Repayment incentives and the distribution of gains from group lending*

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Abstract

Group loans with joint liability are a distinguishing feature of many microfinance programs. While such lending benefits millions of borrowers, major lending institutions acknowledge its limited impact among the very poor and have shifted towards individual loans. This paper attempts to explain this trend by exploring the relationship between borrower wealth and the benefits from group lending when access to credit is limited by strategic default. In our model, individuals of heterogeneous wealth face a given investment opportunity so poor investors demand larger loans. We show that the largest loan offered as an individual contract cannot be supported as a group loan. Joint liability cannot therefore extend credit outreach in the absence of additional social sanctions within groups. We also find that the benefits from group loans are increasing in borrower wealth and that optimal group size depends on project characteristics. By allowing for multi-person groups and wealth heterogeneity in the population, the paper extends the standard framework to analyze joint liability and contributes to an understanding of the conditions under which microcredit can reduce poverty.

Keywords: microcredit, joint-liability, group lending, repayment incentives, social sanctions. JEL codes: I38, G21, O12, O16

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1 Introduction

The ideology and practice of poverty alleviation has been deeply influenced by the idea that access to credit can empower the poor. Microfinance programs around the world cover millions of borrowers and are provided under a variety of different institutional arrangements. The overall gains from such lending are widely acknowledged but there is growing concern about their capacity to reach those at the bottom of the income distribution. This has resulted in a lively debate, but little consensus on how credit contracts can be better designed to improve credit access and the welfare of poor borrowers. A recurring question within this debate is whether group loans with joint liability can effectively achieve these objectives.

Group loans were first popularized by the Grameen Bank of Bangladesh in the 1970s. It was believed that joint liability would generate social pressure on borrowers to repay loans and create a financially sustainable model of lending. This approach was questioned in the late nineties when natural disasters triggered widespread default and borrowers protested against rigidities in the lending program. The Grameen Bank responded by introducing Grameen II which made all members individually liable for their loans (Yunus, 2004; Kalpana, 2006). Within a couple of years, membership doubled, suggesting that individual loans catered to a previously unmet demand for credit.1 A similar switch to individual contracts was made by Banco Sol of Bolivia, another pioneer in group lending.

This trend towards individual contracts is far from universal. A majority of the 663 institutions reporting to the Microfinance Information Exchange (MIX) in 2009 relied on some form of joint liability and many large microfinance institutions offer a combination of group and individual loans.2 An interesting contrast to the Grameen case is provided by the microfinance sector in India which is dominated by village-based groups that strictly adhere to joint liability.3

In this paper, we attempt to explain some of these patterns by modeling the relationship between

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1Wright et al. (2006) report that “Grameen took 27 years to reach 2.5 million members and then doubled that in the full establishment of Grameen II”.

2De Quidt et al. (2012b) classify institutions reporting to MIX by the types of contracts used and Ghate (2008) does this for 129 recognized microfinance institutions in India. Many of these organizations use both individual and joint liability contracts. The Bank for Agriculture and Agricultural Cooperatives (BAAC) in Thailand also allows its members to choose between group and individual loans (Ahlin and Townsend, 2007).

3There are currently an estimated 77 million microfinance clients in India and over two-thirds of these are in what are called Self-Help Groups or more popularly, SHGs. (Srinivasan (2012), p 3, Table 1.1).
borrower wealth and the benefits from group lending. We allow individuals to be in groups of arbitrary size and characterize loan contracts available to them as a function of their initial wealth. To focus on the role of wealth heterogeneity, we assume all borrowers face identical investment opportunities and the poorest investors therefore need the largest loans. Our first set of results compares joint liability contracts with individual loans assuming that groups have no additional social sanctions to discipline their members into repayment. We show that the largest loan available under a group lending contract with joint liability is strictly smaller than the largest individual loan. For borrowers with access to both types of contracts, the benefits from group loans are increasing in wealth.

We next ask whether social sanctions within groups can increase borrower benefits from group loans and also whether they can extend credit to unbanked households. We show that social sanctions always improve welfare for those who already have access to group loans. This is because, unlike bank sanctions, they are not imposed in equilibrium. They do not necessarily extend credit outreach. For a reasonable set of parameter values, individual contracts support larger loans than joint liability contracts even when social sanctions are arbitrarily large.

Our analysis reveals that the two-person groups that have dominated the theoretical literature on group lending are quite particular and many of our results on the interaction of wealth and welfare are obtained by generalizing this framework. When groups have only two members, the fraction contributing to loan repayment is at least one half. This fraction can vary more in large groups and this variation generates a positive relationship between the wealth of borrowers and their benefits from investment. Our last result explores optimal group size. We find that two-member groups are never optimal for small loan sizes but that the benefits from group lending are not generally monotonic in the number of members. We illustrate this non-monotonicity with a numerical example.

Our work contributes to a well-established theoretical literature on the mechanisms through which joint liability can affect investment decisions and borrower welfare. It is most closely related to Besley and Coate (1995) who were the first to demonstrate its ambiguous effects on repayment rates. Several subsequent papers have shown how relaxing the informational or contractual assumptions of the traditional joint liability model can be welfare improving. Rai and Sjostrom (2004) assume borrowers with unsuccessful projects can report the project outcomes of other members to the bank. Bhole and Ogden (2010) allow co-payments for defaulting

4Useful surveys of this literature can be found in Ghatak and Guinnane (1999) and Armendariz de Aghion and Morduch (2005).
projects to be asymmetric. Bond and Rai (2008) and De Quidt et al. (2012a) both leverage social sanctions within groups without joint liability. Our work is also related to models of income dynamics in which the poor have either no access to credit or face unfavorable interest rates (Galor and Zeira, 1993; Banerjee and Newman, 1993; Matsuyama, 2000). These papers use individual credit contracts while we show that the relative disadvantage to poor borrowers may be heightened when banks also offer group loans.

Taking models such as ours to the data on group lending programs is a challenge. Microfinance institutions operate in many different environments and their objectives vary from pure profit-maximization to broader social missions. Besides, empirical work on lending contracts typically estimates average impact whereas we are concerned with the relationship between benefits and loan size. Our theoretical results are however broadly consistent with patterns found in experimental and observational data.

