THE ETHICS OF INTERTEMPORAL DISTRIBUTION IN A WARMING PLANET

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Abstract This paper evaluates, from the ethical viewpoint, current work by economists on intergenerational resource allocation in the presence of global warming. We begin by attempting to elucidate the debate that has recently occurred on the appropriate choice of the discount rate. We offer three justifications for maximizing the discounted sum of generational utilities, and find only one of these to be a satisfactory justification of that practice: the possibility that the human species may become extinct. This implies that a very small discount rate (large discount factor) should be used. We argue that the justification for discounting, inherent in the approaches taken by many economists, is that of 'the present generation of hegemon,' which is unacceptable. The role of the Ramsey equation in deducing the discount rate in these theories is explained. As an alternative to discounted utilitarianism, we propose a principle of sustainability; we describe optimal paths that have been calculated for the sustainabilitiarian (Rawlsian) objective function, and paths that will sustain growth in welfare, at a positive rate. We report results concerning optimal paths when the uncertainty of existence of future generations is taken into account. In sharp contrast to the utilitarian model, it turns out that under some conditions, the 'sustainabilitarian' can ignore the uncertainty regarding the date at which humans become extinct. There is a striking difference between the solutions of the discounted utilitarian program and the sustainabilitarian program under uncertainty.

Keywords Climate change · Global warming · Intertemporal optimization · Social discount rate · Discounted utilitarianism · Rawlsian justice · Sustainability

JEL Codes D63 · D90 · H43

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1 The Urgency of the Matter

Nicholas Stern (2008) writes, "Greenhouse gas (GHG) emissions are externalities and represent the biggest market failure that the world has seen." The simplest proxy for the damage caused by GHG emissions is global temperature. It is not temperature, as such, that is the problem: rather, it is the consequences of higher temperature concerning water that can be disastrous. A rise of global temperature by 5°C. would bring about a massive increase in the intensity of storms and of drought, as well as a rise of the sea level by perhaps 10 m. It is not unlikely that at least half of earth's species would be wiped out.

Global temperature is a function of the stock of CO_2 in the atmosphere. Currently, that stock is 390 parts per million. If the stock is stabilized at 450 ppm, then climate scientists estimate that global temperature will stabilize at somewhere between 1°C and 3.8°C above the current temperature, where this represents a 90% confidence interval. (That is, with probability 0.9, the temperature will stabilize somewhere in the interval [1°, 3.8°] above where it is now.) If stabilization occurs at 550 ppm, the 0.9 confidence interval for global temperature is [1.5°, 5.2°]. The large lengths of these intervals reflect the uncertainty in our knowledge of the effect of carbon emissions on temperature, which is a highly complex process, involving feedbacks of various kinds of uncertain magnitudes. (How much do the deep oceans absorb carbon from the air?) The world's largest computers are currently being used to simulate these effects.

If we continue with 'business as usual,' that is, without controlling emissions more than we currently do, we will reach *at least* 750 ppm CO_2e by the end of this century. The 0.9 confidence interval associated with 750 ppm is [2.2°C, 6°C] warmer than today. Life as we know it would end on the planet if temperatures at the upper end of this interval are realized.

Currently climatology maintains that it is impossible completely to stabilize the level of carbon concentration in the atmosphere unless emissions become zero, something which is not feasible. [See Matthews and Caldeira (2008).] The key question therefore becomes how rapidly concentration increases. If emissions are cut drastically, then the stock of atmospheric carbon will increase slowly, and human society will have time to adjust. If we reach an increase in temperature of 5°C. in 5,000 years, we will have plenty of time to move our cities to the polar regions, which by then will be pleasantly warm (snow and glaciers will have disappeared, along with polar bears).

These facts raise two important ethical issues—of intertemporal and intratemporal justice. Carbon emissions are a public bad: How should rights to produce that bad be allocated over the present and future generations of man? That is the intertemporal issue; the intratemporal one is how the rights to emit carbon should be allocated across nations today, in the present and next generation. The proper solution of the second problem presupposes the solution to the first. If we solve the intertemporal problem, then we know the emissions ethically permitted for our generation. We then can ask how that permitted amount should be allocated across nations today. In this paper, I focus upon the intertemporal ethical issues and their consequences for policy.

2 Arguments for Discounted Utilitarianism

The topic of the evaluation of social welfare in an intertemporal context is a well-worked one in welfare economics; the literature, of a formal sort, begins with Frank Ramsey's (1928) famous paper. I will summarize Ramsey's contribution presently as it has been applied to the climate-change problem.

The problem in its general form can be stated as follows. First, we adopt a model where each generation is represented by a single agent: that is, we abstract away from distributional issues of humans who live contemporaneously. One can think of the single agent as being the 'average person.'

Let us represent the amenities available to an agent at date t as a vector z_t. Components of this vector are consumption, leisure, health, education level, biospheric quality, and so on. Suppose the welfare that the agent enjoys or sustains with this vector is $u(z_t)$, where u is a 'utility function' common to all agents (i.e., it reflects human nature). An *amenity path* is an infinite sequence of these vectors, one for each generation: thus, $(z_1, z_2, z_3, \ldots)$. Such a path may eventually become an unending sequence of zero vectors, reflecting the fact that the species has disappeared and consumes nothing.

Current scientific knowledge can be summarized by saying that we believe that there is a certain set of *feasible consumption paths*, denoted Z. Any path in Z can be implemented, from the technological and physical/biological viewpoint. Of course, different paths implement different trade-offs regarding the consumptions of the present and future generations. What does 'feasible' mean in this sentence? Feasibility takes account of how much we invest in research and development, and how much technology improves as a consequence of that investment. All aspects of technological change, environmental damage, and human knowledge are endogenous, in the sense of being generated by investments in research, the environment, and education at earlier dates. To specify the set Z we will normally write down a set of dynamic equations which relate how all data about the economy at any date $t + 1$ appear as a result of what has happened (e.g., investment of various kinds) at previous dates. Heroically, we will assume, here, that all these dynamic relationships are known. Thus, although we do not know what the technology will be in 200 years, we assume that we can estimate how much labor productivity will increase if we follow a given path of investment in education and in research and development. These estimates are provided by historical data, which enable us to track how labor productivity has increased in the past as a result of these investments. A more complex problem would be posed if we wanted to build in uncertainty in our knowledge of these relationships, so that the set Z would only be specified up to what that uncertainty would imply.

