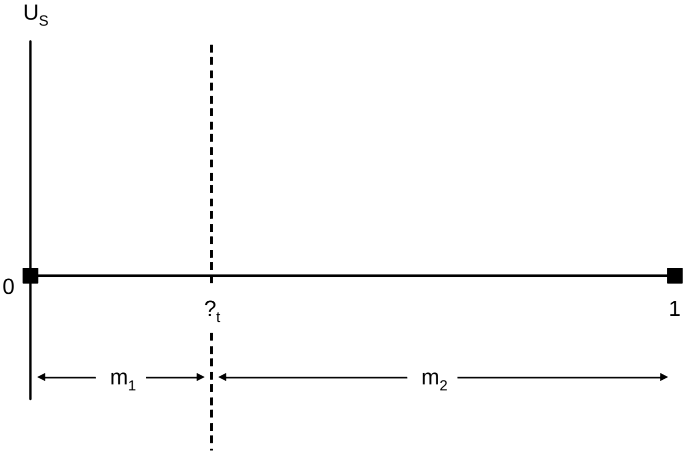
- R's strategy:

$$
\begin{aligned}
a(m) &= \frac{\theta_t}{2} \text{ for } m = m_1 \\
&= \frac{1+\theta_t}{2} \text{ for } m \neq m_1
\end{aligned}
$$

- Best response for S:

$$
\begin{aligned}
\theta + b - \frac{\theta_t}{2} &\lesseqgtr \frac{1+\theta_t}{2} - \theta - b \text{ for } \theta \lesseqgtr \theta_t \\
\theta &\lesseqgtr \frac{1}{4} + \frac{\theta_t}{2} - b \text{ for } \theta \lesseqgtr \theta_t
\end{aligned}
$$

Locating the Threshold



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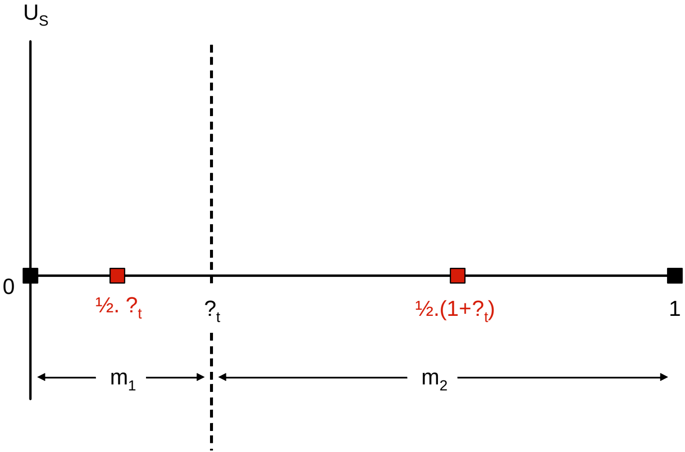

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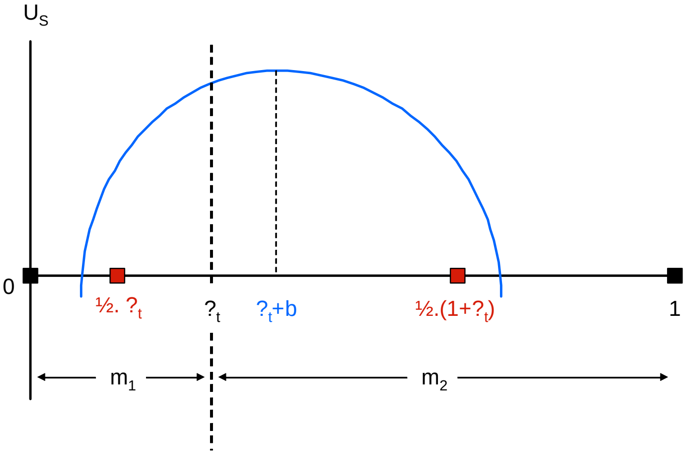

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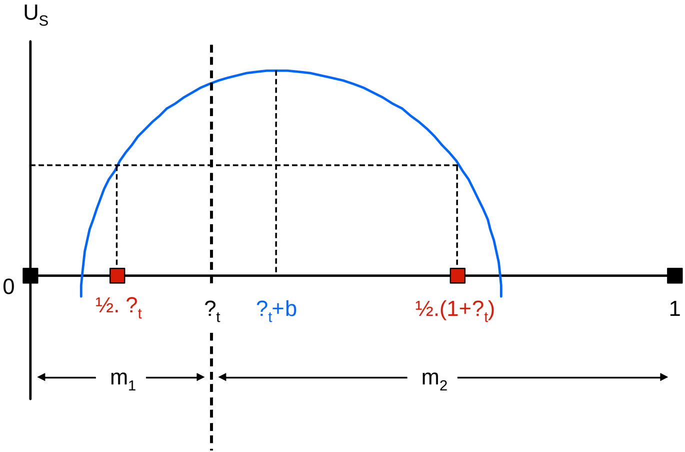


Binary Message Equilibrium

- Solving for the equilibrium cutoff:

$$\theta_t = \frac{1}{4} + \frac{\theta_t}{2} - b \Rightarrow \theta_t = \frac{1}{2} - 2b$$

- Condition for the existence of a 2-message equilibrium:

$$\frac{1}{2} - 2b \geq 0 \Rightarrow b \leq \frac{1}{4}$$

- If available information *must* be partitioned into two intervals, what is the optimal partition? Answer: equal sized intervals.

$$\theta^* = -\arg\max_x \left[\int_0^x \left(\theta - \frac{x}{2}\right)^2 + \left(\theta - \frac{1+x}{2}\right)^2 \right] = \frac{1}{2}$$

Binary Message Equilibrium: Observations

- The equilibrium is informative but "coarse": R learns *something* from the conversation but not *everything*.
- For a 2-message equilibrium, it is necessary but not sufficient (remember babbling) that S's bias be "not too large."
- Useful information exchange depends on *both* incentives (how much bias) and coordination (which equilibrium to select).
- Equilibrium interval lengths are right-skewed. Information gets more garbled in the direction of bias.
- An additional source of welfare loss: unequal intervals.

General Partition Equilibrium

- A different message is sent for each of the intervals $[\theta_0, \theta_1], (\theta_1, \theta_2], \ldots, (\theta_{n-1}, \theta_n]$, where $\theta_0 = 0$ and $\theta_n = 1$.
- When the message corresponds to the k-th interval , R's optimal action choice is

$$a_k = \frac{1}{2}\left(\theta_{k-1} + \theta_k\right)$$

- The borderline type θ_k must be different between the adjacent actions a_{k-1} and a_k:

$$\theta_k + b - \frac{1}{2}\left(\theta_{k-1} + \theta_k\right) = \frac{1}{2}\left(\theta_k + \theta_{k+1}\right) - \theta_k - b$$

- Define interval lengths:

$$y_k = \theta_{k+1} - \theta_k$$

Solution

- The interval lengths $\{y_k\}_{k=1}^{n}$ satisfy a first order difference equation plus an aggregate constraint:

$$y_{k+1} = y_k + 4b$$

$$\sum_{k=1}^{n} y_k = 1$$

- Solution to the system of equations:

$$y_k = \frac{1}{n} + 2b(2k - n - 1)$$

Equilibrium Properties

- Necessary and sufficient condition for an n-interval equilibrium ($n \geq 2$) is

$$y_1 > 0 \Rightarrow b < \frac{1}{2n(n-1)}$$

- For every b, there exists a highest integer $n(b)$ such that an n-interval equilibrium exists for all $n \leq n(b)$ (note: RHS is $\downarrow$ in n).
- $n(b)$ is decreasing in b.
- $n(b) \longrightarrow \infty$ as $b \longrightarrow 0$.
- $n(b) = 1$ for $b < \frac{1}{4}$.