Several studies have highlighted targeting deficiencies in microfinance programs. Morduch (1998) shows that eligibility criteria for membership in the Grameen Bank were frequently violated and the non-poor were admitted as members. With the introduction of Grameen II in 2002, the Grameen Bank explicitly acknowledged that the poor are often best served outside groups.\(^5\) Dewan and Somanathan (2011) and Coleman (2006) find participation rates rising in income in very different regional contexts.\(^6\) Sebstad and Cohen (2001) and Hermes and Lensink (2011) survey programs in multiple countries and emphasize the trade-off between the financial sustainability of microfinance institutions and their outreach among the poor.

In our model, larger bank sanctions allow groups to achieve higher repayment rates. These sanctions could take the form of a rise in the future cost of credit. Karlan and Zinman (2009) collaborate with a South-African bank in an unusual field experiment and show that commitments of lower future interest rates improve repayment on current loans. Abbink et al. (2006) and Cassar et al. (2007) provide experimental evidence on the importance of social ties in repayment decisions. Giné and Karlan (2009) study a microfinance institution in the Philippines

\(^5\)According to their website (Grameen Bank, 2009):

A destitute person does not have to belong to a group...Bringing a destitute woman to a level where she can become a regular member of a group will be considered as a great achievement of a group.

that randomly assigned new areas to either joint or individual liability. As in the Grameen case, the introduction of individual liability attracted many more clients. There was no significant difference in repayment rates across the two types of contracts but it is hard to interpret this finding since repayment rates were close to 100% under both regimes.

This empirical literature suggests that the benefits from microfinance do vary with household wealth. We hope this paper encourages more specific tests of the mechanisms underlying this relationship. The rest of the paper is organized as follows. The next section describes our model and derives optimal contracts for individual and group loans. Section 3 compares credit outreach and borrower welfare under the two types of loans in the absence of social sanctions. Section 4 extends the model to explore the effect of social sanctions on group contracts. Section 5 discusses optimal group size and Section 6 concludes.

2 The model

Our principal unit of analysis is a set of risk neutral households, each of whom can choose to invest in a project. The project requires one unit of capital and no other inputs.\(^7\) It returns \(\rho\) with probability \(\pi\) and zero otherwise. Wealth, which we denote by \(w\), varies continuously over the \((0, 1)\) interval and is used for investment whenever a loan of \((1 - w)\) can be obtained. We are interested in characterizing the set of wealth levels over which households are eligible for individual and group loans and the benefits from these two types of contracts as a function of initial wealth.

The banking system is competitive and offers depositors a gross return \(r\), equal to the opportunity cost of bank funds. Banks lend either to individuals or to groups of size \(n\) under joint liability. We assume that groups are homogeneous in the wealth of their members. This allows us to abstract from questions of redistribution within groups and focus on the risk-pooling function of joint liability contracts. In practice, many successful group lending programs have encouraged the creation of groups with similar members.\(^8\) Interest rates for each contract vary

\(^7\)While we have in mind the self-employment projects financed by many microfinance organizations, we do not explicitly model effort decisions. The returns to the project in our model can be interpreted as net of effort costs.

\(^8\)For example, when promoting micro credit through the commercial banking system, India’s central bank explicitly recommended that savings and credit groups be formed with households of “homogeneous background
with repayment rates to equate the expected return from all lending to $r$. Project returns are never observed by the bank. Under group lending they are observed by members of the group.

The assumption of a single indivisible investment project requires some justification. One might expect poorer borrowers to invest in projects requiring smaller loans. In the Grameen case, for example, the poorest borrowers are often vegetable vendors or basket makers while slightly richer households invest in looms or small shops. Our purpose is not to model investment choices and the distribution of wealth in an economy as a whole, but rather to illustrate that for a given project, access to and benefits from joint liability are systematically related to initial wealth through differences in required loan sizes.

Banks have available non-pecuniary sanctions $\bar{K}$ which they can use to induce borrowers to repay their loans. These sanctions could represent harassment by bank officials, unfavorable future contracts or a damaged reputation. We simply think of $\bar{K}$ as the enforcement capacity of a bank. All loan contracts specify a loan amount, a gross interest rate and the value of the sanction $K \in (0, \bar{K}]$ that is levied if a borrower defaults. For individual contracts, the sanction is levied on a single borrower in case of non-repayment. For group contracts, all members are sanctioned $K$ if the group fails to repay what it collectively owes. Since bank sanctions provide no direct benefits to the bank and are costly to the borrower, competition ensures that the value of $K$ is all chosen to maximize borrower benefits for both types of contract and each loan size.

We refer to a contract as feasible if it induces repayment by borrowers whenever they are able to do so. For individual loans this means that all borrowers with successful projects repay their loans. For group contracts, the group reimburses the loan whenever enough projects within the group succeed. We will specify this more precisely below. An optimal contract for each loan type and loan size is the feasible contract which maximizes welfare. Since banks always break even, this corresponds to the one with the highest benefits per borrower. We denote these benefits by $U^i$ and $U^9$ for individual and group contracts respectively.

Under individual contracts, those with unsuccessful projects necessarily default on their loans. If banks use sanctions to ensure all successful borrowers repay their loans, the repayment rate and interest” (National Bank for Agriculture and Rural Development, 1992).  

9Timely repayment could produce future benefits in the form of lower interest rates, less monitoring and larger loans. The role of dynamic incentives is controversial. Karlan and Zinman (2009) find them to be quantitatively important in lending experiments while Bulow and Rogoff (1989) argue that the financial gain to the borrower from default can often compensate for reputational losses. For our purpose, it is sufficient that banks have some enforcement capacity in any form which makes lending possible.
is $\pi$ and the competitive gross interest rate charged by banks must be $\frac{r}{\pi}$ for all loans. The smallest sanction that induces repayment for a loan of size $L$ is thus

$$K = \frac{r}{\pi}L.$$  \hspace{1cm} (1)

A loan contract is then given by $(L, \frac{r}{\pi}, K)$, where $K$ depends on $L$ according to (1). Expected benefits from investment depend on project returns, sanctions and the opportunity cost of the borrower’s own funds and are given by

$$U^i = \pi \left( \rho - \frac{r}{\pi}L \right) - (1 - \pi)K - rw.$$  \hspace{1cm} (2)

