

Issues in Economic Systems and Institutions: Part V: Reputation

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Reputational Herding (Scharfstein and Stein 1990)

- ▶ Variation on the theme of informational cascades.
- ▶ Agents have no direct payoff from the decision—they are investing other people's money.
- ▶ Agents want to enhance their reputation for expertise.
- ▶ A critical assumption:
 - ▶ signals of good experts are correlated (great minds think alike)
 - ▶ signals of bad experts are uncorrelated (fools often differ)
- ▶ Reputation is enhanced by: (a) taking the right decision (b) agreeing with other experts.
- ▶ (b) may be so strong that all experts mimic the choices of their predecessors.

The Model

- ▶ Two fund managers: A and B .
- ▶ Each manager chooses to invest (I) or not (N).
- ▶ Return on investment is either high ($x_H > 0$) or low ($x_L < 0$), with equal likelihood.
- ▶ Manager A chooses first, then manager B .
- ▶ Each manager receives a private signal, which is either good (s_G) or bad (s_B).
- ▶ The second manager can observe the first manager's action but not his signal.

Information

- ▶ Each manager is either smart (prob θ) or dumb (prob $1 - \theta$).
- ▶ Managers do not know the quality of their own signals.
- ▶ Smart manager's signal (informative but noisy):

	s_G	s_B
x_H	p	$1 - p$
x_L	$1 - p$	p

- ▶ Dumb manager's signal (pure noise):

	s_G	s_B
x_H	$\frac{1}{2}$	$\frac{1}{2}$
x_L	$\frac{1}{2}$	$\frac{1}{2}$

- ▶ Smart signals perfectly correlated, dumb signals independent.

Reputation

- ▶ $\Pr[s_G | \text{smart}] = \Pr[s_G | \text{dumb}]$, hence each signal in itself conveys no information about expertise.
- ▶ Two signals together convey some information about expertise (matched signals good news).
- ▶ Market observes each manager's action but not signal.
- ▶ Also learns the state of the world (x_H or x_L) eventually.
- ▶ Revises the probability that an expert is smart to some $\hat{\theta}$.
- ▶ Experts are interested in maximizing expected value of reputation ($\hat{\theta}$) because their future salaries are linked to it.

Benchmark 1A: Single Investor, One Signal

- ▶ Suppose there is a single investor who invests his own money, i.e., cares about returns, not reputation.
- ▶ Conditional probabilities after each signal:

$$\mu_G = \Pr[x_H | s_G] = \theta p + (1 - \theta) \cdot \frac{1}{2}$$

$$\mu_B = \Pr[x_H | s_B] = \theta(1 - p) + (1 - \theta) \cdot \frac{1}{2}$$

- ▶ Assume:

$$\mu_B x_H + (1 - \mu_B) x_L < 0 < \mu_G x_H + (1 - \mu_G) x_L$$

- ▶ Optimal decision is dependent on the signal.

Benchmark 1B: Single Investor, Two Signals

- ▶ Suppose the investor knows two signals.
- ▶ Given previous assumption, the optimal decision rule (which maximizes returns) is:
 - ▶ if (s_G, s_G) , choose I .
 - ▶ if (s_B, s_B) , choose N .
 - ▶ if (s_G, s_B) or (s_B, s_G) , depends.
- ▶ If the signals are opposite they wash out, i.e., $\Pr[x_H | s_G, s_B] = \frac{1}{2}$.
- ▶ Then I if $x_H + x_L > 0$, and N if $x_H + x_L < 0$.
- ▶ It is not optimal to simply follow the first signal.

Benchmark 2: Single Manager

- ▶ Suppose there is a single manager who cares about reputation, not returns.
- ▶ If the expert is expected to invest under s_G and not invest under s_B , will he behave accordingly (i.e. reveal his signal)?
- ▶ The manager's reputation goes up when he is correct and goes down when he is wrong:

$$\hat{\theta}(s_G, x_H) = \hat{\theta}(s_B, x_L) > \theta > \hat{\theta}(s_G, x_L) = \hat{\theta}(s_B, x_H)$$

- ▶ The manager thinks the state is more likely to be what his signal indicates:

$$\Pr[x_H|s_G] = \Pr[x_L|s_B] > \frac{1}{2} > \Pr[x_H|s_B] = \Pr[x_L|s_G]$$

- ▶ The expected reputation is greater if he reveals his signal than if he misreports it.