Economic Interpretation

- With conflict as well as common interest, speakers will not be able to credibly convey "fine" information, but they may be able to convey "coarse" information.
- As the bias increases, equilibria become coarser in two senses: (i) fewer distinctions (i.e., intervals) can be made (ii) the distinguished categories (i.e. intervals) are less even.
- As the bias goes to zero, the most informative cheap talk outcome converges to full revelation (Spector 2002).
- For high enough bias, only babbling is possible.
- Interesting questions: (i) equilibrium selection (ii) welfare.

Receiver's Welfare

- Measure of welfare: a player's ex ante expected payoff calculated before θ becomes known.
- Consider an interval of length x and uniform distribution. Action choice (mid-point) $= \frac{x}{2}$.
- Conditional expected utility is the (negative of) variance of θ within the interval:

$$\sigma^2 = \int_0^x \left(\theta - \frac{x}{2}\right)^2 . \frac{1}{x} . d\theta = \frac{1}{12} . x^2$$

- Expected payoff is the average of the conditional expected payoffs from each interval, weighted by the probability of θ falling in that interval.

Receiver's Welfare

- Ex ante expected payoff of R:

$$
\begin{aligned}
|U_R| &= \sum_{k=1}^{n} y_k . \frac{1}{12} . y_k^2 \\
&= \frac{1}{12} \sum_{k=1}^{n} \left[\frac{1}{n} + 2b(2k - n - 1) \right]^3 \\
&= \frac{1}{12n^2} + \frac{b^2 (n^2 - 1)}{3}
\end{aligned}
$$

Sender's Welfare

- Consider again an interval of length x and uniform distribution.
- S's conditional expected utility within the interval is:

$$\int_0^x \left(\theta + b - \frac{x}{2}\right)^2 . \frac{1}{x} . d\theta = \frac{1}{12} . x^2 + b^2$$

- *Ex ante* expected utility of S:

$$\begin{aligned} |U_S| &= \sum_{k=1}^{n} y_k . \frac{1}{12} . y_k^2 + \sum_{k=1}^{n} y_k b^2 \\ &= |U_R| + b^2 \end{aligned}$$

Welfare Properties

- For equilibria of fixed number of intervals, welfare decreases as bias increases.
- The most informative equilibrium generates highest welfare.
- As bias increases, a second source of welfare loss is fewer possible intervals in the partition.
- If S could *commit* to speak honestly, or R could *commit* to let S take the decision (delegation), *ex ante* payoffs would be $-b^2$ and 0.
- Honesty is the best policy *ex ante* but not *ex post*. Power is valuable *ex post* but not *ex ante*.

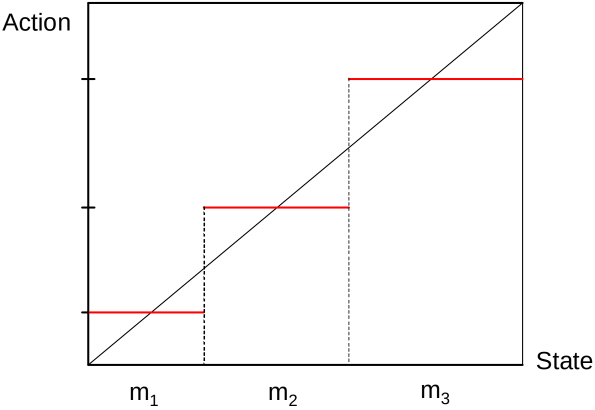
Rights as Self-Interested Gifts

- When **information** and **authority** are vested in different parties, each may be better off by giving up control over information or actions.
- Power is useless without information, information is useless without power.
- This requires some mechanism of **commitment** (rights) due to a tension between *ex ante* and *ex post* motives.
- Rights here viewed as
  - Pragmatic instruments, not moral imperatives.
  - Furthering the interests of not only recipients but also donors.
  - Achievable without conflict.

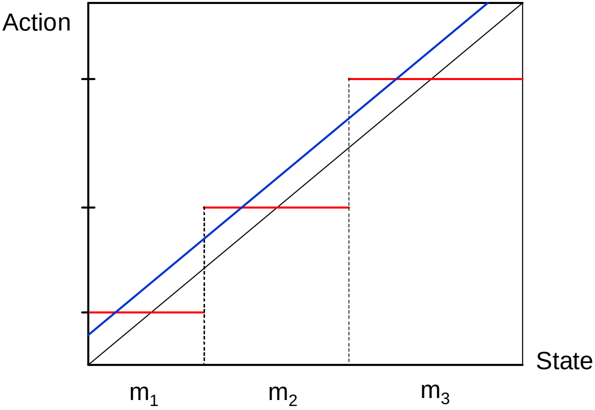
Delegation

- Delegation means R relinquishes authority and gives S the **right** to choose the action.
- Requires a **commitment** mechanism: cannot be overturned *ex post*. Tension between *ex post* and *ex ante* incentives.
- Is R better off delegating than retaining authority and resorting to cheap talk?
- Trade-off between:
  - making a more informed decision (delegation).
  - making the decision agree with R's preferences (cheap talk).
- As b becomes smaller, the cost of delegation becomes smaller but cheap talk also becomes more informative. Comparison is non-trivial.

Cheap Talk



Cheap Talk vs. Delegation



Comparison of Payoffs

Theorem

(Dessein 2004) In the uniform-quadratic case, delegation is better for the receiver than retaining authority and resorting to cheap talk whenever there is a non-babbling equilibrium in the cheap talk game ($b \leq \frac{1}{4}$). It also dominates for part of the babbling range ($\frac{1}{4} < b \leq \frac{1}{\sqrt{12}}$). Delegation is infinitely better than cheap talk as the bias becomes arbitrarily small, i.e.

$$\lim_{b \to 0} \frac{\mathbf{E}(|a^* - \theta|)}{b} = \infty$$

Comparison of Payoffs

- Expected disutility from an n-partition cheap talk equilibrium:

$$\frac{1}{12n^2} + \frac{b^2(n^2-1)}{3}$$

- Expected disutility from delegation:

$$b^2$$

- Expected disutility from making an uninformed decision $= \frac{1}{12}$.
- Delegation payoff beats cheap talk whenever $n \geq 2$. It beats uninformed decision whenever $b \leq \frac{1}{\sqrt{12}}$.

Intermediation

- Suppose R can delegate the decision to players other than S.
- The intermediary's bias b' can be strategically chosen.
- Assume a rich set of options: all possible values of b' allowed.
- S communicates to the intermediary, who who chooses a.
- Trade-off: choosing a more biased intermediary leads to
  - more distorted decisions
  - more informed decisions
- If $b \in \left[\frac{1}{6}, \frac{1}{2\sqrt{2}}\right]$, intermediation is optimal, i.e., $b' < b$.
- If $b < \frac{1}{6}$, delegation is optimal, i.e., $b' = b$.