Our ethical view is summarized by an intertemporal welfare function, which aggregates the welfare levels of the various generations into an index of social welfare.[1] The arguments of this function, W, are the vectors of generational utilities induced by an amenity path. Thus, at the path $\mathbf{z} = (z_1, z_2, z_3, \ldots)$ the social welfare will be

$$W(u(z_1), u(z_2), u(z_3), \ldots).$$

It is important to underscore that W embodies our *ethical views* about how the welfare of one generation should be traded off against the welfare of other generations, and the set Z embodies *facts about the world*—that is, facts of nature (biology, physics) and facts about human knowledge and learning. (I would say that the utility function u also embodies a fact about the world, namely, how goods aggregate to produce human welfare.) Many discussants in the climate-change debate do not respect this division of labor—they want the function W to embody facts about the world, not only ethical views.

[1] More generally, we may order infinite utility streams in a way that cannot be represented by a social welfare function. This issue has been much studied in the recent literature, but we ignore it for our purposes here. See, for example, Basu and Mitra (2003).

The problem of the social planner is to choose that feasible amenity path (in Z) which maximizes social welfare: that is to choose $\mathbf{z} = (z_1, z_2, \ldots)$ to

$$\text{maximize } W(u(z_1), u(z_2), \ldots) \text{ subject to } \mathbf{z} \in Z.$$

In terms of the just-stated division of labor, the problem is to attain the highest possible ethical index subject to the constraints placed upon us by facts about the world.

Let us take an example. Consider the *utilitarian* social welfare function, which is given by

$$W^U(u_1, u_2, \ldots) = \sum_{t=1}^{\infty} u_t,$$

where u_t is the utility of generation t. The problem for a utilitarian is to find the amenity path that maximizes the sum of utilities over all future dates.

If, indeed, W should be justified by purely ethical arguments, then we would expect that W should be *symmetric* in its arguments: that is, it should treat all generations equally. Why, indeed, should any generation have a *right* to more utility than another—why should we place a higher social value on one generation's receiving more utility than another? For example, symmetry means that if there were just two generations, then it should be the case that the planner is indifferent between giving the first generation 7 and the second one 10 units of utility, and the other way around: that is, for any numbers u and v it should be true that

$$W(u, v) = W(v, u).$$

Indeed, this is true of the utilitarian social welfare function: it does not matter in what order we list the utility numbers, their sum will be the same.

I can now state the social welfare function which is employed by almost all the economists working in the area of climate change: it is $W^{\text{DU}}(u_1, u_2, \ldots) = \sum_{t=1}^{\infty} \rho^{t-1} u_t$, where ρ is a number less than one, called the *discount factor*.[2] The superscript 'DU' stands for 'discounted utilitarian.' Sometimes the discount factor is written as $\rho = \frac{1}{1+\delta}$, and δ is called the *discount rate*.

One immediately notes that this social-welfare function is *not* symmetric: it places less value on the utility of generations, the farther distant they are from the present. What could justify such a move? There are a number of possible justifications for adopting a discounted-utilitarian intertemporal ethic. I will partition them into three categories, which I discuss in turn.

A. The present generation as hegemon

I present two justifications in this category.

A.1 The infinitely lived consumer

Suppose we think of the generations of mankind as ethically analogous to different periods in the life of a single individual. Suppose this individual discounts her own future utility at a rate ρ: that is, if the path of her consumption of amenities is given by $\mathbf{z} = (z_1, z_2, \ldots)$ then her 'lifetime' utility will be $u(z_1) + \rho u(z_2) + \rho^2 u(z_3) + \cdots$. The number ρ reflects the individual's degree of impatience: if $\rho = 0.9$, this individual would trade-off one unit of utility next year for slightly more than 0.9 units of utility today.

[2] This is the discount factor on *utility,* to be distinguished from a *consumption* discount factor, introduced below.

If we choose ρ to reflect an individual's subjective rate of time preference (impatience) in this manner, and if we use the discounted-utilitarian social welfare function, then the different generations of human society are being treated as if they were different periods in the life of this individual. In particular, if ρ is 'small,' the individual is 'impatient,' and we will discount the welfare of future generations just as a quite impatient individual discounts his own future welfare.

A.2 The altruistic parent

Suppose that each adult has one child and cares about the utility of her child. Suppose the utility of the parent, who is an adult at date t, is a function of the amenities she enjoys and of her child's utility, and takes a linear form, where she places a weight of $\rho < 1$ on her child's utility:

$$U_1(z_1, U_2) = u(z_1) + \rho U_2. \tag{1}$$

Here, U_1 is the *overall* utility function of the parent, which is a function of what we might call her personal standard of living, which is a function u of the amenities she consumes, and of the *overall* utility of her child.

But the same equation holds for generation 2:

$$U_2(z_2, U_3) = u(z_2) + \rho U_3. \tag{2}$$

Substituting this expression for U_2 into (1), we have:

$$U_1(z_1, U_2) = u(z_1) + \rho u(z_2) + \rho^2 U_3. \tag{3}$$

Indeed, we can continue to expand the right-hand side of this equation indefinitely, which gives the formula:

$$U_1 = \sum_{t=1}^{\infty} \rho^{t-1} u(z_t). \tag{4}$$

The right-hand side of Eq. (4) is just the discounted-utilitarian formula.

In other words: If all parents are altruistic, and conceive of their overall welfare as their own standard of living plus a number ρ times their child's (overall) welfare, then maximizing the overall welfare of the first generation implies maximizing the sum of discounted standards of living of all generations.

The reason I place justifications A.1 and A.2 of discounted utilitarianism in the category 'present generation as hegemon' is that they both amount to *maximizing the utility of the first generation*. For the utility in (4) is just the utility of the generation—one parent.

B. Uncertain existence of future generations

Suppose that the social evaluator is uncertain when the human species will become extinct. We make an extremely simple assumption, that there is a probability of p that each generation in the future will be the last generation; there is an independent draw at each date, which, with probability p, will extinguish mankind. This probability, we furthermore assume, is independent of the amenity path that is chosen (a highly unreasonable but simplifying assumption).