Equilibrium

Theorem

In equilibrium, manager A always invests if he gets s_G and does not invest if he gets s_B .

- ▶ We will show that manager B always mimics manager A , regardless of his own signal.
- ▶ Then, A 's reputation is affected only by his own actions.
- ▶ A single manager will always reveal his signal (previous slide).
- ▶ There is also a “perverse” equilibrium where he reveals his signal by taking the wrong action. Rule it out (suppose he has a small stake in returns).

Equilibrium

Theorem

There is no equilibrium in which manager B always reveals his signal.

- ▶ Suppose there is a revealing equilibrium.
- ▶ Suppose (w.l.o.g) A 's signal is revealed as s_B , but manager B 's signal is s_G .
- ▶ We will show that manager B will misreport his signal as s_B .

Outline of Proof

- ▶ Two conflicting signals cancel each other out (nobody knows who is the dumb one!)

$$\Pr[x_H | s_B, s_G] = \frac{1}{2}$$

- ▶ Incentive constraint for truthfully reporting s_G :

$$\begin{aligned} & \frac{1}{2} \hat{\theta}(s_B, s_G, x_H) + \frac{1}{2} \hat{\theta}(s_B, s_G, x_L) \\ & \geq \frac{1}{2} \hat{\theta}(s_B, s_B, x_H) + \frac{1}{2} \hat{\theta}(s_B, s_B, x_L) \end{aligned}$$

- ▶ In words:

expected reputation (truth-telling) \geq expected reputation (lying)

Outline of Proof

- ▶ However

$$\underbrace{\hat{\theta}(s_B, s_G, x_H)}_{\text{agrees with state alone}} < \underbrace{\hat{\theta}(s_B, s_B, x_L)}_{\text{agrees with both}}$$

$$\underbrace{\hat{\theta}(s_B, s_G, x_L)}_{\text{disagrees with both}} < \underbrace{\hat{\theta}(s_B, s_B, x_H)}_{\text{disagrees with state alone}}$$

- ▶ When signals conflict, either prediction has the same chance of being correct.
- ▶ Ceteris paribus, agreeing with other expert will increase the likelihood of being perceived smart.
- ▶ It is better to be wrong with others than to be wrong alone!

Equilibrium

Theorem

There is an equilibrium in which manager B always does the same thing as manager A, regardless of his own signal..

- ▶ “Reasonable” off-the-equilibrium-path beliefs: if B disagrees with A , then his signal is s_G if he chose I and s_B if he chose N .
- ▶ Incentive constraint:

$$\theta \geq \frac{1}{2}\hat{\theta}(s_B, s_G, x_H) + \frac{1}{2}\hat{\theta}(s_B, s_G, x_L)$$

$$\text{or, } \theta - \hat{\theta}(s_B, s_G, x_L) \leq \hat{\theta}(s_B, s_G, x_H) - \theta$$

- ▶ Intuition: reputation loss from being wrong alone is greater than reputation gain from being right alone.

Political Correctness (Morris 2001)

- ▶ Listeners are often unsure about the bias of speakers (are they sexist/racist/casteist)?
- ▶ What is said reveals something about both the speaker's information as well as his motives.
- ▶ Message affects reputation and vice versa.
- ▶ Political correctness: even unbiased speakers lie! They lie in a direction opposite to the suspected bias.
- ▶ In the extreme, political correctness leads to babbling: everyone says the "safe" thing.
- ▶ PC may be socially inefficient due to informational loss. Important policy issues are not discussed frankly.
- ▶ Possibility of multiple equilibria: different speech cultures.

The Model

- ▶ Two periods: $t = 1, 2$. State-of-the-world at date t : $\omega_t \in \{0, 1\}$. Equi-probable and time-independent.
- ▶ Two players: decision maker (D) and advisor (A).
- ▶ D chooses action $a_t \in \mathbf{R}$ each period. D 's payoff:

$$U_D = -x_1(a_1 - \omega_1)^2 - x_2(a_2 - \omega_2)^2$$

- ▶ A can be good (prob λ_1) or bad (prob $1 - \lambda_1$).
- ▶ Good advisors have the same preference as D , bad advisors always want higher action:

$$U_A^b = y_1 a_1 + y_2 a_2$$

- ▶ $\frac{x_2}{x_1}$ and $\frac{y_2}{y_1}$ represent relative importance of the future.

