902. Issues in Economic Systems and Institutions
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Solutions to Practice Problems.

ESSAY TYPE QUESTIONS

Do not read these as complete, model answers, but as outlines summarizing the main points. You should flesh them out with a little more detail.

1. With a single expert, the Crawford-Sobel model illustrates the truth of the statement. With multiple experts with opposite biases, however, Krishna and Morgan show that (almost) full revelation is possible in equilibrium. Further, if the information is multi-dimensional, Battaglini has proved that almost always (i.e., whenever biases are linearly independent) there are equilibria that are fully revealing. In sum, whether or not information loss is inevitable depends on whether there is a single biased source or multiple ones, and in the latter case, on further details of the problem.

2. Decentralization is essentially the practice of authorities with decision making power delegating the decision to their subjects. For example, the central government may leave it to state governments how the state's developmental budget will be spent or where projects will be located. The state government may in turn allocate funds to local panchayats and let them decide how to spend it locally (e.g., more roads or more irrigation canals?). And so on. Contrast this with a top down approach where the Centre micro-manages everything, deciding how every paisa gets spent in every village or town.

   Refer to the welfare analysis of Crawford Sobel in lecture notes. In the uniform-quadratic case, we showed that both receiver's and senders expected payoffs are higher compared to their equilibrium payoffs if (i) the receiver delegates the decision to the sender (ii) the receiver retains decision making authority but the sender can somehow commit to tell the truth (e.g. by legally agreeing to full disclosure rules, like a company promising to make public its audit reports or being bound by law to do so). Of course we also saw that the receiver will want to withdraw delegated authority after the sender reveals what action he is about to choose, and the sender may want to be dishonest after he learns the exact state. Therefore, for either of these practices to work, there has to be some commitment mechanism that ensures that delegation or transparency will not be oveturned later.

3. In a buyer seller interaction, the seller's preference is state independent and monotonic. This means that regardless of the true quality of the good, the seller wants the buyer to think that the quality is as high as possible and would like to receive a higher rather than a lower price. Relating it to the Crawford-Sobel model, we may say the seller is an extremist, i.e., always prefers a higher action (price) to a lower action. In this situation, cheap talk can only produce a babbling equilibrium—the seller cannot credibly communicate any quality information.

   If the seller can produce verifiable information about quality (his messages can be vague but must contain the true state), we saw that the result is dramatically different—equilibrium is characterized by "unraveling", i.e., the buyer will be able to figure out the true quality *exactly.* The reasoning is based on a sort of iterated dominance argument. If the good were of highest quality, the seller has every reason to reveal this unambiguously. If it is of the second highest quality, there is again an incentive to reveal (by concealing or being vague, there is no chance buyers can be fooled into believing that it is of highest quality). This logic works all the way to the bottom.

   The unraveling result suggests that commitment to disclose information may be redundant—even in the absence of such commitment, the information will spill out anyway as long as claims are verifiable and not just cheap talk. However, the paper on modesty by Harbaugh et al shows that in more limited circumstances, the unraveling result may break down. This could happen, for example, if the information the seller has is itself coarse or noisy (e.g., cannot distinguish between some states) and the buyer has access to an additional exogenous source of noisy information. For example, the seller may have information whether the quality falls in a low, medium or high band, and the buyer may be learning independently whether or not it is likely to belong in the highest band. In such situations, the seller may strategically withold information and exercise "modesty". A mandatory disclosure rule may benefit the buyer and/or the seller in such circumstances.

4. Disagreements over collective choice arise from two possible sources: (i) people have conflicting objectives or values, and (ii) people evaluate options based on differential and private information.

   Consider the issue of free trade vs. protectionism. In an advanced industrial economy like the USA, the relative scarcity of labour means that trade liberalization is likely to lower wages and increase returns to capital. Therefore unions will oppose it in their self interest, while corporations and Wall Street will generally lobby in its favour. However, in a recessionary environment, openness will generally have macroeconomic impacts that affect labour and capital similarly. However, since nobody knows the macroeconomic fundamentals for sure, people may hold different opinions about what policy is desirable right now, even though their interests in the outcome are not divergent (everyone wants the domestic recession not to deepen further).