A More General Mechanism Design Problem

- R **commits** to choose action $a^*(m)$ after hearing message m.
- By the **revelation principle**, we can restrict attention to truth-telling mechanisms, where it is incentive compatible for S to report $m(\theta) = \theta$.
- Define the set of permissible actions under the mechanism:

$$A = \{a | a(\theta) = a \text{ for some } \theta \in [0, 1]\}$$

- S can induce any action in A by manipulating his message. Therefore any mechanism amounts to letting S choose any action from the set A (constrained delegation).

A More General Mechanism Design Problem

- For the cheap talk game, A is the set of equilibrium actions $\{a_1, a_2, \ldots, a_n\}$.
- For unconstrained delegation, $A = \mathbf{R}$.
- What is the optimal mechanism, i.e., the set A^* that maximizes R's expected payoff?
- Does it look anything like delegation, i.e., giving S the right to choose whatever he wants? Ans: qualified yes.

The Optimal Mechanism

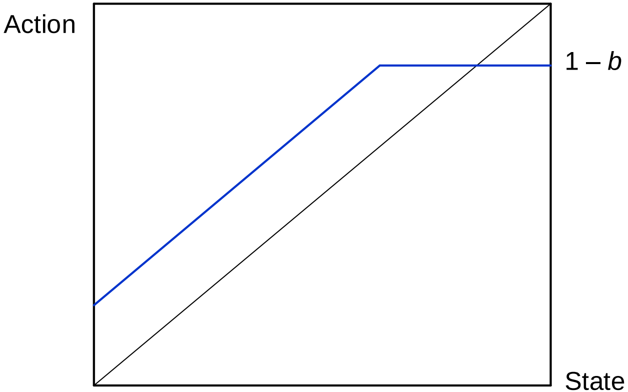
Theorem

(Holmstrom 1977, Goltsman and Pavlov 2007) In the uniform-quadratic case, the optimal mechanism is characterized by

$$\begin{aligned} A^* &= [0, 1-b] \text{ if } b \leq \frac{1}{2} \\ &= \left\{\frac{1}{2}\right\} \text{ if } b > \frac{1}{2} \end{aligned}$$

That is, the optimal mechanism is either delegation with an upper bound on actions, or completely uninformed decision making.

The Optimal Mechanism



Optimal Upper Bound

- Suppose R delegates subject: $a \in [0, \overline{a}]$. Then

$$\begin{aligned} a(\theta) &= \theta + b \ \text{ if } \theta \leq \overline{a} - b \\ &= \overline{a} \ \text{ otherwise} \end{aligned}$$

- R's expected payoff:

$$\begin{aligned} \left| U_R(\overline{a}) \right| &= \int_0^{\overline{a}-b} b^2 d\theta + \int_{\overline{a}-b}^1 (\theta - \overline{a})^2 d\theta \\ &= (\overline{a} - b)\, b^2 + \frac{1}{3} \left[(1 - \overline{a})^3 + b^3 \right] \end{aligned}$$

- Optimal ceiling derived from FOC:

$$b^2 - (1 - \overline{a})^2 = 0 \Rightarrow \overline{a} = 1 - b$$

Generalization of the Results

- Dessein (2004) shows that delegation is better than cheap talk for sufficiently low bias, with uniform distribution and disutility a convex function of distance from bliss point:

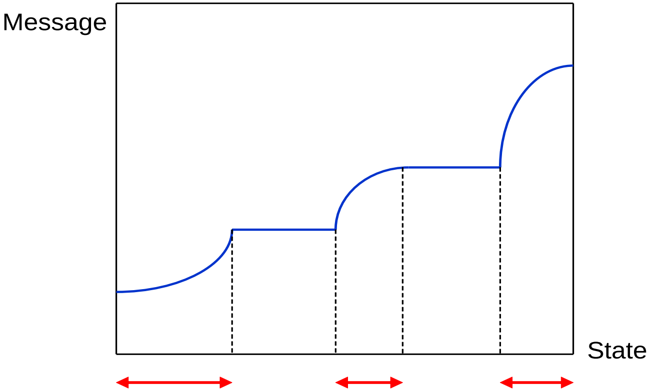
$$\begin{aligned} U_R(a,\theta) &= c_R\left(|a-\theta|\right) \\ U_S(a,\theta) &= c_S\left(|a-b-\theta|\right) \end{aligned}$$

- Distributional assumption not innocuous. Delegation is most attractive when priors are most diffused.
- The interval characteristic of the optimal mechanism is proved only for the uniform-quadratic case as far as I know.

Commitment Mechanisms for Disclosure

- Suppose S can **commit** to *any* function $m(\theta)$.
- If $m(\theta)$ is strictly monotone, it represents full disclosure.
- Alternatively, $m(\theta)$ could be a step function or a non-monotone function (partial disclosure).
- Restrict attention to weakly monotone functions.

A Disclosure Policy



Optimality of Full Disclosure

Theorem

The optimal disclosure policy under commitment is full disclosure.

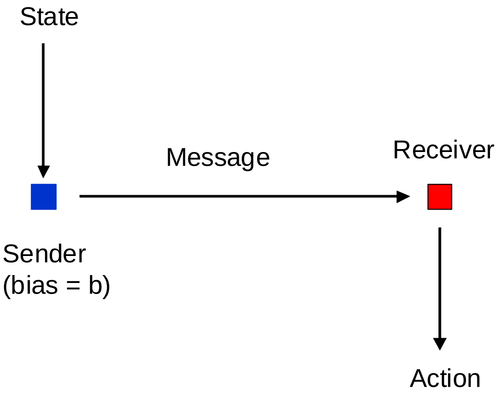
- S's expected disutility in the cheap talk equilibrium: $|U_R| + b^2$
- S's expected disutility under full disclosure: b^2
- Full disclosure dominates cheap talk. Can be applied also to any undislosed sub-interval.
- Under any information structure, the action chosen will be b distance away from S's ideal point on average.
- Incomplete disclosure adds more variance to this gap, which S dislikes if he is risk averse.

Optimality of Silence

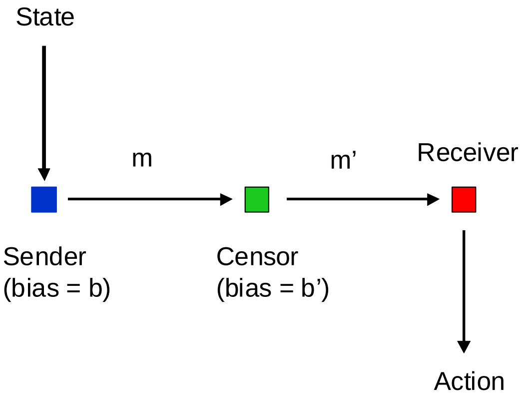
	a_1	a_2	a_3
θ_1	1, 4	3, 3	0, 0
θ_2	0, 0	3, 3	1, 4

- Both states are equally likely ex ante.
- There is a revealing equilibrium; S's payoff is 1.
- S prefers the babbling equilibrium (payoff $= 3$).
- Note: **choosing** to be silent is not the same thing as **committing** to silence.

Free Speech



Controlled Speech



The Censorship Game (Ambrus et al. 2010)

- There is a third player, the censor (C), with bias b'.
- S sends a private message m to C, who in turn sends a message m' to R.
- S's strategy: $m(\theta)$.
- C 's belief $f(\theta|m)$ and strategy $m'(m)$.
- R's belief $g(\theta|m')$ and strategy $a(m')$.
- PBE: strategies must be best responses and beliefs must be Bayesian.