The Model

- ▶ The advisor gets an independent noisy signal of the state each period: $s_t \in \{0, 1\}$.
- ▶ Accuracy of the signal is $\gamma \in (\frac{1}{2}, 1)$:

	$s = 0$	$s = 1$
$\omega = 0$	γ	$1 - \gamma$
$\omega = 1$	$1 - \gamma$	γ

- ▶ Each period, A sends message $m_t \in \{0, 1\}$.
- ▶ After date 1, D learns ω_1 and updates his belief to λ_2 .
- ▶ $\lambda_2(\lambda_1, m_1, \omega_1)$ is the advisor's new reputation.

Full Information Benchmark

- ▶ If A was known to be good, messages would be truthful and credible.
- ▶ Optimal actions equal to expected value of the state:

$$a^*(0) = \Pr[\omega = 1 | s = 0] = 1 - \gamma$$

$$a^*(1) = \Pr[\omega = 1 | s = 1] = \gamma$$

- ▶ Good advisor wants exactly these actions.
- ▶ Bad advisor always wants $a = 1$, regardless of state.
- ▶ Without reputational concerns, good advisor reports true value of signal. Bad advisor lies and reports 1 even if signal is 0.

Last Period

- ▶ Message strategies

	$s_2 = 0$	$s_2 = 1$
Good	0	1
Bad	1	1

- ▶ Optimal actions, $a_2(m_2; \lambda_2)$:

$$a_2(0; \lambda_2) = 1 - \gamma$$

$$a_2(1; \lambda_2) = \Pr[\omega_2 = 1 | m_2 = 1]$$

$$= \frac{1 - \lambda_2(1 - \gamma)}{2 - \lambda_2}$$

- ▶ $a_2(1; \lambda_2) \uparrow$ as $\lambda_2 \uparrow$ and $\rightarrow \gamma$ as $\lambda_2 \rightarrow 1$.

Value of Reputation

- ▶ Let $v_G(\lambda_2)$ and $v_B(\lambda_2)$ denote expected payoff to good and bad advisor in the last period.
- ▶ The bad advisor always reports $m_2 = 0$:

$$v_B(\lambda_2) = y_2 a_2(1; \lambda_2) \uparrow \text{ in } \lambda_2$$

- ▶ The good advisor always reports truthfully:

$$v_G(\lambda_2) = -x_2 \cdot \frac{1}{2} \left[\mathbf{E} \left((a_2 - \omega_2)^2 \mid s_2 = 0 \right) + \mathbf{E} \left((a_2 - \omega_2)^2 \mid s_2 = 1 \right) \right]$$

- ▶ First term is minimized at $a_2 = 1 - \gamma$, second term at $a_2 = \gamma$.
- ▶ $a_2(1; \lambda_2) \uparrow$ towards γ as $\lambda_2 \uparrow$. Both types value reputation.

First Period

- ▶ Assume equilibrium with no political correctness, i.e., good A reports truthfully.
- ▶ Bad A must falsely report the truth sometimes. Suppose not:

	$s_1 = 0$	$s_1 = 1$
Good	0	1
Bad	0	1

- ▶ Then $\lambda_2(\lambda_1, m_1, \omega_1) = \lambda_1$ for any m_1 and ω_1 . Message does not affect reputation.
- ▶ Also, message is believed completely:
 $a_1(0) = 1 - \gamma < \gamma = a_1(1)$. Bad advisor will lie and always report 1.

First Period

- ▶ Assume good advisor always reports truthfully; bad advisor sometimes lies.

	$s_1 = 0$	$s_1 = 1$
Good	0	1
Bad	0 (prob $1 - v$) 1 (prob v)	1

- ▶ Then $m_1 = 0 \Rightarrow s_1 = 0$, but $m_1 = 1$ does not $\Rightarrow s_1 = 1$.
- ▶ Equilibrium actions $a_1(m_1)$:

$$a_1(0) = 1 - \gamma$$

$$a_1(1) \in \left(\frac{1}{2}, \gamma \right)$$

Reputation

- ▶ Reputation is enhanced if message is
 - ▶ (i) factually correct
 - ▶ (ii) politically correct.
- ▶ Ceteris paribus, initial reputation positively affects final reputation.
- ▶ Comparison for $\lambda_2 (\lambda_1, m_1, \omega_1)$:

$$\lambda_2 (\lambda_1, 0, 1) = \lambda_2 (\lambda_1, 0, 0) > \lambda_1 > \lambda_2 (\lambda_1, 1, 1) > \lambda_2 (\lambda_1, 1, 0)$$

- ▶ Exact expressions involve v and can be calculated using Bayes' Rule.