On substituting for $K$ and $w$, this expression simplifies to

$$U^i = \pi \rho - r - \frac{(1 - \pi)}{\pi}rL$$  \hspace{1cm} (2)

Those with smaller loans face lower bank sanctions and therefore enjoy higher benefits. Since banks only lend if $K \leq \bar{K}$, the largest loan offered is

$$\bar{L} = \frac{\pi \bar{K}}{r}.$$  \hspace{1cm} (3)

$\bar{L}$ is a measure of credit outreach for individual loans. We assume that $\bar{L} < 1$ so not all households have access to credit. We also assume that the poorest borrower with such access finds investment worthwhile. From (2) and (3), we see that this requires the project return $\rho$ to be at least

$$\bar{\rho} = \frac{1 - \pi}{\pi} \bar{K} + \frac{r}{\pi}.$$  \hspace{1cm} (4)

We turn now to group contracts. A group of size $n$ and homogeneous wealth $w$ requires a loan of size $n(1 - w) = nL$. For each loan size $L$, we would like to verify whether there is a gross interest rate and bank sanction that makes such a loan feasible and if so, compute borrower benefits to arrive at the optimal contract. While unsuccessful projects cannot contribute to
loan repayment, successful members honor the group’s obligation to the bank as long as the contribution required from each member is less than the bank sanction. The repayment rate for group loans therefore depends on the probability of this event.

Let $B(n, j, \pi)$ be the binomial probability of $j$ or more successes in $n$ independent trials, with a success probability of $\pi$ on each trial. We denote this by $B_j$ when there is no ambiguity about the $n$ and $\pi$ in question. Suppose the loan is repaid whenever there are $j$ or more successful projects within the group, and not otherwise. Then the repayment rate is $B_j$ and banks break even when charging a gross interest rate $\frac{r}{B_j}$. The total loan obligation is $nL\frac{r}{B_j}$. The contribution per successful project depends on the number of successful projects. When exactly $j$ projects succeed, each project contributes $\frac{n}{j}\frac{r}{B_j}L$. For more favorable states, this contribution is smaller as shown below. A group lending contract with per-member loan size $L$ is feasible if there exists an integer $j \leq n$ for which:

$$\frac{n}{j} \frac{r}{B_j} L \leq \bar{K} \tag{5}$$

The bank sanction which induces repayment by each of the $j$ successful members is

$$K_j = \frac{n}{j} \frac{r}{B_j} L. \tag{6}$$

The largest loan size for which (5) is satisfied gives us the marginal household with access to group loans. This is determined by the value of $j$ for which the function $jB_j$ is maximized. The following lemma characterizes this function and is useful in deriving many of our results:

**Lemma 1.** The function $jB_j$ is single-peaked. It starts at zero, takes the value $n\pi^n$ at $j = n$ and attains a maximum at $j^* \leq \lceil n\pi \rceil$, the smallest integer greater than or equal to $n\pi$. If $\pi < \frac{1}{n}$, it is decreasing for $j \geq 1$ and, if $\pi > \frac{n(n-1)}{1+n(n-1)}$, it is increasing throughout.

**Proof.** See Appendix

We can now derive the benefit from an optimal group contract $(L, \frac{r}{B_j}, K_j)$. If there are fewer than $j$ successes in the group, then each member is sanctioned $K_j$ and the successful members keep the return from their projects. If $j$ or more projects succeed, equal contributions by all successful members are used to repay the loan. Using $\pi_k$ to denote the probability of exactly $k$ successes in $n$ trials, the expected per-member benefit from the group loan is
\[ U^g_j = \sum_{k=0}^{j-1} \pi_k \left( \frac{k\rho - K_j}{n} \right) + \sum_{k=j}^{n} \pi_k \left[ \frac{k}{n} \left( \rho - \frac{nrL}{k B_j} \right) \right] - rw \]

\[ = \frac{\rho}{n} \sum_{k=0}^{n} k\pi_k - r(L + w) - (1 - B_j)K_j \]

\[ = \pi\rho - r - (1 - B_j)\frac{n}{B_j} L \quad (7) \]

Our first result links this benefit to the repayment rate \( B_j \):

**Proposition 1.** Consider the set of feasible group contracts \((L, \frac{r}{B_j}, K_j)\) for a per-member loan of size \( L \). The optimal contract is the one with the highest repayment rate.

**Proof.** See Appendix

This result highlights an important difference between individual and group contracts. For an individual loan, the borrower prefers the contract with the lowest possible sanction since this is the penalty incurred whenever the project fails. In the case of group loans, there is a possible trade-off. On the one hand, a higher sanction imposes additional costs on all group members when the group defaults. But raising the sanction also raises the repayment rate and this lowers the interest rate because it increases the extent to which successful members are willing to subsidize failed projects. With the Binomial distribution, which determines the number of successes in groups in our model, the increase in the repayment rate \( B_j \) is large relative to the increase in the required sanction, \( K_j \), and the net effect of a higher sanction is always positive.

The next section uses the optimal individual and group contracts to describe the equilibrium relationship between borrower wealth and the gains from investment.

### 3 Equilibrium credit contracts

Equilibrium in our model consists of two wealth intervals corresponding to households that are offered individual and group loans respectively. A household, if offered both contracts, chooses the one with the higher benefit. We start by showing that there is always a set of households
who are offered individual loans but have no access to group loans. In other words, joint liability
per se cannot extend credit outreach.

**Proposition 2.** The largest loan available in a group lending contract is strictly smaller than
in an individual contract.

*Proof.* The largest individual loan \( \bar{L} \) is not feasible as a group loan if, for all \( 0 < j \leq n \),

\[
\frac{n}{j B_j} \bar{L} > \bar{K}.
\]

Substituting for \( \bar{L} \) from (3), we can rewrite this condition as

\[
n\pi > j B_j.
\]

The LHS is the expectation of a binomial distribution with parameters \( n \) and \( \pi \). Writing this
as a sum, we see that the above condition always holds:

\[
n\pi = \sum_{k=0}^{n} k \pi_k > \sum_{k=j}^{n} k \pi_k \geq j \sum_{k=j}^{n} \pi_k = j B_j.
\]

This result is driven by the fact that banks extract less from groups than from individual loans
of size \( \bar{L} \). The maximum amount that the bank can recover from any borrower is \( \bar{K} \). Since
only successful borrowers are able to repay their loan, expected repayments are bounded above
by \( n\pi \bar{K} \). With individual loans of size \( \bar{L} \), all successful borrowers do repay the bank and so
this bound is achieved. Under group lending, banks recover nothing from successful projects in
defaulting groups and cannot extract more than \( \bar{K} \) from any project in a group that does not
default. These leakages from the banking system cause the inequality in (5) to fail at \( \bar{L} \).