Equilibria of The Censorship Game

- The censor can at best "lump" several messages into one.
- In that case, S might as well lump those messages himself.
- Without loss of generality, assume that the censor does not censor *in equilibrium*.
- Consider partition equilibria. Two sets of incentive constraints:
  - S's indifference condition at cutoffs as in original CS game.
  - No-censorship condition for censor.
- The set of equilibria in the censorship game are outcome equivalent to a subset of the set of equilibria in the original CS game.

Equilibria of The Censorship Game

- In any n-interval equilibrium of the CS game:

$$\begin{aligned} a_1 &= \frac{1}{2n} - b(n-1) \\ a_2 &= \frac{3}{2n} - b(3n-5) \\ \theta_1 &= \frac{1}{n} - 2b(n-1) \end{aligned}$$

- The censor does not want to falsely claim m_2 when told m_1:

$$\left(a_1 + b'\right) - a_1 \leq a_2 - \left(a_1 + b'\right)$$

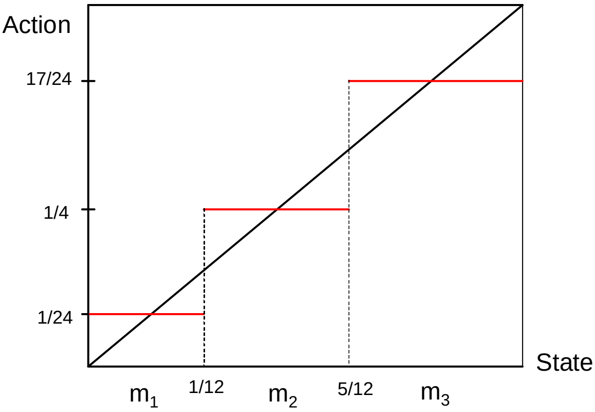
- This incentive constraint implies all other incentive constraints for the censor.

Censorship is Self-Defeating

- The censorship game and the CS game have the same set of equilibria if
$$|b'| \leq \frac{1}{2N(b)} - b(N(b) - 2)$$
- Otherwise, the set of equilibria of the censorship game is a strict subset of equilibria under direct communication.
- Whenever the inequality is violated, the censorship game produces "coarser" equilibria.
- The censor's ex ante expected payoff is (weakly) lower under a censorship regime.
- The censor is better of **committing** to free speech.

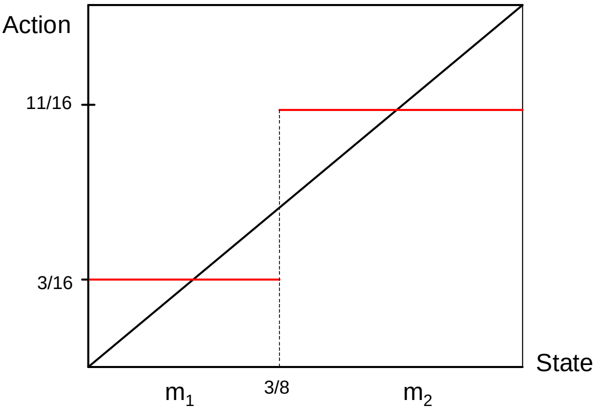
Example: Information Loss due to Speech Control

- Let $b = \frac{1}{16}$, $b' = \frac{1}{8}$. **Free speech**: most informative equilibrium.



Example: Information Loss due to Speech Control

- Let $b = \frac{1}{16}$, $b' = \frac{1}{8}$. **Controlled speech**: most informative equilibrium.



Multiple Speakers, Multiple Dimensions (Battaglini 2002)

- Government budget: guns vs. butter.
- Centre and State Government: infrastructure vs. poverty.
- University and department: teaching vs. research needs.

Main results:

- With two dimensions and single expert, “fine” information can be transmitted along one dimension.
- With multiple dimensions and multiple experts, full revelation is generically possible.
- Full revelation is robust to experts’ information being slightly noisy, or biases being very large.
- In a uni-dimensional problem, full revelation is not robust.

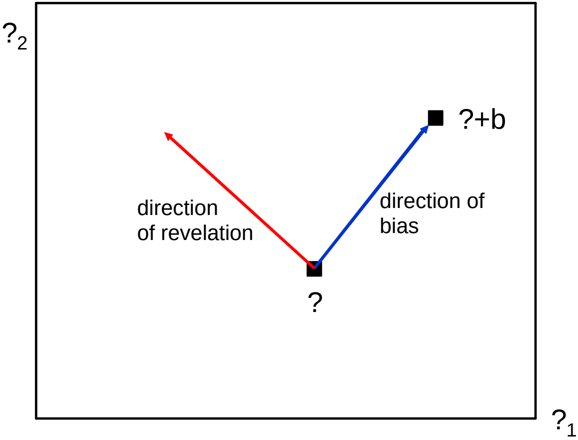
Two Dimensions, One Expert

- State-of-the-world $= \theta = (\theta_1, \theta_2)$, drawn i.i.d from $U[0,1]$.
- Action choice $= a = (a_1, a_2) \in \mathbf{R}^2$.
- Utility functions:

$$\begin{aligned} |U_R| &= (a_1 - \theta_1)^2 + (a_2 - \theta_2)^2 \\ |U_S| &= (a_1 - b - \theta_1)^2 + (a_2 - b - \theta_2)^2 \end{aligned}$$

- Slope of bias vector (b, b) is 1. Orthogonal slope: -1.
- Strategies: $m(\theta)$ and $a(m)$, beliefs: $g(\theta|m)$.
- If b is large, disconnected cheap talk (reporting θ_1 and θ_2 separately) leads to babbling, and action choice $a = \left(\frac{1}{2}, \frac{1}{2}\right)$.
- Comprehensive cheap talk (reporting some relation between θ_1 and θ_2, e.g. their difference) leads to better information.

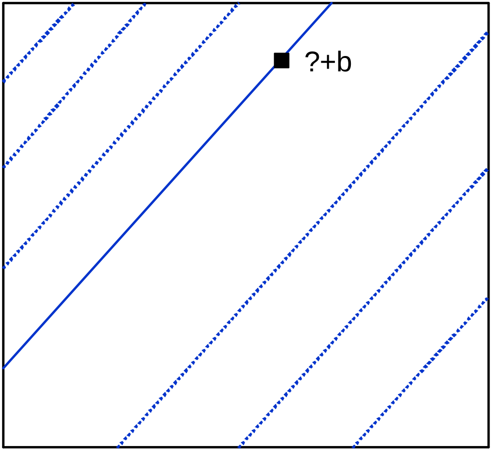
Revealing "Fine" Information: General Idea



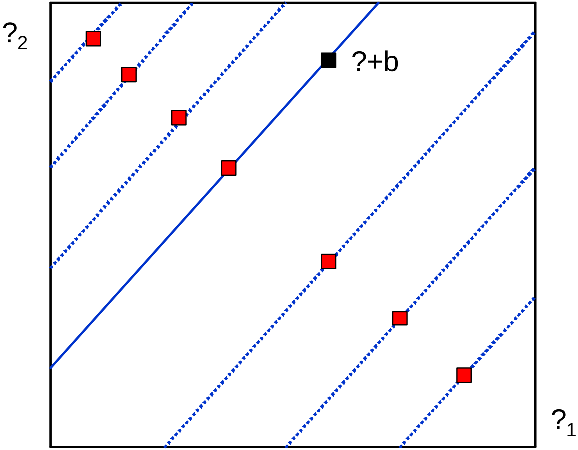
Revealing "Fine" Information: Equilibrium Strategies