   Condorcet's Jury theorem roughly says that decisions made through voting by a large group on a common interest issue will be more accurate than that made by any one individual, and as the number of voters grows to infinity, the probability of error approaches zero. The theorem is discussed in a *statistical* version and a *strategic* version. The statistical version assumes that voters will vote the same way (as a function of their private information) regardless of the decision making context (the size of the jury, the voting rule, etc.) and will therefore have a constant probability of casting a mistaken vote. In the strategic version, voters are assumed to play a Bayesian Nash game with each other and conclusions are based on the properties of the equilibrium of this game. With two options, two states (each option optimal in one of the states) and conditionally independent binary signals, we have seen that the statistical theorem is true if the proportion of votes needed for either decision does not exceed the accuracy of the signal. With strategic voting, the result is more powerful. Conclusions of the Condorcet theorem can be established as long as the voing rule is strictly interior, avoiding extreme rules like a unanimity requirement.

5. When the number of votes required for conviction is raised, there are two effects. If voter's strategies are unchanged, the probability of securing the requisite number of votes diminishes, and hence the probability of conviction (as well as the probability of convicting the innocent) will fall. This is the direct effect. However, the new rule will alter voters' equilibrium strategies. If they vote for conviction more easily, there is an indirect effect pushing towards raising the probability of a wrongful conviction. The net result depends on the relative strengths of the direct and indirect effects, and the Feddersen-Pesendorfer paper illustrates the possibility through numerical examples.

   The reason voters will be more inclined to vote for conviction when the number of required votes is raised has to do with the pivotal voter logic. A vote only matters when it is pivotal, i.e. the tally among the rest of the voters is one short of the threshold. With (say) simple majority rule, pivotality implies the votes are evenly split, which will make a voter sensitive to her private information. With unanimity rule, pivotality means everyone else has voted in favour of conviction, which is a strong signal of guilt and may make the voter ignore her private information (which may indicate innocence) and vote for conviction along with the rest.

6. Positive results supporting the idea of "wisdom of crowds" that we have seen arise from the Condorcet Jury Theorem. This is discussed above and I will not repeat it here. Negative results were obtained in the model of information cascades, where a whole group of investors may ignore their private information and make the same mistaken choice one after the other.

   Cascades may arise in a simple sequential model where later investors observe the choices of the previous investors but not the private information on which these choices were based. Once a succession of investors make the same choice (say invest), it reveals a certain volume of similar information (good news about the stock). An investors who follows this group may then want to replicate their decision even if his private information contradicts that of the earlier investors (i.e., even if it is bad news). Once incentives reach this point, choices no longer reveal private information and the market is stuck with the limited amount of information implied by the choices of a few early investors who started the cascade. Everybody starts ignoring their private information and mimicking the decisions of the early investors.

Even if a large volume of evidence (dispersed privately among several investors) accumulates that the stock may be bad, this does not become public and hence fails to break the cascade. The important lesson of these cascades models is that they illustrate "rational frenzies"—widespread informational breakdown and inefficiency that may arise even if agents are perfectly calculating Bayesian agents.

7. Statistical discrimination in the labour market arises if employers use the average trait of a particular demographic (races, sexes, castes, etc.), in addition to individual specific information (test scores, recommendations, interviews), in evaluating candidates from that group. This might create a positive feedback loop between behaviour and perception. For example, if blacks are perceived to be less skilled on average, they may not be given the benefit of doubt in the hiring process, i.e., they will be hired only if the signals are really strong, not if they are mixed. This may reduce the incentives of most blacks to make costly investments in skills and work habits, making the employers' expectations a self fulfilling prophecy. On the other hand, if whites are perceived to be highly skilled on average, it is likely to set in motion a process whereby most whites have the incentive to invest in skills, again justifying employer expectations. Thus, even if black and white populations are identical at some level (in the distribution of innate ability and cost of skill acquisition), they may get locked into very different equilibria, endogenously giving rise to inter-racial inequality, racial stereotypes and statistical discrimination. This is the crux of Arrow's theory.