Suppose the evaluator is a utilitarian: if he knew that the world would last exactly T generations, then he would choose a consumption path $\mathbf{z}^T = (z_1, z_2, \ldots, z_T)$ to maximize $\sum_{t=1}^{T} u(z_t)$. We now assume that this evaluator has von Neumann–Morgenstern preferences over lotteries—in this case, the lottery is over the number of dates that the species will exist—and that his von Neumann–Morgenstern utility function is given by the utilitarian function. Then he should choose the consumption path to maximize his *expected utility*: that is, the probability-weighted sum of the utilities he receives if the world lasts any finite number of dates. Suppose that the probability that the species lasts exactly T dates is $\pi(T)$. Then the evaluator's problem is to choose the infinite path $\mathbf{z} = (z_1, z_2, z_3, \ldots)$ to maximize:

$$\pi(1)u(z_1) + \pi(2)(u(z_1) + u(z_2)) + \pi(3)(u(z_1) + u(z_2) + u(z_3)) + \cdots . \tag{5}$$

It can easily be shown that $\pi(T) = (1 - p)^{T-1} p$, and so, it follows from a little algebra, that maximizing the expression in (5) is equivalent to maximizing:

$$\sum_{t=1}^{\infty} \rho^{t-1} u(x_t) \text{ where } \rho = 1 - p.$$

That is, the social evaluator is just a discounted utilitarian, where the discount factor is one minus the probability of human extinction at each date.

Note that in this justification, the discount factor ρ has nothing to do with a subjective rate of time preference or a degree of altruism of an agent: it is a technological parameter concerning the universe.

C. Axiomatic characterizations

These justifications are more abstract. They do not begin with utility functions, or social welfare functions, but with general orderings on infinite sequences of utilities $\mathbf{u} = (u_1, u_2, u_3, \ldots)$, to be thought of as describing the utilities experienced by different generations in a history of the world.

A social welfare function, as defined above, generates an order on such sequences, but there may be orders that cannot be represented by a social welfare function. Let us define two axioms on the orderings of these utility streams. The first is *continuity*. Suppose we have a *sequence of infinite utility streams* $\mathbf{u}^1, \mathbf{u}^2, \mathbf{u}^3, \ldots$ which converge to a particular infinite utility stream $\mathbf{u}$ (that is, stream $\mathbf{u}^i$ gets very close to the stream $\mathbf{u}$ as i gets large). Continuity states that if every $\mathbf{u}^i$ is weakly socially preferred to some other infinite utility stream $\mathbf{y}$, then $\mathbf{u}$ must also be weakly preferred to $\mathbf{y}$. Continuity is an axiom that says the social ordering does not make 'jumps'. The second axiom is *monotonicity*. Suppose we have two infinite utility streams $\mathbf{u}$ and $\mathbf{u}'$, and the utility of *every generation* is at least as large in the first stream as in the second stream, and *some generation* has larger utility in the first stream. The monotonicity axiom states that $\mathbf{u}$ should be ranked strictly preferable to $\mathbf{u}'$. This axiom is sometimes called strong Pareto optimality. If no generation is worse off in $\mathbf{u}$ than in $\mathbf{u}'$, and some generation is better off, then the Ethical Observer should rank the $\mathbf{u}$ world as (strictly) better than the $\mathbf{u}'$ world.

Finally, we define a property called *social impatience*. Suppose we have a utility stream $\mathbf{u} = (u_1, u_2, u_3, \ldots, u_t, u_{t+1}, \ldots)$ and then we create another stream by interchanging for some t, u_t and u_1: thus, $\hat{\mathbf{u}} = (u_t, u_2, u_3, \ldots, u_{t-1}, u_1, u_{t+1}, \ldots)$, and suppose that $u_t > u_1$. Social impatience states that the stream $\hat{\mathbf{u}}$ must be (strictly) preferred to $\mathbf{u}$. In other words, the social evaluator would prefer to have the larger utility experienced earlier in history.

Diamond (1965) proved that if a complete social ordering of infinite utility streams satisfies continuity and monotonicity, then it must satisfy social impatience. This means that *something like* discounting of future utilities must occur if continuity and monotonicity are satisfied. The Diamond theorem, however, does not say that the social evaluator maximizes a sum of discounted utilities (that would be a special case of the Diamond conclusion).

Finally, there is a famous theorem of Koopmans (1960) which provides an axiomatization of discounted utilitarianism. It is beyond my scope here to present this result: it says that if a number of axioms are satisfied with respect to the ordering of infinite utility streams, then the social ordering can be represented by the discounted sum of *some* increasing function of the utilities, discounted by *some* positive discount rate. But the theorem does not provide any way for adjudicating among these choices. It merely says that the social preference order lies in a certain class of preference orders.[3]

Let me now evaluate these various justifications of the discounted-utilitarian approach. I find both the justifications of Approach A unconvincing. The reason is simple: the social evaluator is taking into consideration only the utility of the first generation. This is obviously the case in the model of the infinitely lived consumer—for there is only one individual in this model. In particular, if the infinitely lived consumer is very impatient, then the social evaluator discounts heavily the welfare of future generations. A similar criticism applies to the altruistic-parent model. Suppose that the parents across the generations are only *mildly* altruistic, so that the value of ρ is small. Again, the Ethical Observer must discount heavily the welfares of future generations. There is no justification, in my view, of prejudicing the desires of the first generation in the way that these two models do. The subjective views of the first generation, with regard either to impatience or altruism, should not determine our intergenerational ethics. To give the first generation such power completely violates *fairness*, which requires that we give all generations equal consideration. Formally, this is summarized by the requirement that the social-welfare function be symmetric. To put it slightly differently, it is fallacious to assume that either the model of an infinitely-lived consumer, or of the altruistic parent is ethically equivalent to a model of many distinct generations.

Dasgupta (2005), an economist active in climate-change analysis, believes otherwise. He writes, with regard to the model of the infinitely lived consumer:

> An individual's lifetime well-being is an aggregate of the flow of well-being she experiences, while intergenerational well-being is an aggregate of the lifetime well-beings of all who appear on the scene. It is doubtful that the two aggregates have the same functional form. On the other hand, I know of no evidence that suggests we would be way off the mark in assuming they do have the same form. As a matter of practical ethics, *it helps enormously* [my italics—JR] to approximate by not distinguishing the functional form of someone's well-being through time from that of intergenerational well-being.

This is an amazing statement. Dasgupta says, first, and correctly in my view, that there is no reason to believe that the model of the infinitely-lived consumer is a good model for intergenerational ethics, but then says it 'helps enormously' to assume that it *is* a good model. We will argue in this paper that there are very good alternatives to discounted utilitarianism, and hence this pragmatic view is unnecessary.