- S's strategy:
$$m(\theta_1, \theta_2) = \theta_1 - \theta_2$$
- R's beliefs: (θ_1, θ_2) are uniformly distributed along the line
$$\theta_1 - \theta_2 = m$$
- R's strategy (choose mid-point of the line):
$$\begin{aligned} a_1(m) &= \frac{1+m}{2} \\ a_2(m) &= \frac{1-m}{2} \end{aligned}$$
- Locus of equilibrium action choices:
$$a_1 + a_2 = 1$$

Sender's Iso-message Curves



Receiver's Optimal Actions



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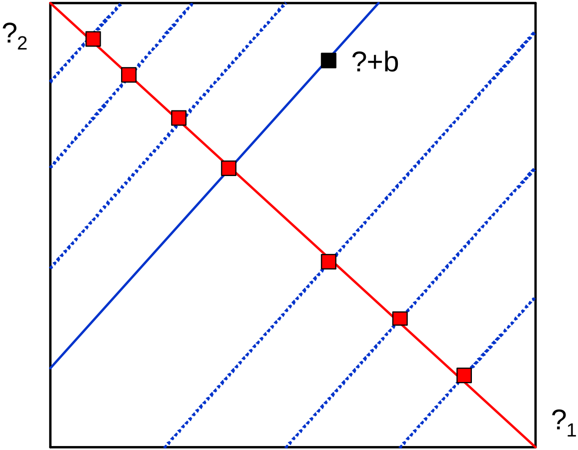


Why Sender's Strategy Is Optimal

- Optimal message, given R's response function:

$$\begin{aligned} & \arg\min_m \left(a_1(m) - b - \theta_1\right)^2 + \left(a_2(m) - b - \theta_2\right)^2 \\ = \quad & \arg\min_m \left(\frac{1+m}{2} - b - \theta_1\right)^2 + \left(\frac{1-m}{2} - b - \theta_2\right)^2 \end{aligned}$$

- First order condition:

$$\begin{aligned} \frac{1+m}{2} - b - \theta_1 &= \frac{1-m}{2} - b - \theta_2 \\ \text{or} \quad m &= \theta_1 - \theta_2 \end{aligned}$$

Why Sender's Strategy Is Optimal

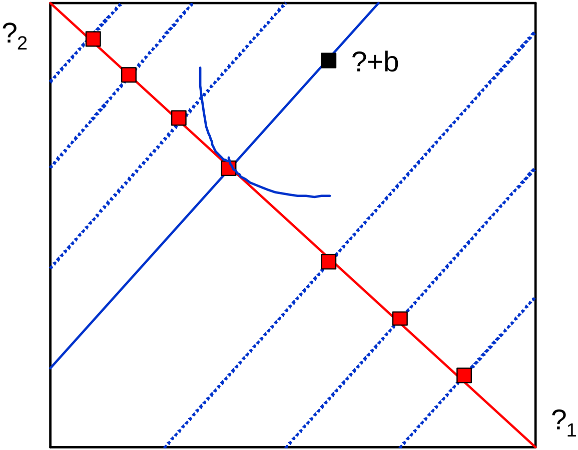
- Same result seen as credible delegation:

$$\min_{a_1, a_2} (a_1 - b - \theta_1)^2 + (a_2 - b - \theta_2)^2 \quad \text{sub to } a_1 + a_2 = 1$$

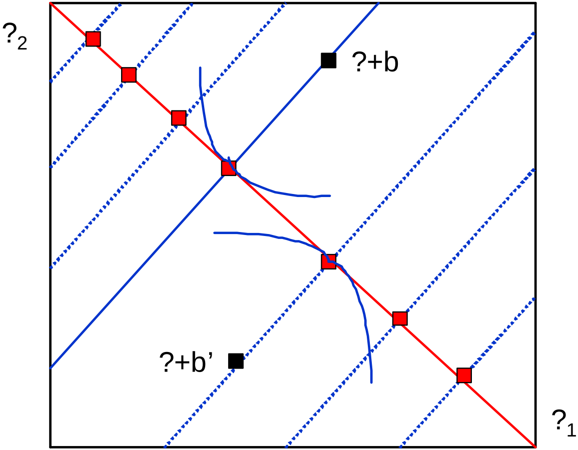
- Using the first-order-condition:

$$a_1 = \frac{1}{2}(1 + \theta_1 - \theta_2), a_2 = \frac{1}{2}(1 - \theta_1 + \theta_2)$$

Why Sender's Strategy Is Optimal



Why Sender's Strategy Is Optimal



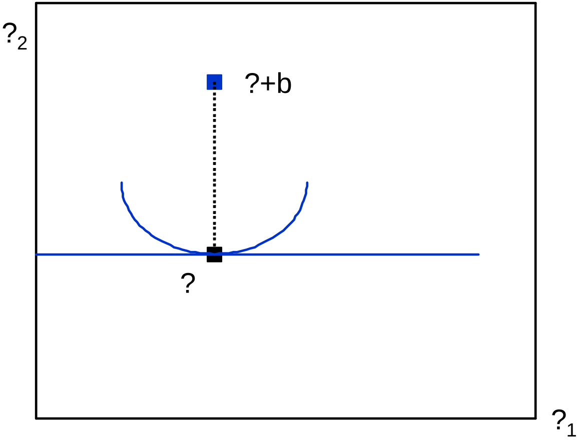
Two Dimensions, Two Experts

- Experts have bias vectors $b = (b_1, b_2)$ and $b' = (b_1', b_2')$.
- Experts simultaneously send messages m and m'.
- R takes action $a = (a_1, a_2)$.
- Experts' strategies: $m(\theta_1, \theta_2)$ and $m'(\theta_1, \theta_2)$.
- R's beliefs are $g(\theta | m, m')$.
- R's strategy is $(a_1(m, m'), a_2(m, m'))$.

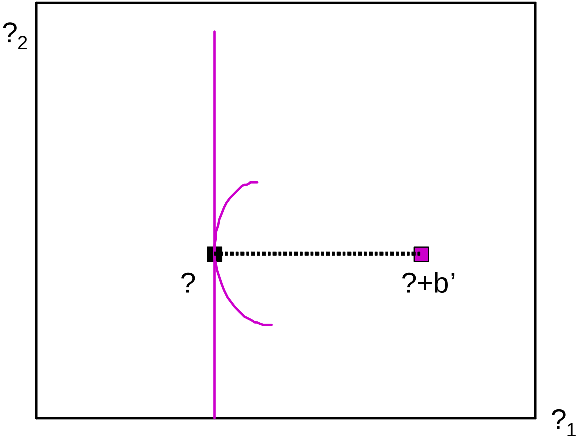
Special Case: Orthogonal Biases

- Each expert is biased only along one dimension.
- Bias vectors are $(0, b)$ and $(b', 0)$.
- Experts speak truthfully on the dimension on which they are unbiased:
$$m = \theta_1, m' = \theta_2$$
- R trusts each expert on the dimension in which she is unbiased:
$$a_1 = m, a_2 = m'$$
- Expert i's optimum is to choose the message
$$\arg\min_{m_i} (m_i - \theta_i)^2 = \theta_i$$