   The policy of affirmative action has theoretically ambiguous effects in such a scenario. If behaviour (in terms of skill acquisition) remained fixed, a hiring quota for blacks will certainly worsen the stereotype of a typical black worker, because blacks are now being hired under "softer" criteria. However, the heart of economic analysis is to track how behaviour responds to changing policy, and this must be taken into account while evaluating quotas and reservations. On the one hand, it may lead to an encouragement effect—blacks, perceiving greater opportunities for themselves if they perform moderately well, may be encouraged to work harder and invest in skills in larger numbers. This will lead to an improvement in black stereotypes over time, and may pull out the black population from a bad equilibrium in the long run, even after the affirmative action ceases to exist. On the other hand, if quotas are so large that getting a job becomes too easy, it may lead to even less effort among the black population, worsening their image among employers over time. Which way the effect will go will depend on details of the situation.

   Directly subsidizing black efforts to acquire skills will have a more predictable positive effect on stereotypes, but since much of this effort is unobservable, implementing such a policy in practice is not easy.

PROBLEM SOLVING QUESTIONS

8. (a) Babbling equilibrium: $[0,1]$.
   Two-interval equilibrium: $[0,\frac{2}{5}],[\frac{2}{5},1]$.
   Three-interval equilibrium: $[0,\frac{2}{15}],[\frac{2}{15},\frac{1}{3}],[\frac{1}{3},1]$.
   (b) Lowest value: $b \leq \frac{1}{24}$.
   (c) Applying the indifference condition at cutoff points:

$$\frac{1}{2}(\theta_{k+1}+\theta_k)-(b+c\theta_k)=(b+c\theta_k)-\frac{1}{2}(\theta_k+\theta_{k-1})$$

   Let y_k be the length of the k-th interval. Then the differential equation can be rewritten as:

$$y_{k+1}=y_k+4b+4(c-1)\theta_k$$

   If $b>0$ and $c>1$, then successive intervals increase in length by at least $4b$. This immediately establishes that for any given b and c, the equilibrium can contain some highest number of intervals and no more, i.e. properties similar to the standard model are obtained. Closed form solutions to the differential equations are harder to obtain and are not economically insightful.

9. In the private message game, equilibria follow exactly the same equations as in the case with quadratic preferences. Note that there will be two different partitions, one in each game with a distinct receiver.

   In the public message game, there will arise only one partition and the sender's message will affect his payoff through changes in the actions of *both* receivers (a_1 and a_2). Assuming b_1 and b_2 close enough in absolute value, the indifference condition implies

$$\begin{aligned} & \frac{1}{2}(\theta_{k+1}+\theta_k)-(b_1+\theta_k)+\frac{1}{2}(\theta_{k+1}+\theta_k)-(b_2+\theta_k) \\ = \quad & (b_1+\theta_k)-\frac{1}{2}(\theta_k+\theta_{k-1})+(b_2+\theta_k)-\frac{1}{2}(\theta_k+\theta_{k-1}) \end{aligned}$$

   This can be rewritten as:

$$y_{k+1}=y_k+4b' \quad \text{where } b'=\frac{1}{2}(b_1+b_2)$$

   Therefore the equilibria of the public message game are the same as those that would arise if the sender were playing against a single receiver with a bias equal to the *average* bias of the two receivers in our game.

   If b_1 and b_2 have the same sign, then the public message game produces coarser information than private messaging with the less biased receiver, but finer information relative to private messaging with the more biased receiver. More interesting is the case where b_1 and b_2 have opposite signs. Then, public messaging may unambiguously provide finer information. In the special case where $b_1=-b_2$, there is an equilibrium with full revelation!