Reputation Rankings: Intuition

- ▶ Politically correct message ($m_1 = 0$) enhances reputation regardless of factual correctness. Reason:
 - ▶ the good type sends $m_1 = 0$ more often than the bad type.
 - ▶ $m_1 = 0, \omega_1 = 1$ is always an honest mistake.
- ▶ Politically incorrect message ($m_1 = 1$) harms reputation even when factually correct!
 - ▶ the bad type sends $m_1 = 1$ more often than the good type.
 - ▶ $m_1 = 1, \omega_1 = 1$ is sometimes dishonest yet accidental accuracy.
- ▶ $m_1 = 1$ causes further damage to reputation when factually incorrect.
 - ▶ $m_1 = 1, \omega_1 = 0$ is sometimes a deliberate mistake.

First Period Equilibrium

- ▶ **Condition 1:** when $s_1 = 0$, the bad advisor must be indifferent between truth-telling and lying:

$$y_1 a_1(0) + \mathbf{E}[v_B(\lambda_2) | 0, 0] = y_1 a_1(1) + \mathbf{E}[v_B(\lambda_2) | 1, 0]$$

$$\underbrace{y_1 [a_1(1) - a_1(0)]}_{\text{gain from lying}} = \underbrace{\mathbf{E}[v_B(\lambda_2) | 0, 0] - \mathbf{E}[v_B(\lambda_2) | 1, 0]}_{\text{expected reputational loss}}$$

- ▶ When $s_1 = 1$, the bad advisor strictly prefers to tell the truth (implication of above).
- ▶ When $s_1 = 0$, the good advisor always wants to tell the truth (better outcome as well as better reputation).
- ▶ **Condition 2:** the good advisor must prefer to tell the truth when $s_1 = 1$ (algebra omitted).

Informative Equilibria: General Properties

- ▶ Good advisor sends $m_1 = 0$ whenever $s_1 = 0$. He announces $m_1 = 1$ with (weakly) positive probability if $s_1 = 1$.
 - ▶ When $s_1 = 0$, both current and reputational payoffs are higher for $m_1 = 0$.
- ▶ Bad advisor sends $m_1 = 1$ more often than the good advisor.
 - ▶ Otherwise there would be no reputational cost to sending $m_1 = 1$.
- ▶ There is a strict reputational incentive to be politically correct:

$$\lambda_2(\lambda_1, 0, 1) = \lambda_2(\lambda_1, 0, 0) > \lambda_1 > \lambda_2(\lambda_1, 1, 1) > \lambda_2(\lambda_1, 1, 0)$$

- ▶ Reasons as before.

Extreme Political Correctness

- ▶ If $\frac{x_2}{x_1}$ is high enough, the only equilibrium is babbling.
- ▶ One way to depict the strategies:

	$s_1 = 0$	$s_1 = 1$
Good	0	0
Bad	0	0

- ▶ Both type of advisors say the “safe” thing so as not to damage their reputation and influence in the future.
- ▶ Reason: the bad advisor’s indifference condition pins down reputational gain from $m_1 = 0$. Good advisor will want to capture this if x_2 is high enough.
- ▶ Under PC, **nothing** is learnt about the state or the speaker!

Welfare

- ▶ Benchmark: a model where D in the 2nd period does not know what happened in the 1st period (no reputational incentives).
- ▶ Without reputation, strategies in both periods are as in last period of the game with reputation.
- ▶ Reputation creates 3 effects:
 1. *Discipline effect*: bad advisor tells the truth more often (announce $m_1 = 0$ when $s_1 = 0$). (+)
 2. *Sorting effect*: decision maker learns something about the speaker and his trustworthiness. (+)
 3. *Political correctness effect*: even the good advisor starts lying sometimes (send $m_1 = 0$ when $s_1 = 1$). (-)
- ▶ When we have an extreme PC (babbling) equilibrium, 3 dominates $1 + 2$.

What's In A Name?