Although joint liability does not extend outreach, it does result in higher welfare for borrowers
with small loans. Using (2) and (7), the difference between the benefit from a group loan and
an individual loan of size \( \bar{L} \) is given by:

\[
U_g - U_i = L \left[ \frac{1 - \pi}{\pi} - \frac{(1 - B_j)n}{j B_j} \right]
\]

(8)
Thus, if a group contract \((L, \frac{r}{B_j}, K_j)\) satisfies the condition

\[
\frac{1 - \pi}{\pi} > \frac{(1 - B_j) n}{B_j} \frac{n}{j},
\]  

(9)

then each member is better off than they would be under an individual contract. The following result identifies one interesting case for which this condition always holds.

**Proposition 3.** If a per-member loan \(L\) is small enough to require only one successful project for group repayment, borrower welfare is always higher under group lending.

**Proof.** Rewriting (9) for \(j = 1\), we get

\[
1 - (1 - \pi)^n - n \pi(1 - \pi)^{n-1} > 0.
\]

The above expression is simply the binomial probability of more than one success in \(n\) trials and is greater than zero for all \(n \geq 2.\)

Group loans are always preferred to individual contracts in this case because groups default on their loan only when all projects fail. But these borrowers could not have repaid their loan with an individual contract. For any other distribution of project outcomes, successful projects in groups cover the unsuccessful ones, resulting in higher overall repayment rates. The condition that one successful project in the group is enough to reimburse the entire group loan is quite restrictive. This is sufficient but not necessary. For medium and large groups, benefits from group loans are higher than from individual contracts under weaker conditions. With small groups however, this condition often binds, as illustrated by the following example. If \(n = 3, \pi = .5, r = 1.2\) and \(\bar{K} = 1\), the largest individual loan is .42 and the largest group loan is .27. As long as the loan size is below .25, only one successful project is required for group repayment and welfare is higher under the group contract. For \(L = .24\), the difference in benefits is maximized with \(U^g = .38\) and \(U^i = .21\). For higher values of \(L\), group repayment requires at least 2 successes and per member benefits fall below those from individual contracts. As \(L\) increases marginally to .25, \(U^g\) falls abruptly to .05 while \(U^i\) is only slightly lower at .20. If \(n = 4\), with all other parameters at the above values, benefits from group loans are higher than under individual loans for all loan sizes offered under both types of contracts. We return to the effects of group size in Section 5.
4 Social sanctions

We have now obtained two main results. First, in the absence of social sanctions, joint liability cannot increase credit outreach. Second, borrowers who need small loans prefer group lending. In this section we explore how these results are modified if groups can augment the enforcement capacity of the bank with social sanctions. Our treatment of these sanctions is standard and follows Besley and Coate (1995). Our contribution is to demonstrate how they interact with borrower wealth to determine the benefits from group loans. Given the popular association of microcredit groups with these types of informal sanctions, we believe this is a useful extension of our model.

We model social sanctions as a utility cost $\gamma$ on members who are able to contribute their fair share of the group loan but choose not to do so. Since the size of this sanction does not depend on the number of contributing members, it is best interpreted as a fall in a borrower’s status within the community or a partial or total exclusion from group activities. Members with unsuccessful projects are not sanctioned since they have zero returns and cannot contribute.

We now modify the feasibility constraint in (5) for a group loan of size $L$:

$$\frac{n}{j} \frac{r}{B_j} L \leq \min(\rho, \bar{K} + \gamma).$$

A group-lending contract with per-member loan size $L$ is feasible if there exists some $j$ for which members with successful projects are both able and willing to pay their share of the loan. This requires that each member’s contribution be less than the return $\rho$ and that it be lower than the sum of bank and social sanctions. We can think of these two constraints as the liquidity and group incentive constraints respectively. We ignored the liquidity constraint in previous sections because $\rho$ was larger than the total sanction $\bar{K}$. This need not be the case when total sanctions are $K + \gamma$.

We show that the effects of social sanctions on credit outreach are ambiguous. They do improve

\textsuperscript{10}There is no distinction between the total sanction and the sanction per contributing member in the Besley-Coate model because with groups of two, each member can only be sanctioned once.

\textsuperscript{11}One might ask whether it is plausible that group members are sanctioned if all members are successful and default collectively. We could allow for no sanctions in this case, but this would complicate the model and for moderately sized groups, this is a low probability event.

\textsuperscript{12}In section 2, our floor on project returns given by (4) and $\bar{L} < 1$ together implied this.
repayment incentives but there are reasonable conditions under which even arbitrarily large sanctions do not support larger loans than those available as individual contracts. In contrast, higher social sanctions always improve borrower welfare for those with group loans because, unlike bank sanctions, they are never imposed in equilibrium. Since project outcomes are observed within groups but not outside them, banks sanction whenever default occurs whereas groups use sanctions to induce repayment without punishing those with failed projects.

Analogous to our previous analysis of group loans, the optimal contract corresponds to the smallest value of \( j \) for which (10) is satisfied. If fewer than this number of successes occur, the group defaults, the bank sanctions all members and no social sanctions are used. In all other cases, the group repays the loan and each successful member contributes an equal share towards repayment. Comparing (5) and (10), we see that social sanctions relax the incentive compatibility constraint and allow a larger share of project returns to be extracted from each successful member. This increases possibilities for risk pooling and the group repays the bank loan in states with fewer successful projects than in the absence of social sanctions.

Our next result outlines the conditions under which social sanctions allow all individual loans to be supported as group loans, thereby extending credit outreach:

**Proposition 4.** Let social sanctions \( \gamma \geq \bar{\rho} - \bar{K} \), where \( \bar{\rho} \) is the minimum project return and \( \bar{K} \) is the maximum bank sanction. Then \( \bar{L} \), the largest individual loan, is a feasible group loan for groups of size \( n = 2 \) or for a project success probability \( \pi \leq 1/4 \). In contrast, for large enough \( \pi \), \( \rho = \bar{\rho} \) and \( n > 2 \), there always exist loan sizes which will be offered as individual loans but not as group loans.