   Are people more honest and forthcoming in public speech than in private communication? This particular model tends to favour public speech, but it is interesting to think of other situations or models where the effect may be opposite. The question is of obvious relevance to organizational design and things like the Right to Information Act.

10. (a) There is no equilibrium where sender 1 fully reveals information. Suppose there is. Then the game between sender 2 and the receiver is exactly the standard game of CS with partition equilibria determined by the size of b. Consider any cutoff point θ_k and a value of θ slightly to the right, i.e. $\theta_k+\epsilon$. For ϵ small enough, this point is closer to the mid-point of the lower interval, $\frac{1}{2}(\theta_k+\theta_{k-1})$, than the mid-point of the higher interval, $\frac{1}{2}(\theta_{k+1}+\theta_k)$. Therefore, when $\theta=\theta_k+\epsilon$, sender 1 will have an incentive to deviate and report a value in the lower interval $[\theta_{k-1},\theta_k]$. This destroys incentive for full revelation by sender 1.

    All equilibria are consequentially equivalent to equilibria of the following form: sender 1 reports a partition to sender 2, and sender 2 reports exactly the same partition to the receiver (two equilibria are consequentially equivalent if each value of θ leads to the same action being chosen). This means that when sender 2 learns the state is in the interval $[\theta_{k-1},\theta_k]$, he prefers its mid-point to the mid-point of any other interval. A sufficient condition for this to hold is that he prefers it to the mid-point of the next higher interval, $[\theta_k,\theta_{k+1}]$. Let x_k denote the mid-point of the k-th interval ($x_k=\frac{1}{2}(\theta_k+\theta_{k-1})$). Equilibria can be described by the set of inequalities:

$$b \leq x_{k+1}-(x_k+b) \quad \text{for all } k$$

$$\text{or} \quad b \leq \frac{1}{2}(x_{k+1}-x_k)=\frac{1}{4}(\theta_{k+1}-\theta_{k-1})$$

    Any partition satisfying these inequalities is an equilibrium. However, let us find the most efficient equilibrium. Recall from our discussion of Crawford-Sobel that for an information partition with n intervals, receiver's expected utility is maximized when the intervals are of equal length. Therefore, the most efficient equilibrium here will have that feature, implying $\theta_{k+1}-\theta_{k-1}=\frac{2}{n}$. Hence it is the symmetric n-interval partition where n is the largest interval satisfying

$$n \leq \frac{1}{2b}$$

(b) The condition for an n-interval equilibrium to exist in the CS model is (see p. 18 of lecture notes; you can calculate it easily if you don't recall the formula but remember the reasoning):

$$b < \frac{1}{2n(n-1)}$$

Comparing the two inequalities, we see that the condition is more stringent in standard CS, i.e., whenever an n-interval asymmetric equilibrium exists there, an n-interval symmetric equilibrium exists in the Chinese Whispers game, but not vice versa. Hence, comparing most efficient equilibria, the Chinese Whispers game produces better information than standard CS.

11. (a) The optimum voting rule is that which induces sincere voting, where each voter votes for conviction when he gets a guilty signal and for acquittal when he gets an innocent signal (see the paper by Persico for details). Because of symmetry, simple majority rule will be optimum here.

(b) Let a voter vote for conviction with probability σ when he sees an i signal. Let γ_G and γ_I be the probabilities of any voter voting for C when the true state is G and I respectively. Mixing implies the posterior, conditional on being pivotal and receiving signal i must equal the threshold of doubt, i.e.

$$\frac{\frac{1}{2}.\frac{1}{3}\gamma_G^2.}{\frac{1}{2}.\frac{1}{3}\gamma_G^2 + \frac{1}{2}.\frac{2}{3}\gamma_I^2} = \frac{1}{2}$$

$$\text{or} \quad \gamma_G^2 = 2\gamma_I^2$$

Now γ_G and γ_I can be written as functions of σ:

$$\gamma_G = \frac{2}{3}.1 + \frac{1}{3}.\sigma$$

$$\gamma_I = \frac{1}{3}.1 + \frac{2}{3}.\sigma$$

Use the last three equations to solve for the equilibrium value of σ.