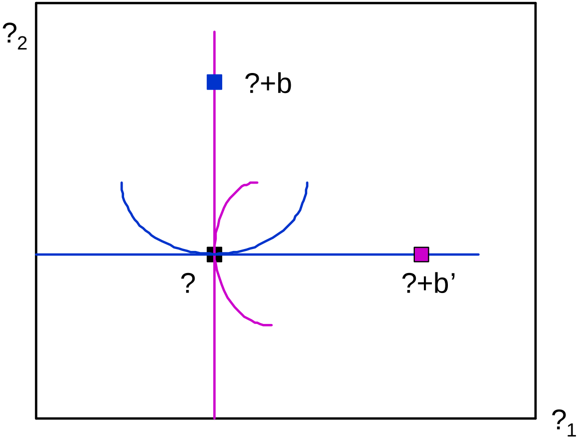
Expert 1's Incentive to Tell the (Partial) Truth



Expert 2's Incentive to Tell the (Partial) Truth



The Whole Truth



Full Revelation With General Bias

- Same idea: each expert has incentive to reveal information orthogonal to her direction of bias.
- If the biases are linearly independent, combining the two messages reveals the exact truth.
- Robust to small noise in experts' information.
- Unlike "shoot the messenger" strategies for truth telling.

Full Revelation: Strategies

- Experts' strategies:

$$\begin{aligned} m(\theta_1, \theta_2) &= b_1'\theta_1 + b_2'\theta_2 \\ m'(\theta_1, \theta_2) &= b_1\theta_1 + b_2\theta_2 \end{aligned}$$

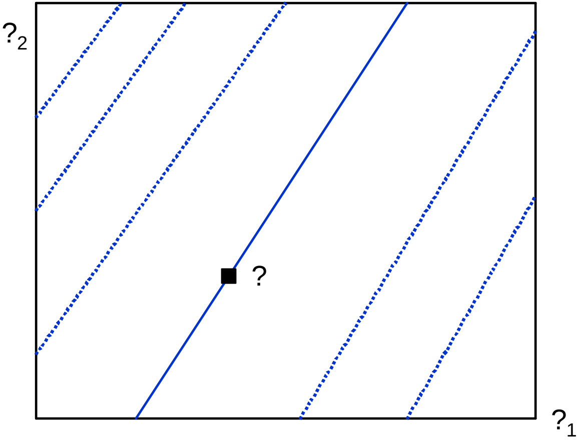
- R's strategy:

$$\begin{aligned} a_1(m, m') &= \frac{b_2 m - b_2' m'}{b_1' b_2 - b_1 b_2'} \\ a_2(m, m') &= \frac{b_1' m' - b_1 m}{b_1' b_2 - b_1 b_2'} \end{aligned}$$

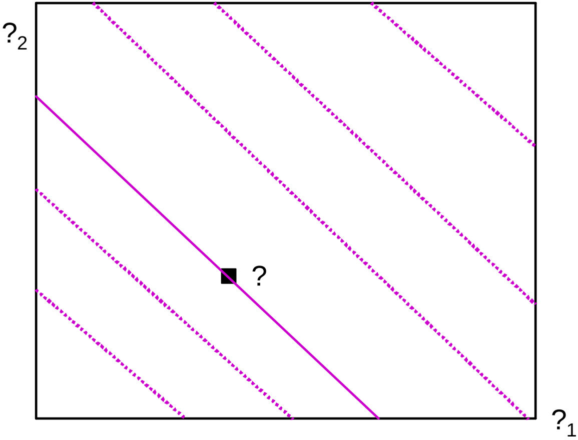
- R's beliefs coincide with his action choices:

$$\theta_i^e = a_i(m, m')$$

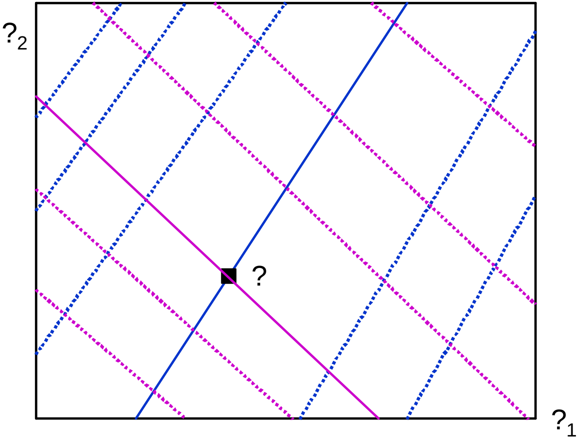
Expert 1's Iso-Message Curves



Expert 2's Iso-Message Curves



Full Revelation!



Why Experts' Strategies Are Optimal

- Each expert takes the other expert's strategy and R's reaction function as given.
- Expert 1's problem

$$\min_m \left(a_1(m, m') - b_1 - \theta_1\right)^2 + \left(a_2(m, m') - b_2 - \theta_2\right)^2$$

$$\equiv \quad \min_m \left(\frac{b_2 m - b_2' m'}{b_1' b_2 - b_1 b_2'} - b_1 - \theta_1\right)^2 + \left(\frac{b_1' m' - b_1 m}{b_1' b_2 - b_1 b_2'} - b_2 - \theta_2\right)^2$$

Why Experts' Strategies Are Optimal

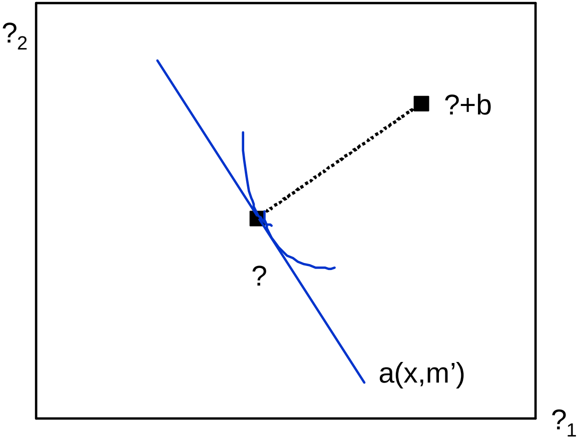
- First order condition:

$$b_2 \left(\frac{b_2 m - b_2' m'}{b_1' b_2 - b_1 b_2'} - b_1 - \theta_1 \right) = b_1 \left(\frac{b_1' m' - b_1 m}{b_1' b_2 - b_1 b_2'} - b_2 - \theta_2 \right)$$

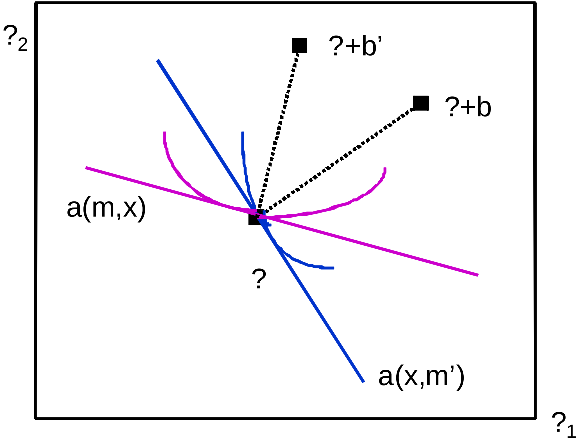
$$b_2 \left(a_1(m, m') - b_1 - \theta_1 \right) = b_1 \left(a_2(m, m') - b_2 - \theta_2 \right)$$

- In equilibrium, $a_i(m, m') = \theta_i$. Both LHS and RHS reduce to $-b_1 b_2$. Hence, each expert's message strategy is a best response to the other expert's message strategy and R's reaction function.