The proof is in the Appendix and is based on the following idea. When \( \gamma \geq \bar{\rho} - \bar{K} \), repayment is constrained by liquidity and not by incentives. The minimum return \( \bar{\rho} \), is decreasing in \( \pi \) because risky projects must have higher returns to make them worthwhile. Social sanctions allow these to be extracted under group lending contracts. If the expected number of successful projects is an integer, the bound on \( \pi \) can be considerably relaxed and \( \bar{L} \) is a feasible group loan whenever \( \pi \leq 1/2 \).13 The proposition once again illustrates that the two-member groups that have been the focus of earlier studies on group lending are a special case for which high enough sanctions always extend outreach.

13When \( n\pi \) is an integer, the mean, median and mode of a binomial distribution coincide and we are able to exploit this feature to relax the condition on \( \pi \). Details are in the Appendix.
While social sanctions do not necessarily improve credit access, they always raise benefits from group loans by reducing the bank sanction required for each loan size. To see this, consider a group contract \((L, \frac{r}{B_j}, K_j)\) that is feasible when \(\gamma = 0\). For small \(\gamma\), the same loan remains feasible, but the bank sanction needed to enforce it is reduced by \(\gamma\) to:

\[
K_j' = K_j - \gamma = \frac{n}{j} \frac{r}{B_j} L - \gamma
\]  

(11)

For \(\gamma \geq K_j\), social sanctions substitute fully for bank sanctions and \(K_j' = 0\).

Social sanctions can also improve welfare through higher repayment rates by relaxing the constraint in (10) and thereby reducing the number of successes in the optimal group contract. Utility from the group loan is now

\[
U^g_j = \pi \rho - r - (1 - B_j) K_j'
\]  

(12)

This discussion is summarized in our next result. A complete proof is in the Appendix.

**Proposition 5.** Social sanctions result in higher welfare for borrowers already using group loans. If the largest per-member loan requires at least two or more successful projects within the group for repayment, social sanctions also raise repayment rates.

5 Group size

The literature on group lending has focussed on two-member groups and fixed loan sizes. Our previous results have shown that group size does influence both access to and the benefits from group loans. In this section, we explore the interaction between borrower wealth, project characteristics and group size. The following result shows that the effects of group size on welfare depend both on the size of the loan and the project success probability.

**Proposition 6.** If \(\pi \geq \frac{n(n-1)}{1+n(n-1)}\), the largest loan available to a group of size \(n\) is both feasible and generates higher per-member welfare in smaller groups. In contrast, for sufficiently small
loans and no social sanctions within groups, a fall in group size lowers welfare and two-member groups are never optimal.

The proof of the second part of the proposition is intuitive. Group loans raise welfare whenever they lead to lower expected sanctions. If a loan is small enough to require only one success for repayment, large groups are favored because the probability of one or more successes, \( B(1, n, \pi) \), is increasing in \( n \). The result will hold for zero or small social sanctions. If social sanctions are large, they could substitute fully for bank sanctions and since no sanctions will then be implemented in equilibrium, larger groups would have no added benefits.

For the first part of the result, we know from Lemma 1 that when the project success probability is greater than \( \frac{n(n-1)}{1+n(n-1)} \), the largest loan offered to a group of size \( n \) requires all members to succeed. This is more likely for smaller groups. To complete the argument, we need to show that loans of this size are feasible for a smaller group. This is done in the Appendix.

Moving away from these limit cases we find non-monotonic effects of group size on repayment rates and borrower welfare for given loan and project characteristics. Figure 1 illustrates this for a loan size \( L = .4 \) and a project success probability, \( \pi = .5 \). Bank and social sanctions are 1 and .5 respectively and the risk free interest rate is 20 per cent. For all group sizes between 2 and 20, we show the repayment rate, the fraction of successful projects and the net benefit, \((U^g - U^i)\), from the optimal group contract. Group sizes in microfinance programs are typically in this range.

In this example, an increase in group size initially lowers welfare. Repayment in a group of 2 requires one or more projects to succeed, and this happens with probability equal to \( \frac{3}{4} \). For a group of 3, repayment occurs when at least 2 projects succeed and this probability is \( \frac{1}{2} \). As group size increases locally around 3, so do repayment rates and benefits from the group loan. When \( n=6 \), repayment rates and net gains fall once more, since repayment now occurs when 3 or more projects succeed and this probability is roughly \( \frac{2}{3} \). The scissor-like patterns in the figure mirror the changes in the value of \( j \) on which the optimal group contract is based.

It is useful to briefly summarize what we have learned about group-size effects. We know that when project success probabilities approach 1, a decrease in group size raises welfare. We have also shown that for sufficiently small loans, two-member groups are never optimal. This is related to Proposition 3 in Section 3. Proposition 4 on the hand, shows that two-member groups often maximize credit outreach. This contrast highlights a central theme in this paper, namely
Figure 1: Group size effects ($\pi = .5, L = .4, \tilde{K} = 1, \gamma = .5, r = 1.2$)

that the characteristics of groups that increase gains from group loans are not always the ones that increase access to them. For other parameter values, the benefit from group lending could either increase or decrease in the size of the group. In small groups, additional members can dramatically change the fraction of successful projects needed for repayment. Changes in this fraction are mirrored in oscillating repayment rates and expected benefits from group loans. General results on group size are elusive and need to be explored further.

6 Conclusion

This paper is motivated by the now common observation that group lending with joint liability is more successful for moderately poor households than for the very poor. We highlight one mechanism which generates this pattern of benefits by examining how credit contracts vary by loan size. We characterize conditions under which individual contracts support larger loans than joint liability contracts. We also show that for those with access to group loans, wealthier households can pool risk more effectively. This results in higher repayment rates, infrequent bank sanctions and greater benefits from group lending.