(c) Suppose unanimity requirement for conviction is the optimum voting rule. Then each voter will vote according to signal and not play mixed strategies. This means conditional on being pivotal, a g signal (which implies 5 out of 5 g signals) should lead to a posterior higher than q, and an i signal (which implies 1 out of 5 signals is i, i.e., 3 net g signals) should lead to a posterior less than q. Calculating these posteriors using Bayes' Rule:

$$\Pr[\theta = G|\text{pivotal, } g] = \frac{\frac{1}{2}\left(\frac{2}{3}\right)^5}{\frac{1}{2}\left(\frac{2}{3}\right)^5 + \frac{1}{2}\left(\frac{1}{3}\right)^5} = \frac{32}{33}$$

$$\Pr[\theta = G|\text{pivotal, } i] = \frac{\frac{1}{2}\left(\frac{2}{3}\right)^3}{\frac{1}{2}\left(\frac{2}{3}\right)^3 + \frac{1}{2}\left(\frac{1}{3}\right)^3} = \frac{8}{9}$$

Therefore, the condition for unanimity to be the optimal rule is

$$\frac{8}{9} < q < \frac{32}{33}$$

(d) Using the same logic as above, unanimity is optimum if

$$\frac{\frac{1}{2}\left(\frac{2}{3}\right)^{n-2}}{\frac{1}{2}\left(\frac{2}{3}\right)^{n-2} + \frac{1}{2}\left(\frac{1}{3}\right)^{n-2}} < q = 0.9 < \frac{\frac{1}{2}\left(\frac{2}{3}\right)^{n}}{\frac{1}{2}\left(\frac{2}{3}\right)^{n} + \frac{1}{2}\left(\frac{1}{3}\right)^{n}}$$

$$\frac{2^{n-2}}{2^{n-2}+1} < 0.9 < \frac{2^n}{2^n+1}$$

Note that as $n \uparrow$, both the right and left side expressions of the inequality becomes larger (tending to 1 in the limit). Therefore the highest n that makes unanimity optimal is the highest integer for which the left side of the inequality is satisfied. Intuitively, as n becomes larger, being pivotal contains stronger and stronger information about guilt if votes are completely responsive to signals.

12. First, calculate the threshold value of the posterior belief that the state is good (call it λ) that makes investment profitable. This is given by

$$\lambda.1 + (1-\lambda)(-2) \geq 0 \Rightarrow \lambda \geq \frac{2}{3}$$

Now, how many *net* positive signals will it take to push the posterior above $\frac{2}{3}$, starting from a prior of $\frac{1}{2}$? Bayes' Rule tells us that after k net positive signals, each of accuracy p, the posterior is given by

$$\frac{\frac{1}{2}p^k}{\frac{1}{2}p^k + \frac{1}{2}(1-p)^k}$$

With $p = \frac{3}{5}$, you can easily calculate that investing is optimal only if $k \geq 2$. Therefore the first investor will not invest even if he gets a positive signal, and everyone following him will do the same. A cascade starts immediately given the parameters of the problem. The probability of it being wrong is $\frac{1}{2}$.

Suppose, instead, that $p = \frac{4}{5}$. Now, investment is optimal for $k \geq 1$, and not investing is optimal for $k \leq 0$. An investment cascade will start after there accumulates public information about 2 net positive signals, and a non-investment cascade will start after there is public information about 1 net negative signal. Due to this asymmetry, calculating probabilities of cascades is slightly trickier than the exercise in the notes.