- ▶ Bertrand and Mullainathan (*AER*, 2004): RCT for testing discrimination in job applications.
- ▶ Fictitious CVs randomly matched with white (Emily, Greg) and black (Lakisha, Jamal) sounding names.
- ▶ 5000 resumes submitted in response to 13000 employment ads in Boston and Chicago.
- ▶ Call back rates: whites = 9.65%, blacks = 6.45%. Whites have 50% higher call back.
- ▶ As resume quality improves, response rate goes up faster for whites than blacks.
- ▶ Blind auditions improve the chances of women violinists significantly (Goldin and Rouse, *AER* 2000).

Other Examples of “Discrimination”

- ▶ Insurance premiums: young drivers (under 25) and older health insurees (over 65) face higher rates.
- ▶ Airport screening: Middle Eastern males more likely to be terrorists than Swiss nuns.
- ▶ Residential choice and segregation: racial composition of a neighbourhood is a predictor of crime. Avoid the Bronx.
- ▶ Credit: 97% of Grameen bank loans are given to women.
- ▶ Racial profiling: Search rates from Knowles, Persico and Todd's (*JPE* 2001) data on 1,590 stop-and-search operations by Maryland police, 1995 - 99:
 - ▶ Blacks = 63%, whites = 29%.
 - ▶ Men = 93%, women = 7%.

Three Notions of Discrimination

- ▶ **Incidental discrimination:** groups are not treated differently, but there is differential impact because groups differ statistically in behaviour (prison population disproportionately male because lawbreakers are disproportionately male).
- ▶ **Statistical discrimination:** groups are treated differently, but only insofar as group affiliation is a statistical predictor of behaviour (young people drive rash, on average).
- ▶ **Taste based discrimination:** groups are treated differently because doing so for its own sake generates utility (racial or caste-based segregation). Prejudice, pure and simple.

How do we empirically separate these strands?

Discrimination in Monitoring

- ▶ Crime as a rational choice: criminals commit an illegal act after weighing benefits against expected punishment cost (Becker, 1968).
- ▶ Two races, $r = A, W$. Observationally distinct to the police.
- ▶ Observable non-racial characteristic c follows distribution $n_r(c)$, with totals

$$N_r = \int n_r(c) dc$$

- ▶ Distribution of x , i.e. legal income opportunities: $F_r(x; c)$.
- ▶ Benefit of carrying drugs = B , penalty if caught = P .
- ▶ Police have enough resources to search only S people.

The Game and Equilibrium

- ▶ Individual decision: carry drugs or enter a legal profession?
- ▶ Police simultaneously choose $\sigma_r(c)$ —what proportion of group (r, c) to search, subject to the budget constraint

$$\sum_{r=A}^W \int n_r(c) \sigma_r(c) dc = S$$

- ▶ Police objective function:

$$U = \sum_{r=A}^W \int n_r(c) \sigma_r(c) [\theta_r(c) + u_r] dc$$

where u_r is the intrinsic utility/disutility of searching race r , and $\theta_r(c)$ is the “hit rate” among group (r, c) .

The Testable Implication

- ▶ Interior equilibrium: police must be indifferent across groups, i.e. $\theta_r(c) + u_r$ is a constant.
- ▶ Expected payoff from drug peddling:

$$q(\sigma_r(c)) = (1 - \sigma_r(c))B - \sigma_r(c)P$$

- ▶ Those whose legal incomes are less, choose to commit crime:

$$\theta_r(c) = F_r(q(\sigma_r(c)))$$

- ▶ Equality is preserved after integrating over non-racial characteristics. Let $\theta_r = \int \theta_r(c)n_r(c)dc$. Then

$$\theta_A + u_A = \theta_W + u_W$$

Testable Implication

- ▶ If police are unprejudiced ($u_A = u_W$), **hit rates** will be equalized across groups, even if **search rates** are very different.
- ▶ If police are prejudiced against blacks ($u_A > u_W$), equilibrium hit rates will be **lower** among blacks.
- ▶ The general idea: compensating differentials.
- ▶ Onerous jobs pay more, attractive cities/neighbourhoods have higher rents.
- ▶ Does not require the econometrician to know or control for all the variables that police condition their decisions on.