Let me turn to Approach C: I will discuss the Diamond result, but the same critique applies to the Koopmans theorem. One problem with requiring certain axioms to hold on very complicated mathematical spaces (here, sets of infinite utility streams), is that the axioms may impose restrictions that are much more severe than our intuitions warrant. For instance, the axiom of continuity is very powerful and abstract. One must be comfortable with the idea of the convergence of a sequence of paths (each of which contains an infinite number of

[3] There is by now a large literature which develops the axiomatic approach to social preferences on abstract sets of infinite utility streams. For recent contributions see, for example, Asheim and Tungodden (2004) and Roemer and Suzumura (2007).

elements) to a path. Do our intuitions really grasp the import of requiring this condition on a social preference order? One of the best ways of evaluating a theorem such as Diamond's is to ask what kinds of social orderings (i.e., theories of intergenerational distributive justice) it declares inadmissible. The following three social orderings do not satisfy the Diamond axioms:

- undiscounted utilitarianism
- maximin
- leximin.

Undiscounted utilitarianism—that is, ranking two infinite utility streams by comparing $\sum_{t=1}^{\infty} u_t$ with $\sum_{t=1}^{\infty} u'_t$—is ruled inadmissible because it does not provide a *complete* ordering—and that is the only reason it is disqualified. It is incomplete, because if the sums of the utilities on two paths are both infinite, then utilitarianism cannot compare these two paths—hence incompleteness (on the domain of utility streams posited in Diamond's theorem). But on the set of paths where the sum of utilities *does* converge, utilitarianism is both monotonic and continuous. So utilitarianism is eliminated only because it is incomplete. Is it, however, incompleteness such a defect of an ethical theory? Where, indeed, do we have an ethical theory that provides convincing answers to all ethical questions?

The 'maximin' ordering says that a stream $\mathbf{u} = (u_1, u_2, \ldots)$ is preferable to a stream $\mathbf{u}' = (u'_1, u'_2, \ldots)$ if and only if the smallest utility in the first stream is greater than the smallest utility in the second stream. This is a very familiar social ethic, from Rawls (although he did not apply it in the intergenerational context). But maximin violates monotonicity: the stream $(1, 2, 2, 2, \ldots)$ is ranked socially indifferent to the stream $(1, 1, 1, \ldots)$, because in both cases the minimum utility is one, which contradicts monotonicity.

Leximin is a refinement of maximin; it is the social ordering which ranks one stream as better than another if and only if the lexicographic minimum of the first is greater than the lexicographic minimum of the second: but this ordering is not continuous [see Roemer (1996)].

If one believes that these three social orderings have some virtue, then one must call into question the usefulness of the Diamond result. It is furthermore the case—and this is of utmost importance in the climate-change discussion—that the Diamond (and Koopmans) axiomatic characterizations give no clue as to what the size of the discount rate should be. They both provide axiomatic characterizations of *classes* of social preference orders, where the class in each case contains orders represented by social-welfare functions with many discount rates.

In my view, Approach B is the only convincing argument for adopting a discounted-utilitarian approach. I do not say it is a *fully convincing* argument, because it depends upon the premise that the Ethical Observer is a *utilitarian,* and that is contestable, but I find it to be a good argument for discounted utilitarianism, *if* one's underlying ethical view is utilitarianism. How do I address the fact that the discounted-utilitarian objective does not, in the final analysis, give equal weights to the welfare of all generations? This comes about because of a naturally asymmetry in facts about the world: to wit, that the probability that the world will last for $T + n$ generations is less than the probability that it will last for T generations, for any positive n. (As the physicists say, time flows in only one way.) Thus, although the Ethical Observer begins with a symmetric social welfare function, she ends up with an asymmetric one.

3 What Climate-Change Economists do

Most economists [e.g., Nordhaus (2008) and many who use cost-benefit analysis, although not Stern (2007)] who use a discounted-utilitarian formula in climate-change analysis adopt the infinitely-lived consumer approach: this can be seen by noting that they justify their choice of discount factor by appealing to the rate of interest.[4] I claim this can only be based on the model of society as one of an infinitely lived consumer.

I will present a model to explain this important point. Suppose that an infinitely lived individual has a utility function for his consumption over the infinite number of periods of his life, which is given by:

$$U(c_1, c_2, c_3, \ldots) = \sum_{t=1}^{\infty} \left(\frac{1}{1+\delta}\right)^{t-1} u(c_t), \text{ where } u(c) = \frac{c^{1-\eta}}{1-\eta}.$$

Here the function u evaluates the consumer's single-period utility: it is a concave function of c for parameter values $\eta > 0$, and it is the utility function that is used by many climate-change researchers (e.g., Nordhaus). His discount rate, which reflects his degree of impatience, is δ; his discount factor is $\frac{1}{1+\delta}$. Diminishing marginal utility of consumption is reflected in the assumption that $\eta > 0$. The individual begins life with a capital stock of s_0. At each period, he devotes his entire energies to work; the amount of output he produces at date t if his capital stock inherited from the last date is s_{t-1} is given by $f(s_{t-1})$, where f is an increasing, concave function. Then his problem of life-time utility maximization is to choose a consumption path $(c_1, c_2, c_3, \ldots)$ and an investment path $(i_1, i_2, i_3, \ldots)$ to

$$\begin{aligned} &\text{maximize} \sum_{t=1}^{\infty} \left(\tfrac{1}{1+\delta}\right)^{t-1} \tfrac{c_t^{1-\eta}}{1-\eta} \\ &\quad \text{subject to :} \\ &\qquad (1)\quad c_t + i_t = f(s_{t-1}), \quad t = 1, 2, 3, \ldots \\ &\qquad (2)\quad s_t = (1-d)s_{t-1} + i_t, \quad t = 1, 2, 3, \ldots \end{aligned} \qquad \text{Program ILC}$$

where i_t is the amount of output he chooses to invest in the capital stock at date t. Constraints (1) simply say that output is partitioned between consumption and investment at each date, and constraints (2) say that the capital stock at date t equals the depreciated capital stock from the last date plus new investment. (Capital depreciates at rate d per period.)

Define the growth rate of consumption at date t along a path, denoted $g(t)$, by the formula:

$$1 + g(t) = \frac{c_t}{c_{t-1}}.$$

Define the *marginal productivity of capital net of the depreciation rate* at date t, denoted $r(t)$, by:

$$r(t) = f'(s_{t-1}) - d,$$

where f' is the derivative of f. Then it can be proved that the solution of Program ILC entails that:

$$(1+\delta)(1+g(t))^{\eta} = r(t) + 1 \text{ for all dates } t.$$

Taking the logarithm of this equation, we have:

[4] This locution may seem strange. Aren't economists up-front about their justification of discounted utilitarianism? No: they rarely state the fundamental view (e.g., the view A or B or C) that would justify the ethic of discounted utilitarianism.

$$\log(1+\delta) + \eta \log(1+g(t)) = \log(1+r(t)).$$

Now suppose that the numbers δ, $g(t)$, and $r(t)$ are all close to zero. Using the fact that if x is a small positive number, then $\log(1+x) \approx x$, we can write this equation as:

$$\delta + \eta g(t) \approx r(t);$$

finally, we can write this as:

$$r(t) \approx \delta + \eta g(t). \quad \text{(Ramsey)}$$

This equation is known as the Ramsey equation, as it was first derived in Ramsey (1928).[5] The reason the notation r is chosen for the quantity it represents is that in a market economy where firms maximize profits, $r(t)$ can be thought of, in economic equilibrium, as the (real) consumption rate of interest.