Conditional on state θ ($\theta = +1, -2$), let $q_0(\theta)$ be the probability of a cascade on investment eventually forming when the current informational state is 0 net positive signals in the public domain (with some abuse of terminology, I'll refer to both the underlying state of the stock return and the current state of public information as "state". Don' get confused!), and let $q_1(\theta)$ be the similar probability when there is 1 net positive signal (since a cascade will eventually form almost surely, the corresponding probabilities of a non-investment cascade are $1 - q_0(\theta)$ and $1 - q_1(\theta)$ respectively). In state 0, if there is one more negative signal, a non-investment cascade starts immediately. If there is one more positive signal, the state bumps up to state 1. In state 1, a negative signal bumps it down to state 0 again, while another positive signal leads to the start of an investment cascade. When the true state is +1, we have the following recursive equations:

$$\begin{aligned} q_0(1) &= \frac{4}{5}.q_1(1) + \frac{1}{5}.0 \\ q_1(1) &= \frac{4}{5}.1 + \frac{1}{5}.q_0(1) \end{aligned}$$

On solving, we get

$$\begin{aligned} q_0(1) &= \frac{16}{21} \\ q_1(1) &= \frac{20}{21} \end{aligned}$$

Thus, at the beginning of the game, there is a $\frac{5}{21}$ probability of a wrong cascade (given by $1 - q_0(1)$) when the state is +1. When the true state is −2, the similar equations are

$$\begin{aligned} q_0(-2) &= \frac{1}{5}.q_1(-2) + \frac{4}{5}.0 \\ q_1(-2) &= \frac{1}{5}.1 + \frac{4}{5}.q_0(-2) \end{aligned}$$

which yields the solution

$$\begin{aligned} q_0(-2) &= \frac{1}{21} \\ q_1(-2) &= \frac{5}{21} \end{aligned}$$

Therefore, when the state is −2, the probability of a wrong cascade (given by $q_0(-2)$) is $\frac{1}{21}$ at the initial date.

Since both states are equally likely, the unconditional probability of a wrong cascade is

$$\frac{1}{2}.\frac{5}{21} + \frac{1}{2}.\frac{1}{21} = \frac{1}{7}$$

13. The basic idea is as follows. Since it is costly to train workers, competitive firms will pay lower wages to workers who are perceived to have higher quit rates. On the other hand, if one member of a household has to stop working and devote time to childcare or housework, it is generally efficient for the less well paid member to do so (lower opprtunity cost). This could lead to a situation where gender stereotypes become deeply entrenched even without overt (taste based) sexism. Firms don't pay women very well because they perceive a high probability they will quit the job very soon, and women quit soon because they are not paid well enough.

    The competitive wage for any worker with a perceived quit rate q (probability of quitting in any given period) is obtained from the zero profit condition

    $$\frac{1-w}{1-\delta(1-q)} = c$$

    $$w = 1 - c\left[1 - \delta(1-q)\right]$$

    where δ is the time discount factor. Let w_m and w_f denote the wages of males and females respectively. Then

    $$\Delta w = w_m - w_f = c\delta(q_f - q_m)$$

    Let the probability of a shock arriving to a family in any period be α, and conditional on the shock, let λ be he probability that it is the woman who will quit. Then

    $$q_m = \alpha(1-\lambda)$$

    $$q_f = \alpha\lambda$$

    Replacing above

    $$\Delta w = \alpha\delta c(2\lambda - 1)$$

    Now turn to household decision making. Given that the man enjoys a wage premium of Δw, it will be efficient for the woman to quit as long as her utility from housework is more than $-\Delta w$, which has probability $1 - F(-\Delta w)$. Thus, if the market expects women's condiional quit probability to be λ, then the actual quit probability will turn out to be

    $$g(\lambda) = 1 - F(\alpha\delta c(1 - 2\lambda))$$

    Any fixed point of this function represents an equilibrium. Clearly, since $F(0) = \frac{1}{2}$, $\lambda^* = \frac{1}{2}$ is one fixed point representing a non-discriminatory equilibrium where men and women are equally likely to quit, and there is no gender wage gap. However, note that the function above is positively sloped, so there can be multiple intersections wih the 45 degree line, representing possibly asymmetric equilibria with $\lambda^* > \frac{1}{2}$ (or $\lambda^* < \frac{1}{2}$ for that matter). These are quilibria where gender stereotypes prevail and there is a gender wage gap.