Highway Search Data: Hit Rates

- ▶ Test of equality of means (Pearson's chi-squared):

$$\sum_{r=1}^R \frac{(\hat{p}_r - \hat{p})^2}{\hat{p}_r} \sim \chi^2(R-1)$$

- ▶ Observed hit rates across racial and gender groups

	Black	White	Hispanic	All Races
Both sexes	.34	.32	.11*	.30
Male	.34	.33	.11	.32
Female	.44*	.22*	-	.36

- ▶ Null hypothesis of equality is not rejected when only whites and blacks are used. Rejected when Hispanics are added.
- ▶ Evidence of taste based discrimination against Hispanics and white females, but not males or blacks.

Efficiency

Theorem

If $u_A = u_W$ and $F_A = F_W = F$ is concave, the equilibrium allocation maximizes rather than minimizes aggregate crime rate.

- ▶ Let $N_A = N_W = N$ and $\sigma = \frac{S}{N}$. Budget constraint:

$$\frac{1}{2} [\sigma_A + \sigma_W] = \sigma$$

- ▶ Aggregate crime rate:

$$F_A(q(\sigma_A)) + F_W(q(2\sigma - \sigma_A))$$

- ▶ First-order condition holds at $f_A = f_W$ ($q'(\cdot)$ is constant)):

$$f_A(q(\sigma_A)) = f_W(q(2\sigma - \sigma_A))$$

- ▶ Since the objective function is concave, this is a maximum!

Equilibrium, Fairness and Efficiency

- ▶ In general, they are described by three different conditions:
 - ▶ Equilibrium: $F_A = F_W$, i.e. crime rates are equal.
 - ▶ Fairness: $\sigma_A = \sigma_W$, i.e. search rates are equal.
 - ▶ Efficiency: $f_A = f_W$, i.e. response elasticities are equal.
- ▶ Mandatory quotas (e.g. $\sigma_A = \sigma_W$) to improve fairness may improve efficiency or worsen it.
- ▶ Intuition: there is a conflict between two different objectives:
 - ▶ discouraging crime (deterrence).
 - ▶ catching as many criminals as possible (retribution).
- ▶ Dynamic inconsistency and problem of commitment.

An Example

- ▶ Extreme case: $F_W(q(2\sigma)) = 1$, i.e., W s are completely unresponsive to higher scrutiny (zero response elasticity).
- ▶ Efficient allocation: $\sigma_W = 0$ and $\sigma_A = 2\sigma$.
- ▶ Equilibrium allocation is fairer but less efficient than this benchmark.

Statistical Discrimination

- ▶ Skills a function of both innate ability and effort.
- ▶ Noisy measurement of skill \Rightarrow priors will affect posteriors.
- ▶ Positive feedback loop between employer perceptions and worker effort:
 - ▶ if employers hold optimistic beliefs, workers may have a strong incentive to become skilled.
 - ▶ if employers hold pessimistic beliefs, workers may have a weak incentive to become skilled.
- ▶ Multiple equilibria are possible for some parameters.
- ▶ Different groups (e.g. blacks and whites) may get locked into different equilibria.
- ▶ Affirmative may improve or worsen stereotypes (beliefs about skill distribution in target population).

The Coate-Loury model

- ▶ Two races: W (proportion λ) and B .
- ▶ Two kinds of tasks: unskilled (0) and skilled (1).
- ▶ Two worker types: qualified (q) and unqualified (u).
- ▶ Every worker (either race) could become qualified at a cost c which is private information. Within each group, $c \sim U[0, 1]$.
- ▶ Payoffs to employers and workers:

	Skilled (1)	Unskilled (0)
Qualified (q)	x_q, w	0, 0
Unqualified (u)	$-x_u, w$	0, 0

- ▶ Noisy test of qualification:

	Pass	Unclear	Fail
Qualified (q)	$1 - p_q$	p_q	0
Unqualified (u)	0	p_u	$1 - p_u$

Employer and Worker Best Response

- ▶ Employers: Pass \rightarrow 1, Fail \rightarrow 0, Unclear \rightarrow ?
- ▶ Let prior $= \pi$. Then posterior

$$\sigma = \Pr(q|\text{Unclear}) = \frac{\pi p_q}{\pi p_q + (1 - \pi) p_u}$$

- ▶ Assigning skilled task after unclear test result is optimum iff

$$\sigma x_q + (1 - \sigma) x_u \geq 0$$

$$\pi \geq \frac{p_u x_u}{p_u x_u + p_q x_q} = \hat{\pi}$$

- ▶ Workers invest in qualification iff cost below a cutoff:

$$\begin{aligned} \phi(\pi) &= \bar{c} = (1 - p_u)w \quad \text{if } \pi \geq \hat{\pi} \\ &= (1 - p_q)w \quad \text{if } \pi < \hat{\pi} \end{aligned}$$