Now what many economists do, in analyzing climate change, is to deduce the value of the *subjective impatience parameter* δ from the Ramsey equation. Assuming that we are in a situation where rates of interest and growth rates of consumption are relatively stable, we can delete the time arguments in (Ramsey), and write the Ramsey equation as:

$$\delta \approx r - \eta g. \quad \left(\text{Ramsey}^*\right)$$

The consumption interest rate r and the growth rate g are all observables from economic data. Economists in the climate-change area (e.g., Nordhaus and Weitzman) typically estimate η by appealing to the idea that it reflects attitudes towards risk, and subjective views of inequality aversion that people have. (A larger value of η reflects a higher degree of inequality aversion. See Dasgupta (2008) for a good discussion.) *Hence δ is estimated,* since all the terms on the right-hand side of (Ramsey*) are observed or conjectured (as with η). It is this value of δ that these economists use as the discount rate in the social welfare function for intergenerational welfare calculations.

In summary, the discount rate that is estimated from equation (Ramsey*) is based upon these assumptions :

(1) that the economy is a competitive market economy, and the marginal productivity of capital net of the rate of depreciation can be taken to be equal to the observed real consumption interest rate;
(2) an infinitely lived consumer, who represents Society, has decided on his consumption path by maximizing a utility function in which δ is his *subjective rate of time preference*.

Nordhaus's (2008) approach is somewhat of a mixture: he takes the parameter η in the utility function u to reflect not the individual's attitude towards his *own* consumption, but the (subjective) attitudes of people about inequality of consumption across generations. But because Nordhaus (and other authors) take r from *market data,* we can only assume that the δ they compute from (Ramsey*) is the *individual's rate of time preference*, because in making their savings decisions, actual market agents are optimizing given their own subjective rates of time preference.

Moreover, I claim that the model of the infinitely-lived consumer is the *only* way to justify this choice of the discount factor. This is, of course, a sweeping claim. Surely, it would be a remarkable coincidence if some other (ethically more appealing) way of justifying this approach turned out to be *equivalent* to the model of the infinitely lived consumer. I cannot

[5] If we write the Ramsey problem in continuous time, then the Ramsey equation is precisely the condition that characterizes the optimal path.

prove that this claim is true (who could ever do so?). Suffice to say that the claims that some have made for *other* justifications of the Ramsey equation are bogus (more on this below).

To show the ubiquity of this practice, let me quote Nordhaus (2008, p. 10):

> In general, we can think of the discount rate as the rate of return on capital investments. We can describe this concept by changing our one-commodity economy from corn to trees. Trees tomorrow (or, more generally, consumption tomorrow) have a different 'price' than trees or consumption today because through production we can transform trees today into trees tomorrow. For example, if trees grow costlessly at a rate of 5% a year, then from a valuation point of view, 105 trees a year from now is the economic equivalent of 100 trees today. … Therefore, to compare different policies, we take the consumption flows for each policy and apply the appropriate discount rate. We then sum the discounted values for each period to get the total present value. ….The choice of an appropriate discount rate is particularly important for climate-change policies because most of the impacts are far in the future. The approach of the DICE model [Nordhaus's model] is to use the estimated market return on capital [the interest rate] as the discount rate. The estimated discount rate in the model averages 4% per year over the next century.

In fact, in Nordhaus (2008, p. 178) chooses:

$$r = .055, \quad g = .02 \quad \eta = 2 \quad \delta = .015.$$

If we substitute these values into the Ramsey equation, we compute that

$$r = \eta g + \delta.$$

His value of $\rho = \frac{1}{1+\delta}$ is 0.985, and so the utility of individuals one century from now is discounted by a factor of $(.985)^{100} = 0.22$; their welfare counts about one-fifth as much as ours in his social calculus.

How does this jibe with his claim quoted above that 'the estimated discount rate in the model averages 4% per year'? This is because there are in fact *two* discount factors in Nordhaus's locution: the discount factor on utility, which is $1/(1+\delta)$, and the *implied* discount factor on *consumption,* which is a consequence of solving the optimization problem (ILC). At the solution to that problem, one can ask, "How much would the consumer require as an increment in consumption at date t (call this increment Δc_t) in order to compensate him for a small decrease in consumption (call this decrement Δc_{t+1}) at date $t+1$? It can be shown, for the given utility function, that this amount is approximately $\frac{1}{1+r_t}\Delta c_{t+1}$. (See Dasgupta (2008, fn.12) for the derivation.) So r_t can be viewed as the *consumption discount rate,* as opposed to δ, which is the *utility* discount rate. Note, however, that δ is a primitive of the model—it comes with the objective function—while r_t is endogenous, a number which is deduced as a consequence of solving the consumer's optimization problem. For conceptual clarity, it is the discount rate on *utility* that is important for the theory, because its value must be stipulated before the optimization is carried out.

Let me respond in another way to the above citation of Nordhaus, for his argument may sound appealing to the reader: Why, indeed, should we not discount future welfare by a great deal, if the marginal productivity of capital is so large, so that 100 trees today can be transformed into $100 \times (1.04)^{100} = 5050$ trees a century from now? The answer is that this fact of capital's marginal productivity will be properly incorporated into the specification of the set Z of amenity paths—and that is the only way it should appear in the problem. Our social welfare function must be founded on ethical judgments only, not on technological facts. Our problem is to maximize a welfare function justified by *general ethical principles*, which properly incorporates the relative worth of all generations, subject to constraints which are determined by *facts about the world* (technological progress, the influence of emissions on biospheric quality, etc.).

The same conceptual error occurs, in a slightly different form, in the next citation. Partha Dasgupta writes as follows [Dasgupta (2008, p. 145)]:

> There are two reasons why it may be reasonable [to discount future consumption at a positive rate]. First, an additional unit of consumption tomorrow would be of less value than an additional unit of consumption today *if society is impatient to enjoy that additional unit now* (my italics—JR). Therefore, impatience is a reason for discounting future costs and benefits at a positive rate. Second, considerations of justice and equality demand that consumption should be evenly spread across the generations. So, if future generations are likely to be richer than us, there is a case for valuing an extra unit of their consumption less than an extra unit of our consumption, other things being equal. Rising consumption provides a second justification for discounting future consumption costs and benefits at a positive rate.