Why Experts' Strategies Are Optimal



Why Experts' Strategies Are Optimal



Good Language, Bad Language

"I returned and saw under the sun, that the race is not to the swift, nor the battle to the strong, neither yet bread to the wise, nor yet riches to men of understanding, nor yet favour to men of skill; but time and chance happeneth to them all."

–Ecclesiastes, 9:11-13.

"Objective considerations of contemporary phenomena compel the conclusion that success or failure in competitive activities exhibits no tendency to be commensurate with innate capacity, but that a considerable element of the unpredictable must invariably be taken into account."

–George Orwell's spoof, *Politics and the English Language*.

Political Use of Bad Language

- Video clip from George Carlin's stand-up comedy.

Euphemism, Code, Vagueness

- Language is sometimes deliberately vague or indirect.
- Plausible deniability, or lack of common knowledge (Pinker)
  - polite mannerisms
  - bribery attempts
  - romantic overtures
  - veiled threats
- Dog whistle effect: language that has dual meaning to appeal to multiple audiences (Krugman/Brooks)
- Speaker may want to shift beliefs, but retain *some* uncertainty in the listener's mind (Stalnaker).

Strategic Ambiguity?

"When the dollar is at a lower level it helps exports, and I think exports are getting stronger as a result."
– US Treasury Secretary John Snow, May 2003.

"The Bush administration secretly welcomes the dollar's decline."
– *Wall Street Journal*, May 13, 2003.

"[Snow] isn't yet fluent in the delicate language of dollar policy."
– Anonymous currency strategist.

"When the secretary of the Treasury says something like that, it gets imbued with deep meaning whether he wants it to or not."
– Alan Blinder.

An Illustrative Example (Stalnaker)

	a_1	a_2	a_3	a_4	a_5
θ_1	$9, -5$	$0, -5$	$8, 5$	$3, 0$	$6, 0$
θ_2	$0, -5$	$9, -5$	$3, 0$	$8, 5$	$6, 0$

- The two states are equally likely. Absent any information, R chooses a_5 and S gets 0.
- If R knew the state perfectly, he would choose either a_1 or a_2 and S would get -5.
- S does better than either scenario if he could commit to an ambiguous speech strategy ($\frac{3}{5} \leq p \leq \frac{4}{5}$):

	θ_1	θ_2
m_1	p	$1-p$
m_2	$1-p$	p

Silence is Eloquent

Watson: "Is there any point to which you would wish to draw my attention?"

Holmes: "To the curious incident of the dog in the night-time."

Watson: "The dog did nothing in the night-time."

Holmes: "That was the curious incident."

– Arthur Conan Doyle, *The Hound of the Baskervilles*.

Disclosing Evidence (Milgrom and Roberts 1986)

- Messages can be vague but not false. Must contain a "grain of truth".
- Receiver wants to take an action that is optimally increasing in the state-of-world value.
- Sender always wants the receiver to take a higher action, regardless of the state.
- The model has a sender has infinite bias. Only babbling equilibrium possible under cheap talk.
- Under evidence disclosure, there is full revelation!
- Robustness issues:
  - Noise in the sender's information
  - Receiver has independent sources of information

A Buyer-Seller Model

- Nature chooses quality $\theta \in \Theta$, probability distribution $p(\theta)$.
- Seller observes θ, buyer does not.
- Seller sends message $m \in S(\Theta)$, where S is the set of subsets of Θ.
- The message must have a "grain of truth": $\theta \in m$.
- Buyer hears m and chooses action (quantity purchased) $a \in \mathbf{R}_+$.
- Payoff functions: $u(a, \theta)$ for buyer and $v(a, \theta)$ for seller.

Strategies and Beliefs

- **Assumption:** $v(a, \theta)$ is strictly increasing in a, i.e., seller has extreme (infinite) bias.
- In the cheap talk version of the game, the only equilibrium is babbling.
- Seller strategies: $m(\theta) \in \overline{S}(\theta)$ where $\overline{S}(\theta) = \{m \in S | \theta \in m\}$.
- Buyer's posterior beliefs: $p(\theta | m)$ for every $m \in S$.
- Buyer strategies: $a(m)$.
- Full information purchase: $a^*(\theta)$ is the optimal purchase if the buyer knew θ, i.e.,

$$a^*(\theta) = \arg\max_a u(a, \theta) \Rightarrow u'(a^*(\theta), \theta) = 0$$

- If $u(a, \theta)$ is strictly concave in a, $a^*(\theta)$ exists uniquely.

Perfect Bayesian Equilibrium

1. Seller maximization:

$$m(\theta) = \arg \max_{m \in \overline{S}(\theta)} v\left(a(m), \theta\right) \equiv \arg \max_{m \in \overline{S}(\theta)} a(m)$$

2. Buyer maximization:

$$a(m) = \arg\max_{a} \sum_{\theta \in \Theta} u(a, \theta).p(\theta | m)$$

3. Sequentially rational beliefs: if $m_0 = m(\theta)$ for some θ

$$p(\theta | m_0) = \frac{p(\theta)}{p\left(m^{-1}(m_0)\right)} \text{ for all } \theta \in m_0$$

4. Consistency:

$$p\left(\theta | m_0\right) = 0 \text{ for all } a \notin m_0$$

Main Results

- First order condition for buyer maximization:

$$\sum_{\theta \in \Theta} u'(a(m), \theta) p(\theta | m) = 0$$

Theorem

There exists an unraveling equilibrium where (i) the buyer always buys the full information quantity: $a(m(\theta)) = a^*(\theta)$ *(ii) the buyer's beliefs are skeptical, i.e., for every* m_0, $p(\theta | m_0)$ *is such that it minimizes buyer's optimal purchase among all sequentially rational and consistent beliefs.*

Main Results

Theorem

If $u(a, \theta)$ is differentiable and strictly concave in a, the only PBE involves unraveling.

- $a(m(\theta)) \geq a^*(\theta)$, otherwise seller can profitably deviate to $m'(\theta) = \{\theta\}$. This implies:

$$u'(a(m(\theta)), \theta) p(\theta | m(\theta)) \leq 0 \text{ for all } \theta \in \Theta$$

- Combined with buyer maximization condition (previous slide):

$$u'(a(m(\theta)), \theta) p(\theta | m(\theta)) \leq 0 \Rightarrow a(m(\theta)) = a^*(\theta) \text{ for all } \theta \in \Theta$$

- If beliefs are not skeptical, it is possible to get more than $a^*(\theta)$ for some θ.

How Robust is Unraveling?

- The unraveling result seems too strong.
- The model does not allow some realistic features:
  1. The receiver may be somewhat naive or credulous.
  2. The sender may not have hard evidence in his possession.
  3. The sender's evidence may be noisy.
  4. The receiver may have other (noisy) sources of information.
- Milgrom Roberts (1986) show unraveling survives (1).
- Shin (1998) shows unraveling may break down under (2), Harbaugh and To (2006) under (3) and (4).