Multiple Equilibria

- ▶ **Equilibrium with liberal beliefs:** suppose $\pi \geq \hat{\pi}$ in equilibrium \Rightarrow unclear test result leads to skilled task.
Condition:

$$\pi_l = \phi(\pi_l) = (1 - p_u)w \geq \hat{\pi}$$

- ▶ **Equilibrium with conservative beliefs:** suppose $\pi < \hat{\pi}$ in equilibrium \Rightarrow unclear test result leads to unskilled task.
Condition:

$$\pi_c = \phi(\pi_c) = (1 - p_q)w < \hat{\pi}$$

- ▶ Multiple equilibria exist if

$$\pi_c < \hat{\pi} \leq \pi_l$$

$$\text{or } (1 - p_q)w < \frac{p_u x_u}{p_u x_u + p_q x_q} \leq (1 - p_u)w$$

Numerical Example

- ▶ Stability: suppose employer beliefs evolve the following way:

$$\pi^{t+1} = \phi(\pi^t)$$

The both equilibria are locally stable.

- ▶ Payoffs to employers and workers:

	Skilled	Unskilled
Qualified	1, 1	0, 0
Unqualified	-1, 1	0, 0

- ▶ Noisy test of qualification:

	Pass	Unclear	Fail
Qualified	$\frac{1}{4}$	$\frac{3}{4}$	0
Unqualified	0	$\frac{1}{2}$	$\frac{1}{2}$

Employer's Hiring Strategy

- ▶ Employers: pass \rightarrow manager, fail \rightarrow clerk, unclear \rightarrow ?
- ▶ Let fraction of qualified workers = π . After unclear test result

$$\sigma = \Pr(s|\text{Unclear}) = \frac{\pi \cdot \frac{3}{4}}{\pi \cdot \frac{3}{4} + (1 - \pi) \cdot \frac{1}{2}} = \frac{3\pi}{2 + \pi}$$

- ▶ Assigning as manager after unclear test result is optimum iff

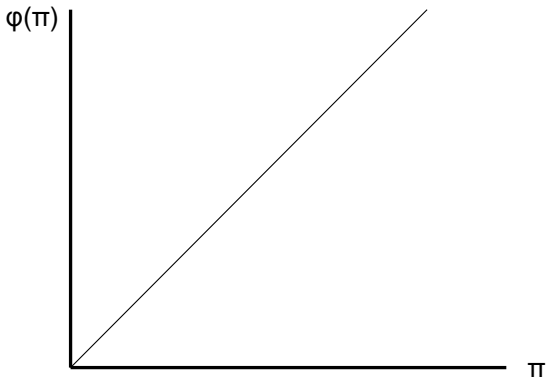
$$\sigma \cdot 1 + (1 - \sigma)(-1) \geq 0 \Rightarrow \sigma \geq \frac{1}{2}$$

$$\text{or } \pi \geq \frac{2}{5}$$

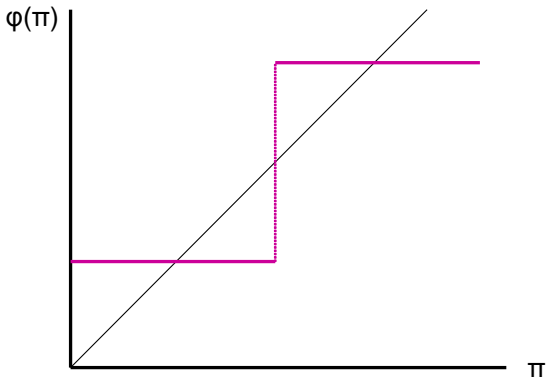
Worker's Investment Strategy

- ▶ Let $\phi(\pi)$ = fraction of workers who **actually** acquire skills when employers **think** this fraction is π .
- ▶ **Optimistic beliefs:** $\pi \geq \frac{2}{5}$ (unclear \rightarrow manager).
 - ▶ Hiring probability if qualified = 1.
 - ▶ Hiring probability if unqualified = $\frac{1}{2}$.
 - ▶ Expected gain from skill = cost threshold for investment = fraction of qualified workers = $\phi(\pi) = \frac{1}{2}$.
- ▶ **Pessimistic beliefs:** $\pi \geq \frac{2}{5}$ (unclear \rightarrow clerk).
 - ▶ Hiring probability if qualified = $\frac{1}{4}$.
 - ▶ Hiring probability if unqualified = 0.
 - ▶ Expected gain from skill = cost threshold for investment = fraction of qualified workers = $\phi(\pi) = \frac{1}{4}$.
- ▶ In equilibrium, employers' beliefs must be fulfilled: $\phi(\pi) = \pi$.