Dasgupta's first reason for positive discounting of the consumption of future generations, which I have italicized, is clearly based on the model of the infinitely-lived consumer. Who is 'society' in his phrase? It is *the present generation*. Surely, if 'society' included all future generations, his phrase would be nonsensical. (How could we say that our great-grandchildren are impatient to enjoy an extra unit of consumption *now*?) But society, for the Ethical Observer must comprise *all* generations of humans who will ever live. Dasgupta's second reason for discounting future utility is incoherent, for it puts the cart before the horse. We do not know that future societies will be 'richer' than we are: whether or not that occurs will be an *outcome* of the policies we decide to implement—it cannot be taken as a premise. And if those societies are indeed 'richer,' because of the technological progress that takes place—which will depend, inter alia, on the resources we pass down to the next generation through education—and because we have saved the global commons for them, and it turns out that the optimal policy has their consuming more than we do, their *average unit* of consumption will not receive as much weight in the social-welfare function as our average unit as long as $\eta > 0$, which implements diminishing marginal utility. Why *further* discount their utility with positive discount rates? This is, again, an issue of mixing the ethics (which determine what social welfare function we maximize) with the facts about the world (which determine the feasible set of paths over which maximization takes place).

Weitzman (2007) also bases his analysis on the Ramsey equation, and proposes the parameter values:

$$r = 0.06 \quad g = 0.02 \quad \eta = 2 \quad \delta = 0.02.$$

Thus Weitzman would discount the utility of individuals one century from now by a factor of $\left(\frac{1}{1+\delta}\right)^{100} = .138$; their utility is worth about one-seventh of ours, in his social calculus.

The only economist recently active in climate-change research who is a discounted utilitarian and uses a discount factor very close to one is Stern (2007), who chooses $\delta = 0.001$ per annum. This gives a discount factor of $\rho = 0.999$ per year. Stern would therefore discount the utility of those living a century from now by the factor $\left(\frac{1}{1.001}\right)^{100} = 0.90$, that is, by only about 10%. And Stern (2008) writes explicitly that the only justification for discounting utilities of future generations is because of the possibility that they might not exist. So Stern is clearly adopting Approach B described here, and assuming that the probability that the species becomes extinct is $p = 0.001$ in each year.

Indeed, Stern (2007) has been criticized by Nordhaus (2007) and Weitzman (2007) for using a discount factor too close to one. Although the arguments of these critics are stated in terms of opportunity costs, and are not transparent to the non-economist, they all boil down to a fundamental claim that the right discount factor is derived from the observed marginal productivity of capital. Since the marginal productivity of capital in a competitive economy is equal to the interest rate, which is the 'opportunity cost' of using resources today instead of tomorrow, the arguments are stated in terms of opportunity cost. The critics would be more forthright if they simply argued that the right model for intergenerational welfare analysis is the one of the infinitely-lived consumer, rather than our Approach B.

Lest readers think that I put too fine a point on the issue of the meaning of the discount rate, and therefore on its size, I quote the abstract from Nordhaus (2007):

> How much and how fast should we react to the threat of global warming? The Stern Review argues that the damages from climate change are large, and that nations should undertake sharp and immediate reductions in greenhouse gas emissions. An examination of the Review's radical revision of the economics of climate change finds, however, that it depends decisively on the assumption of a near-zero time discount rate [i.e., $\delta = .001$] combined with a specific utility function. The Review's unambiguous conclusions about the need for extreme immediate action will not survive the substitution of assumptions that are consistent with today's marketplace real interest rates and savings rates.

Although Frank Ramsey (1928) pioneered the analysis of intertemporal social welfare, he wrote in the introduction of that paper:

> One point should perhaps be emphasized more particularly; it is assumed that we do not discount later enjoyments in comparison with earlier ones, a practice which is ethically indefensible and arises merely from weakness of the imagination; we shall, however, in Sect. 2, include a rate of discount in some of our investigations.

Ramsey is perfectly clear, and correct, on the ethics. So why did he nevertheless introduce a rate of discount later in his paper? Because often it is the case that the maximum of the *undiscounted* utilitarian welfare function is infinite, and this infinite value is achieved by many possible consumption paths, so the utilitarian view provides no way of choosing an optimal path! This, indeed, is why many economists today use discounted-utilitarianism: not because it has a sound ethical foundation—at least for the discount rates commonly employed—but because it *gives a unique answer* to the problem. There is no good justification for this practice: it is an example of looking for the lost diamond ring under the street lamp, because that is the only place one can see! If (undiscounted) utilitarianism does not enable us to find the optimal policy in all problems, that means only that it is an incomplete ethical doctrine—not something to be ashamed of. Recall how Diamond's axiomatization eliminated utilitarianism *ab initio* because it is an incomplete ordering. This does not imply that the way of 'completing' utilitarianism is to discount future utilities with sufficiently small discount factors so that unique optimal paths exist. An *argument* with valid ethical foundations must be given to justify such a practice.

Bad ethics—adopting the model of the infinitely—lived consumer as ethically equivalent to a sequence of generations of human beings—are prevalent among economists on this important issue, and this is reflected in the focus of discussion being 'what is the proper discount rate to choose,' as if this were a subjective issue. According to Approach B, there is a fact of the matter as to what the discount rate should be, and it has nothing to do with (subjective) rates of time preference of individuals, or subjective degrees of intergenerational altruism.

It must be said that Weitzman (2008), although he adopts the view of the infinitely-lived consumer, has another argument for the *irrelevance* of the discount rate—that is, that consequences of not taking serious measures to control emissions today will be so disastrous (with large probability), that the precise value of the discount rate is of only secondary importance. In terms of the mathematics, he is saying that with insufficient abatement, the utilities of generations not far in the future may be so low that even a discounted utilitarian who chooses ρ to be fairly small would decide upon serious abatement. I agree with Weitzman's conclusion, but it does not substitute for doing the ethics properly.

I do not wish to misrepresent Dasgupta's position. I have quoted him as advocating the model of the infinitely-lived consumer, and deducing the discount factor from market data. But his position is ambivalent. In Dasgupta (2008, p. 157), he also writes:

> One influential school of philosophers has argued that societal impatience is ethically indefensible. They say that to set $\delta > 0$ is to favour policies that discriminate against the well-being of future generations merely on the grounds that they are not present today. They also say that values frequently in use among economists, ranging as they do between 2 and 3% a year, are way too high. I find their arguments hard to rebut.