Our paper extends the standard approach in the group lending literature in two important
respects. First, we move away from two-person groups to arbitrary group sizes. Second, we allow the nature of the credit contract to vary by project and borrower characteristics. We believe these extensions are significant, not least because both credit contracts and group sizes observed in the microfinance sector vary considerably. To focus on repayment incentives we ignored the effects of group lending on borrower selection and on project and effort choices. There may be interesting interactions between wealth and credit contracts along these dimensions that remain unexplored, so generalizing existing models to address these aspects of group lending could be fruitful.

It is difficult to link our model tightly to existing empirical studies on group lending because these rarely focus on the relationship between loan size and impact. Our analysis suggests that this is a useful direction to pursue in empirical research. In terms of policy, our results suggest that effective strategies to expand rural credit require a mix of contractual arrangements, and an exclusive focus on group lending may be misplaced.

Appendix

Proof of Lemma 1

Let \([n\pi]\) denote the largest integer below \(n\pi\) and \([n\pi]\) the smallest integer above it. The function \(jB_j\) takes the value zero at \(j = 0\) and is positive for all \(j > 0\). We begin by showing that it is decreasing to the right of \([n\pi]\). We then establish that it is single-peaked by showing that, if for some \(j^* < [n\pi]\), \(jB_j > (j + 1)B_{j+1}\), then this relationship holds for all \(j\) in the range \(j^* \leq j \leq [n\pi]\). Finally, for the case where \([n\pi] = n\), we derive the lower bound on \(\pi\) given in the lemma for which \(jB_j\) is increasing throughout.

1. \(jB_j\) is decreasing to the right of \(j = [n\pi]\):

Consider \(n > j \geq [n\pi]\). The function \(jB_j\) is strictly decreasing at \(j\) if \(jB_j > (j + 1)B_{j+1}\). The LHS can be written as \(jB_{j+1} + j\pi_j\) and rearranging terms gives us the condition:

\[
j\pi_j > B_{j+1}
\]

The binomial probabilities, \(\pi_j\) and \(\pi_{j+1}\) are respectively

\[
\pi_j = \pi^j(1 - \pi)^{n-j} \frac{n!}{j!(n-j)!}
\]
and
\[ \pi_{j+1} = \pi^{j+1}(1-\pi)^{n-j-1} \frac{n!}{(j+1)!(n-j-1)!}. \]

Writing \( \pi_j \) in terms of \( \pi_{j+1} \) and multiplying by \( j \), we get
\[ j\pi_j = \left( \frac{j}{n-j} \frac{1-\pi}{\pi} \right) (j+1)\pi_{j+1} \]

But \( \frac{j}{n-j} \frac{1-\pi}{\pi} \geq 1 \) if \( j \geq n\pi \), which implies
\[ j\pi_j \geq \pi_{j+1} + j\pi_{j+1}. \]

In the same way, we can express \( j\pi_{j+1} \) as a function of \( \pi_{j+2} \):
\[ j\pi_{j+1} = \left( \frac{j}{n-j-1} \frac{1-\pi}{\pi} \right) (j+2)\pi_{j+2}. \]

Since \( \frac{j}{n-j-1} \frac{1-\pi}{\pi} > 1 \) for \( j \geq n\pi \),
\[ j\pi_{j+1} > \pi_{j+2} + j\pi_{j+2} \]

Combining the expressions for \( j\pi_j \) and \( j\pi_{j+1} \) we get
\[ j\pi_j > \pi_{j+1} + \pi_{j+2} + j\pi_{j+2}. \]

For all \( k \leq n-1 \), we can continue expressing \( j\pi_k \) as a function of \( \pi_{k+1} \), and we obtain:
\[ j\pi_j > \sum_{k=j+1}^{n} \pi_k = B_{j+1}. \]

As a result, \( jB_{j} \) is strictly decreasing to the right of \( \lceil n\pi \rceil \).

2. \( jB_{j} \) is single-peaked to the left of \( \lfloor n\pi \rfloor \):

We use the property of a Binomial distribution that the mode \( M \) of the distribution is either \( \lfloor n\pi \rfloor \) or \( \lceil n\pi \rceil \) (Kaas and Buhrman, 1980).

Let us first consider any value \( j \), \( 1 \leq j \leq \lfloor n\pi \rfloor \) for which \( jB_{j} \) is increasing at \( j \). From (13), this implies that \( j\pi_j \leq B_{j+1} \). But since \( \pi_j \) is maximized at the mode, which is at least as large as \( j \), the LHS \( j\pi_j \) is increasing for all \( j < j \). The upper tail probability, \( B_{j+1} \) is strictly decreasing in \( j \) throughout. It follows that \( \forall j < j \), \( jB_{j} \) must be increasing at \( j \).

Now suppose that \( j'' \leq \lfloor n\pi \rfloor \) is the smallest value of \( j \) at which \( B_{j} \) is decreasing, or equivalently, \( j\pi_j \geq B_{j+1} \). Since \( j'' \) is less than the mode, this inequality must hold for all \( j \) between \( j'' \) and the mode by the same reasoning given above, i.e. \( j\pi_j \) is increasing in \( j \) until the mode and \( B_{j+1} \) is strictly decreasing in \( j \). If no such value exists, \( jB_{j} \) is increasing throughout.
3. For \( jB_j \) to be increasing throughout, we require the mode to be greater than \((n - 1)\) and in addition, using (13), \((n - 1)\pi_{n-1} < \pi_n\). We can rewrite this inequality as

\[
n(n - 1)\pi^{n-1}(1 - \pi) < \pi^n
\]

or

\[
\pi > \frac{n(n - 1)}{1 + n(n - 1)}
\]

However, at the values of \( \pi \) satisfying the above condition, \( n\pi > n - 1 \) so that \( M \geq n - 1 \).

**Proof of Proposition 1**

Recall that \( U^g_j \) is the benefit to the group from a contract in which repayment occurs whenever there are \( j \) or more successes. The contract is given by \((L, \frac{r}{B_j}, K_j)\), where \( \frac{r}{B_j} \) is the interest rate charged and \( K_j = \frac{n}{j} \frac{r}{B_j} L \), is the sanction applied to each member when the group defaults. We will show that \( U^g_j > U^g_{j+1} \) whenever both contracts are feasible.