Noisy Evidence and Unraveling

- Let $\theta \sim U[0,1]$, $a \in [0,1]$ and

$$|U_R| = (a-\theta)^2; \quad U_S = a$$

- With probability λ, S has hard evidence on the exact value of θ. With probability $1-\lambda$, he does not (private information).
- If S has evidence (informed), he can send $\{\theta\}$ or no message.
- If S has no evidence (uninformed), he cannot send a message.
- Equilibrium in cutoff strategy: presents the evidence iff he has it and $\theta \geq \overline{\theta}$.
- Bayesian updating:

$$\Pr\left(S \text{ informed} \middle| \text{no message}\right) = \pi = \frac{\lambda\overline{\theta}}{\lambda\overline{\theta}+1-\lambda}$$

Noisy Evidence and Unraveling

- If R sees no message, his expectation of the state is:

$$\pi.\frac{\overline{\theta}}{2}+(1-\pi).\frac{1}{2}$$

- Using the value of π:

$$\phi\left(\overline{\theta}\right)=\frac{1}{2}\left[\frac{\lambda\overline{\theta}^{2}+1-\lambda}{\lambda\overline{\theta}+1-\lambda}\right]$$

- The equilibrium cutoff is given by the fixed point: $\phi\left(\overline{\theta}\right)=\overline{\theta}$:

$$\overline{\theta}=\frac{\sqrt{1-\lambda}-(1-\lambda)}{\lambda}$$

- Check: $\overline{\theta}\uparrow$ as $\lambda\downarrow$, and as $\lambda\to 1$, $\overline{\theta}\to 0$. There is always **partial unraveling** whenever there is some noise ($\lambda<1$), but equilibrium converges to full unraveling as the noise vanishes.

False Modesty (Harbaugh and To 2006)

- People sometimes withold unambiguously “good news”:
  - the old rich do not flaunt wealth, celebrities dress down.
  - great scholars act humble, avoid titles.
  - high end brands do “soft selling”.
- There are sources of information apart from self disclosure.
- Witholding good news serves as a signal of confidence in the outcome of extraneous signals.
- “Bragging” separates the mediocre from the bad, “modesty” separates the good from the mediocre.
- Welfare: mandatory disclosure may make everyone better off.

A Simple Model

- S is a worker whose productivity (θ) is unknown.
- R is a competitive employer. Action a is the offered wage.
- Ability distribution is common knowledge:

Type	Low	Medium	High
Productivity (θ)	0	$\frac{1}{2}$	1
Proportion	α	$1-\alpha-\beta$	β

- Standard preferences:

$$|U_R| = (a-\theta)^2; \quad U_S = a$$

A Simple Model

- M, H can produce some evidence E (e.g., PhD), L cannot.
- M, H possess the evidence with probability 1.
- Some exogenous noisy signal of H type will be received by R:

	h	ml
H	p	$1-p$
ML	$1-p$	p

- Sender does not know the realization of this signal.
- Sender can choose to disclose E or not.

A Separating Equilibrium

- M discloses E, H does not.
- Expected payoff from disclosure $= \frac{1}{2}$.
- Action choices (wage offers), contingent on observed signal:

$$a(h) = \pi(h) = \frac{\beta p}{\beta p + \alpha(1-p)}$$

$$a(ml) = \pi(ml) = \frac{\beta(1-p)}{\beta(1-p) + \alpha p}$$

- Expected wage is a lottery over these contingent wages.

A Separating Equilibrium

- Expected action (wage) from non-dislosure (H-type):

$$a_H^e = pa(h) + (1-p)a(ml)$$

$$= \frac{\beta p^2}{\beta p + \alpha(1-p)} + \frac{\beta(1-p)^2}{\beta(1-p) + \alpha p}$$

- Expected action (wage) from non-dislosure (H-type):

$$a_M^e = (1-p)a(h) + pa(ml)$$

$$= \frac{\beta p(1-p)}{\beta p + \alpha(1-p)} + \frac{\beta p(1-p)}{\beta(1-p) + \alpha p}$$

Conditions for Existence

- Expected non-disclosure wage is higher for H type:

$$a_H^e - a_M^e = \frac{\alpha\beta(2p-1)^2}{[\beta p + \alpha(1-p)]\,[\beta(1-p) + \alpha p]} > 0$$

- Condition for separating equilibrium:

$$a_M^e \leq \frac{1}{2} \leq a_H^e$$

- As $p \to \frac{1}{2}$, both a_M^e and $a_H^e \to \frac{\beta}{\alpha+\beta}$. No "modest" equilibrium for very uninformative exogenous signals.
- As $p \to 1$, $a_M^e \to 0$ and $a_H^e \to 1$. "Modest" equilibrium always exists for very informative exogenous signals.
- As $\alpha \to 0$, $a_H^e \to 1$ and $a_M^e \to 1 > \frac{1}{2}$. No "modest" equilibrium when Ls are scarce.
- Without refinements, pooling equilibrium always exists.

Dr. Ghosh Knows Econometrics!

- In the University of California system, faculty members with PhDs are less common in non-doctoral universities.
- Is self description as “Dr.” or “PhD” more common in voicemail greetings and course syllabus in these places?
- Faculty composition:

	% full time	% PhD (full)	% PhD (part)
Non-doctoral	55.6	80.1	24.5
Doctoral	80.0	98	-

- Behaviour differences:

	Voicemail title	Syllabus title*
Non-doctoral	26.667	77.612***
Doctoral	3.876	52.419***

Welfare Analysis: Should Disclosure be Mandatory?

- There is a trade-off. Mandatory disclosure adds information (L is perfectly identified) but also subtracts information from signaling (M is no longer perfectly identified).
- Let $\alpha = \beta = \frac{1}{3}$. Conditional wages:

$$\begin{aligned} a(h) &= p \\ a(ml) &= 1 - p \end{aligned}$$

- Separating equilibrium exists:

$$\begin{aligned} a_H^e &= p^2 + (1-p)^2 > \frac{1}{2} \\ a_M^e &= a_L^e = 2p(1-p) < \frac{1}{2} \end{aligned}$$

Welfare Analysis

- R's expected payoff:

$$|U_R| = \frac{2}{3}p(1-p)$$

- Under mandatory disclosure, conditional wages:

$$\begin{aligned}
\widehat{a}(h) &= p.1 + (1-p).\frac{1}{2} = \frac{1}{2}(1+p) \\
\widehat{a}(ml) &= (1-p).1 + p.\frac{1}{2} = \frac{1}{2}(2-p) \\
\widehat{a}(l) &= 0
\end{aligned}$$

Welfare Analysis

- R's expected payoff:

$$
\begin{aligned}
|U_R| &= \frac{1}{3}\left[p\left(1-\widehat{a}(h)\right)^2 + (1-p)\left(1-\widehat{a}(ml)\right)^2\right] \\
&\quad + \frac{1}{3}\left[(1-p)\left(\frac{1}{2}-\widehat{a}(h)\right)^2 + p\left(\frac{1}{2}-\widehat{a}(ml)\right)^2\right] \\
&= \frac{1}{6}p(1-p) < \frac{2}{3}p(1-p) \quad \text{(non-disclosure payoff)}
\end{aligned}
$$

- R is better off if disclosure is made mandatory.
- Laws about compulsory vs. voluntary product labeling, financial disclosure, etc. may have an impact on outcomes.