Dasgupta does not seem satisfied with any of the resolutions on offer of the problem of choosing the 'right' values for δ and η. I submit that the *true* problem may be in the adherence to the (discounted) *utilitarian* social welfare function. Dasgupta (and surely Nordhaus and Weitzman as well) do not consider abandoning utilitarianism as the basis for intergenerational ethics. I will argue in the next section that one should consider doing so.

4 Sustainability

I have argued that, if the Ethical Observer is a utilitarian with von Neumann–Morgenstern preferences under uncertainty, then he should choose a utility discount rate very close to zero, to reflect the very small probability that the human species will end at any given date. (Of course, this argument is *modulo* the simplifying assumption that the consumption path chosen will not affect that probability.) But are there alternatives to being a utilitarian? It is noteworthy, as I have pointed out, that neither side of the 'discount rate debate' that has taken place in the last two years in the economics journals challenges utilitarianism as an ethical view.

Although Rawls mounted a pervasive critique of utilitarianism, this seems to have had little effect on economists, who routinely adopt utilitarian objective functions. This is not the place to review the arguments pro and con. Let us just note the simple fact that utilitarianism treats society as a vessel without boundaries between persons—in the precise sense that all that matters is the *total* stuff (utility) in the vessel, not how it is distributed among the individuals who comprise it. In this sense, it seems quite at odds with liberalism, which prizes individual distinctiveness. Utilitarians attempt to temper the indifference of utilitarianism to distribution by choosing values of η in the utility function $u(c) = \frac{c^{1-\eta}}{1-\eta}$ which are greater than one, which builds in an aversion to inequality between individuals (here, between generations). I believe this is an ad hoc way of addressing the problem.

Indeed, there is an obvious alternative to utilitarianism, which draws upon the popular view that intergenerational ethics should require *sustainability*. How can we interpret this view? I propose that it means that the Ethical Observer should choose a consumption path which maximizes the level of human welfare that can be *sustained forever* (or for as long as the human species exists). Suppose the human species were to last forever: then the problem would be to choose $\mathbf{z} \in Z$ to maximize the number Λ such that, for every date t, $u(z_t) \geq \Lambda$. That is—human welfare is sustained at a level of at least Λ for all time, where the path is chosen so that Λ is the largest number making this statement true.

I note that this is distinctly anthropomorphic conception of sustainability: it differs from *strong sustainability,* which maintains that the biological capital of the earth should be sustained forever. The anthropomorphic conception is less radical and is often called 'weak sustainability': it is theoretically possible that humans could deplete the earth's biological capital and maintain their own welfare. [For an extensive discussion of weak and strong sustainability, see Neumayer (2003).]

In other parlance, this is just the 'maximin' social welfare function: that is, Λ is just the welfare of the worst-off generation. My collaborators and I find this an attractive ethic, and we have investigated its consequences for intertemporal distribution in Llavador et al. (2008), hereafter LRSa. Sustainability was first introduced into the economic literature soon after John Rawls's book was published: early contributions were Arrow (1973), Dasgupta (1974), and Solow (1974). For more recent contributions to the topic, see the footnote 7 below.

The simplest ethical justification of sustainability, so defined, is that the date at which a person is born should be viewed as arbitrary from the moral viewpoint, and therefore no generation should be better off than any other generation, unless such a utility difference comes for free, in the sense that it comes without lowering the utility of the worst off generation. This does not necessarily mean that utilities will be equal for all generations—that is, the 'maximin' path may Pareto dominate the highest equal-utility path. We are familiar with such a possibility from Rawls; however, the reason this may occur here is not due to the familiar incentive effects in Rawls. It is, rather, that there may exist intergenerational public goods which would be produced by early generations for their own welfare, but which impact positively upon the welfare of future generations. (For instance, our generation invents the steam engine for ourselves, and future generations reap the benefits as well—for free.) So maximin may well dominate equality in the intergenerational context, when innovation is taken into account.[6] For an extended discussion of this point, see Silvestre (2002), which characterizes exactly when the maximin solution engenders an increase in welfares over generations.

The social welfare function which models sustainability is

$$W^S(u(z_1), u(z_2), u(z_3), \ldots) = \min_t u(z_t). \tag{6}$$

Solving the problem

$$\max W^S(u(z_1), \ldots) \text{ subject to } \mathbf{z} \in Z \tag{7}$$

is the same as finding the maximum Λ and $\mathbf{z}$ such that $u(z_t) \geq \Lambda$ for all t.

In LRSa,[7] we present a model calibrated to our world with climate change (this amounts to being very precise about the nature of the set Z, deriving it from physical constraints

[6] Suppose we applied the reasoning of Dasgupta (2008). Suppose that the solution of the intergenerational maximin program produces increasing utilities as time progresses. Should we then go back and alter our maximin objective to penalize later generations because they did well in the first formulation? Hardly.

[7] Geir Asheim, and scholars with whom he has collaborated, has written extensively on sustainability in abstract models: see, for example, Withagen and Asheim (1998), Asheim et al. (2001), and Buchholz (1997).

concerning production, education, the impact of emissions on biospheric quality, etc.) and we solve problem (7)—that is, we find the path that would sustain welfare at the highest possible level, given current knowledge about the relationship between production, emissions, and global warming. It turns out that the utilities of all generations are *equal* along that path—that is, maximin and equality indeed recommend the same path in this case.

Some may be disturbed by this outcome, and thereby reject sustainability as an ethic, because it appears to relegate us to a world with no growth (of human welfare).[8] Would it not be desirable to have some growth in human welfare—and if so, to use a different social-welfare function? Let us recognize what the trade-offs are. If we wish to have positive growth in human welfare, that means that the early generation(s) will have lower welfare than they have in the sustainable solution—growth is purchased at this cost. But why should early generations sacrifice for later ones? There is no obvious answer.

There is a second defense of the sustainability model. In the intertemporal problem, we have ignored *intra*temporal inequality. A strong argument for growth is that it (usually) improves the standard of living of the very poor, an improvement with high ethical value. If there is no intratemporal inequality, this justification disappears. Thus, properly (and consistently) to consider the plight of the very poor requires another model, where there is heterogeneity of individuals at each generation—the simplest model would have one poor individual and one rich individual at each date. In that model, we could advocate a preference for paths that bring the welfare of the present-day poor up to the welfare of the present-day rich, over a period of time, and then sustain that welfare level forever.