We restrict our consideration to contracts with \( j + 1 \leq \lceil n\pi \rceil \) (the smallest integer greater than or equal to \( n\pi \)) since we know by Lemma 1 that contracts with higher values of \( j \) are never optimal. We can write the difference in benefits from \( U^g_j \) and \( U^g_{j+1} \) as

\[
U^g_j - U^g_{j+1} = \Delta_{j-1} + \Delta_j + \Delta_{j+1}.
\]

\( \Delta_{j-1} \) is the difference in benefits from the two contracts for states with fewer than \( j \) successes, \( \Delta_j \) is this difference when exactly \( j \) successes occur and \( \Delta_{j+1} \) is the difference in benefits when more than \( j \) successes occur. In the first case, the group is sanctioned under both contracts, in the third case, it is not sanctioned under either contract, and in the second case, when exactly \( j \) successes occur, it is sanctioned under \((L, \frac{r}{B_{j+1}}, K_{j+1})\) but not under \((L, \frac{r}{B_j}, K_j)\). These three differences are given by

\[
\Delta_{j-1} = \sum_{l=0}^{j-1} \pi_l \left[ (l\rho - nK_j) - (l\rho - nK_{j+1}) \right] = n(1 - B_j)(K_{j+1} - K_j)
\]

\[
\Delta_j = \pi_j \left[ (j\rho - \frac{nLr}{B_j}) - (j\rho - nK_{j+1}) \right] = \pi_j nK_{j+1} - \pi_j \frac{nLr}{B_j}
\]

\[
\Delta_{j+1} = \sum_{l=j+1}^{n} \pi_l \left[ \frac{nLr}{B_{j+1}} - \frac{nLr}{B_j} \right] = nLr \frac{B_j - B_{j+1}}{B_j B_{j+1}} B_{j+1} = \pi_j \frac{nLr}{B_j}
\]
Summing the three terms we obtain

\[ U^g_j - U^g_{j+1} = n(1 - B_j)(K_{j+1} - K_j) + \pi_j nK_{j+1} \]

and substituting for \( K_j \) and \( K_{j+1} \), we see that \( U^g_j - U^g_{j+1} > 0 \) as long as

\[ n^2 L r \left[ \frac{\pi_j}{(j+1)B_{j+1}} + (1 - B_j) \frac{jB_j - (j+1)B_{j+1}}{jB_j (j+1)B_{j+1}} \right] > 0 \]

or

\[ \pi_j + (1 - B_j) \frac{jB_j - (j+1)B_{j+1}}{jB_j} > 0 \]

Using \( B_{j+1} = B_j - \pi_j \) we need

\[ (j + 1)\pi_j > B_j \left[ 1 - B_j + \pi_j \right] \]

or

\[ (j + 1)\pi_j > B_j \left[ \pi_0 + \pi_1 + \cdots + \pi_j \right] \]

The sum of the \((j + 1)\) terms on the RHS of the above inequality is always smaller than the LHS if \( \pi_j \) is increasing in \( j \). Since any contract under consideration must have \((j + 1) < \lfloor n\pi \rfloor\), we always have \( j \leq \lfloor n\pi \rfloor \) and we know that \( \pi_j \) is increasing in this range. So \( U^g_j > U^g_{j+1} \).

**Proof of Proposition 4**

We start with the first part of the proposition and show that under the stated conditions on \( n \), \( \pi \) and \( \gamma \), the largest individual loan \( \bar{L} \) can be offered as a group contract even when the project return takes its minimum value of \( \bar{\rho} \).

Let \( \gamma \geq \bar{\rho} - \bar{K} \) and \( \rho = \bar{\rho} \). We know from (10) that repayment is constrained by the project return \( \bar{\rho} \). \( \bar{L} \) is therefore feasible if there exists \( j \) such that

\[ \frac{n}{j} \frac{r}{B_j} \bar{L} \leq \bar{\rho} \]

Substituting for \( \bar{L} \) from (3) and for \( \bar{\rho} \) from (4), this condition can be rewritten as

\[ \frac{n}{j} \frac{\pi}{B_j} \bar{K} \leq \frac{1 - \pi}{\pi} \bar{K} + \frac{r}{\pi} \]
Now, since $\bar{L} = \frac{\pi K}{r} < 1$, $\frac{r}{\pi} > \bar{K}$. It is therefore enough to show that
\[ \frac{n \pi}{j B_j} K \leq \frac{1 - \pi}{\pi} K + K, \]
or equivalently
\[ \frac{n \pi}{j B_j} \leq \frac{1}{\pi}. \] (14)

If $n = 2$, set $j = 2$ and since $B_2 = \pi^2$, (14) holds with equality, so a loan of size $\bar{L}$ is always feasible for a two-person group.

The rest of the proof uses a well-known result on the relationship between the mean ($n\pi$) and the median ($m$) of a binomial distribution (Kaas and Buhrman, 1980). This states that the median is either $\lceil n\pi \rceil$, the smallest integer above the mean or $\lfloor n\pi \rfloor$, the largest integer below the mean. For integer values of the mean, $m = n\pi$.

We will first prove the result for integer values of $n\pi$ and then consider non-integer values.

If $n\pi$ is an integer, set $j = n\pi = m$. The inequality in (14) is now
\[ \pi \leq B_m \]
But since $m$ is the median, $B_m \geq \frac{1}{2}$ so with $\pi \leq \frac{1}{2}$ this is always true. For integer values of $n\pi$, we therefore have an upper bound for $\pi$ of $\frac{1}{2}$, which is larger than stated in the proposition.

Now consider non-integer values of the mean. The median in this case must either be $\lceil n\pi \rceil$ or $\lfloor n\pi \rfloor$. If the median $m = \lceil n\pi \rceil$ then set $j = \lceil n\pi \rceil$ in (14). Since $n\pi < \lceil n\pi \rceil$, the LHS of (14) is smaller than when $n\pi$ is an integer and the result goes through a fortiori.

If instead $m = \lfloor n\pi \rfloor$, consider first $j = \lfloor n\pi \rfloor = 1$. In this case $n\pi$ is strictly less than 1 and (14) holds if $\pi < B_1$. But $B_1 = 1 - (1 - \pi)^n$ so this is always true. If $\lceil n\pi \rceil > 1$, set $j = \lfloor n\pi \rfloor = m$. We can now rewrite (14) as
\[ \frac{n \pi}{m B_m} \leq \frac{1}{\pi}. \]
But since $n\pi < \lfloor n\pi \rfloor$, it is enough to show that
\[ \pi \leq \frac{m}{m + 1} B_m \]
The ratio $\frac{m}{m + 1}$ is increasing in $m$ and takes its minimum value of $\frac{1}{2}$ when $j = 2$. The minimum value taken by $B_m$ is also $\frac{1}{2}$, so the above inequality holds whenever $\pi \leq \frac{1}{2}$. 