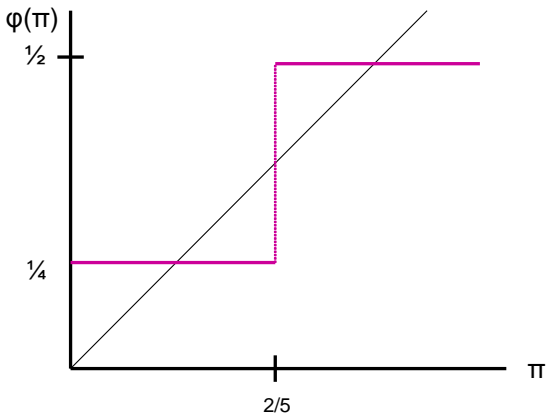
Multiple Equilibria



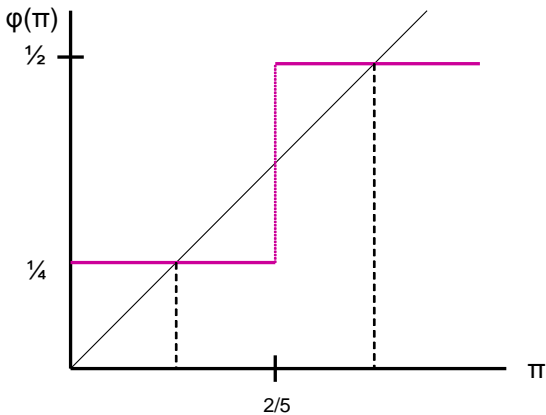
Multiple Equilibria



Multiple Equilibria



Multiple Equilibria



Affirmative Action, Incentives and Stereotype

- ▶ Does affirmative action destroy incentives, investment in skills, etc.?
- ▶ Does affirmative action worsen stereotypes of beneficiaries?
- ▶ Theoretically, it can go either way:
 - ▶ since AA makes entry easier, fewer members may invest.
 - ▶ under statistical discrimination, if entry is too hard, it may discourage investment.
- ▶ The effect can only be determined empirically.
- ▶ Our model illustrates potential positive effect on stereotypes in two senses:
 - ▶ the bad equilibrium improves locally (temporary effect).
 - ▶ the bad equilibrium is destroyed altogether (permanent effect).

Affirmative Action, Incentives and Stereotype

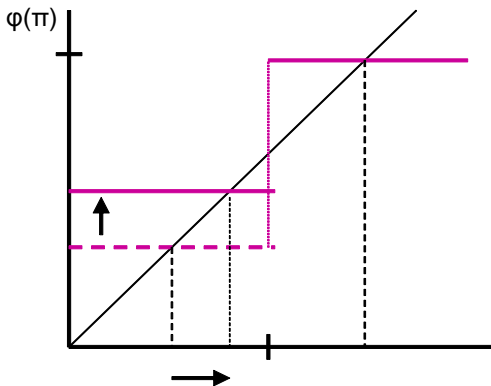
- ▶ Suppose A 's are in bad equilibrium, W 's are in good equilibrium.
- ▶ If quotas are introduced, employers may be forced to hire a fraction α of A 's with unclear test results.
- ▶ For given α , a A worker invests if

$$\underbrace{(1 - p_q + \alpha p_q) w - c}_{\text{qualified payoff}} \geq \underbrace{\alpha p_u w}_{\text{unqualified payoff}}$$

$$\text{or, } c \leq [1 - p_q + \alpha(p_q - p_u)] w$$

- ▶ If $p_q > p_u$ (condition for multiple equilibria), the cost threshold for investment increases with α .
- ▶ Incentives and stereotype improve in the bad equilibrium.

Case 1: Local Improvement, Temporary Effect



Case 2: Global Improvement, Permanent Effect