20
We now turn to the second part of the proposition and show that for large enough \( \pi \) and \( n > 2 \), there always exist loan sizes that will be offered as individual but not group loans.

By Lemma 1, when \( \pi > \frac{n(n-1)}{1+n(n-1)} \), the function \( jB_j \) is maximized at \( j^* = n \). The largest feasible group loan therefore has a repayment rate \( B_n = \pi^n \). If the group loan is equal to \( \bar{L} \), each successful member in the group is required to contribute \( \frac{r}{\pi^n} \bar{L} \), while the minimal return to the project \( \bar{\rho} \) is

\[
\bar{\rho} = (1 - \pi) \bar{K} + \frac{r}{\pi} = \frac{(1 - \pi) r \bar{L}}{\bar{L}} + \frac{r}{\pi}
\]

Required payments are therefore higher than \( \bar{\rho} \) whenever

\[
\bar{L} > \frac{\pi^n}{\bar{L} - \pi^n - 1 + \pi^n}
\]

If \( n > 2 \), the RHS of the above expression is smaller than one and there exist loan sizes \( \bar{L} \) that are not feasible group loans even for arbitrarily high social sanctions.

#### Proof of Proposition 5

Let \((L, \frac{r}{B_j}, K_j)\) be the optimal group contract for a loan of size \( L \) when \( \gamma = 0 \). This contract results in repayment when there are \( j \) or more successful projects in a group so the repayment rate is \( B_j \).

Denote by \( \tilde{j} \), the number of successes in the optimal contract with \( \gamma > 0 \). Without loss of generality, we can restrict ourselves to values of \( \gamma \) less than \( \rho - \bar{K} \). Beyond this value, contributions are constrained by project returns \( \rho \) and higher social sanctions have no effect on incentives.

Since the increase in \( \gamma \) relaxes (10), we know that \( \tilde{j} \leq j \). If, for all feasible loans and \( \gamma = 0 \), the parameters \( \rho \) and \( \bar{K} \) are large enough to enable repayment with only one success in the group, \( \tilde{j} = j = 1 \) and repayment rates cannot change since \( B_1 \) is the maximum possible repayment rate. We see from (12) that the benefit from a group loan increases.

If, on the other hand, some group loans require more than one success for repayment, repayment rates also increase, or equivalently, \( \tilde{j} < j \) for some loan sizes. Denote by \( L_{\gamma}^1 \) the largest loan per member that can be repaid with only one success in the group when social sanctions are equal to \( \gamma \). From (10), this is given by
\[ L_1^g = (\bar{K} + \gamma) \frac{B_1}{nr} \]

Moving from zero to positive social sanctions, this increases by \( \gamma \frac{B_1}{nr} \). All those with loans in the interval \((\bar{K} \frac{B_1}{nr}, (\bar{K} + \gamma) \frac{B_1}{nr}]\) have higher repayment rates than in the absence of social sanctions, since they now require only one success to be reimbursed while they previously required at least two successes. For these loans \( j = 1 < j \). More generally, groups with loan sizes in the interval \((\bar{K} \frac{iB_i}{nr}, L_j^g(\gamma)]\) required at least \((j + 1)\) success for repayment in the absence of social sanctions and now require at most \(j\) successes. Since wealth has a continuous distribution on \((0, 1)\), all these intervals are non-empty. Repayment rates therefore go up for some loan sizes, causing an increase in the overall repayment rate.

**Proof of Proposition 6**

For a group of size \(n\), we know from Lemma 1 that if \(\pi \geq \frac{n(n-1)}{1+n(n-1)} = \bar{\pi}(n)\), the largest loan requires all \(n\) members to succeed. Since \(\pi(n)\) is increasing in \(n\), smaller groups will also require all members to succeed if \(\pi \geq \bar{\pi}(n)\). For these values of \(\pi\), we see from (5) that the largest loan available to groups with \(n\) members is

\[ \bar{L}_n^g = \frac{\bar{K} \pi^n}{r} \]

which is decreasing in \(n\). The largest loan available to a group of size \(n\) is therefore also available to a group of size \(n-k\).

For a loan of size \(\bar{L}_n^g\), the difference in borrower benefits for groups with \((n-k)\) members and \(n\) members is at least

\[ r \bar{L}_n^g \left[ \frac{1 - \pi^n}{\pi^n} - \frac{1 - \pi^{n-k}}{\pi^{n-k}} \right] > 0. \]

This is a lower bound on the relative benefits of the smaller group since, for a loan of size \(\bar{L}_n^g\), a group with \((n-k)\) members may need less than \((n-k)\) successes for repayment and benefits increasing in the repayment rate.

We now show that borrower benefits are increasing in group size for very small loans. Consider a loan size \(L\) which is repaid by a group of size \(n\) and a group of size \(n-1\) with only one success.
The difference in benefits to a borrower in an \( n \) member group relative to an \((n - 1)\) member group is given by

\[
U_n^g - U_{n-1}^g = (1 - B(1, n - 1, \pi)) \frac{(n - 1)Lr}{B(1, n - 1, \pi)} - (1 - B(1, n, \pi)) \frac{nLr}{B(1, n, \pi)}
\]

\[
= Lr \left[ \frac{(1 - \pi)^{n-1}}{1 - (1 - \pi)^{n-1}}(n - 1) - \frac{(1 - \pi)^n}{1 - (1 - \pi)^n} \right]
\]

\[
= Lr \frac{(1 - \pi)^{n-1}}{(1 - (1 - \pi)^{n-1})(1 - (1 - \pi)^n)} [n\pi - 1 + (1 - \pi)^n]
\]

The term in parenthesis can be written as

\[
n\pi - (1 - (1 - \pi)^n) = \sum_{l=0}^{n} l\pi_l - \sum_{l=1}^{n} \pi_l
\]

which is strictly positive for all \( n \geq 2 \).

References