Nevertheless, Llavador, Silvestre, and I believe that the issue of sustainability versus growth (of human welfare) cannot be settled completely at the abstract level: it requires discovering what the actual trade-offs are—that is, how much the early generation(s) would have to sacrifice to permit positive rates of growth of human welfare. We investigate in LRSa, besides the sustainable solution, a model which allows for *sustainable growth*: that is, growth of welfare over time. In this case, the problem becomes:

$$\text{choose } \mathbf{z} \in Z \ \text{ to maximize } \Lambda \ \text{subject to } u(z_t) \geq \Lambda(1+g)^{t-1}. \tag{8}$$

Here, we specify *exogenously* a rate of growth g, and require human welfare to grow at a rate of at least g at every date. If $g = 0$, (8) reduces to the pure sustainability problem. A possible justification for choosing g greater than zero is that humans want their children to be better off than they are; indeed, they are willing to sacrifice their own welfare to make this possible—or, to state this less personally (so that childless adults are included) each generation wants 'human development' to take place, in the sense of increasing generational welfare. Program (8) will have a solution for an interval of positive values of g.

It is important to note that in program (8), we specify exogenously the rate of growth g. Solving (8) is *not* equivalent to solving the discounted-utilitarian problem. We do not propose an ethical theory directing us how to choose g. We will present some of our results of these two formulations of the intergenerational social-welfare problem below in Sect. 6. We will argue that small positive values of g deliver solutions that may be ethically preferable to the

Footnote 7 continued
I believe, however, that LRSa is the first attempt to compute sustainable optimal paths in a concrete model, parameterized with real data.

8 Indeed, the path that solves the optimization problem (7) is the best feasible path according to both the maximin and the leximin criterion. Thus, one can choose which ethic one prefers to justify it. (Following the discussion of Approach C, one can thereby choose either to justify it by an ethic that violates continuity, or one that violates monotonicity.)

solution with $g = 0$ (pure sustainability), because the cost borne by the first generation to sustain a small positive g seems acceptable.

5 The Consumptionist Fallacy

In the global-warming literature, economists usually specify the vector of amenities as comprising only one component: consumption of a single commodity. For example, Nordhaus (2007) chooses $u(c) = \frac{c^{1-\eta}}{1-\eta}$ where η is a positive number, and c is the consumption of the representative individual of a produced commodity. Thus, human welfare depends only upon commodity consumption, the commodity being produced from capital and labor, and welfare exhibits diminishing marginal utility in consumption.

There is an argument of mathematical simplicity that favors choosing only one amenity, but I believe that it constitutes a 'consumptionist fallacy' to conceptualize human welfare as depending only upon consumption of produced goods. LRSa postulate a vector of amenities that comprises four elements: consumption, educated leisure, the stock of human knowledge, and the quality of the biosphere. Thus, there are four avenues that can be traveled to increase welfare: increasing consumption, increasing education, and so on. As well as being, we feel, more psychologically realistic, this allows for the possibility that we maintain (or increase) human welfare by shifting from emissions-producing commodities to other activities (like production of knowledge and education) which are less emission intensive, but also affect welfare positively.

Nordhaus (2008) states that taking utility to depend only on one consumption good is a permissible abstraction, and that his reader may think of consumption as comprised of various goods. But this is not right. The essential point is that consuming the four goods in the utility function of LRSa involves quite different impacts on the biosphere. We may be able to sustain human welfare by, for example, substituting education (a low-emissions activity) for automobiles (a high emissions activity). The consumptionist fallacy is committed when the model does not represent consumption alternatives for people as implying different effects on scarce biospheric resources.

Consumption and leisure are private goods, while the stock of human knowledge and biospheric quality are public goods. We defend modifying (multiplying) leisure by the individual's level of education by saying that education enables more uses for one's leisure time—hence it increases welfare. It is perhaps contestable to include the *stock* of human knowledge as a factor in *individual* welfare: we argue that our lives are better, to the extent that our science and art and more developed—not because science improves the quality of commodities we consume (that will be reflected in consumption)—but for perfectionist reasons. Knowing how the world works is an intrinsic good in human experience—how else to explain our quest for abstract knowledge, which has no immediate application? Finally, including biospheric quality in the welfare of individuals models the value that species-variety and other biological and environmental goods have for humans, intrinsically, quite apart from the medicines that they may generate (although health is also related to biospheric quality).

On this last point, the reader may be puzzled: if economists who study global warming place only commodity consumption in the basket of amenities, and not biospheric quality, how do they model the bad effects of GHG emissions? The answer is that these emissions are assumed to increase the costs (capital and labor) of producing commodities—so they impact negatively on consumption. The only way in which a thing can be bad, if the only amenity is commodity consumption, is if it reduces the possibilities of consuming!

In sum, the larger the number of amenities postulated, the more strategies there are for maintaining human welfare besides maintaining commodity consumption. The consumptionist fallacy would seem to force too dismal a set of choices upon human society—not to speak of the truncated conception of human well-being that it models.

6 Sustainability: Some Results

Here, I summarize some of the results of analyzing the sustainability model of LRSa. The utility of a given generation is a function of commodity consumption, leisure time multiplied by the level of the individual's education, the stock of knowledge, and the quality of the biosphere (taken to be inversely related to the stock of carbon in the atmosphere). The labor of the single individual at each generation is divided into four uses: educating the next generation, participation in commodity production, working in the 'knowledge industry' (research and development, the arts, university research), and enjoying leisure time. The model begins at year 2000, with four initial stocks: the average level of education of the consumer-producer, the stock of physical capital, the stock of knowledge, and the quality of the biosphere. There are four production functions: the production of educated children from adult teaching, the production of commodities from educated labor, capital, and knowledge, the production of knowledge from educated labor and capital, and the production of biosphere quality from the current stock of biospheric quality and emissions. Emissions are modeled as an input in the production of commodities, as is biospheric quality: thus, substitution of emissions for other inputs (labor, capital) is the way abatement is accomplished. The commodity produced is used for human consumption and investment (increasing the capital stock). In turn, capital depreciates, as does knowledge (the knowledge of how to make mechanical watches is of little value today). The stocks and flows are illustrated in Fig. 1.

There are many parameters in the model: parameters which specify the four production functions, those which specify the relationship between production and emissions, those which model the rate of technical change, those which describe the way the amenities combine into an index of quality-of-life, and the two rates of capital depreciation. These parameters are estimated from data using familiar econometric techniques. Unfortunately, we had to use, for the most part, US data, and so we cannot argue that our results are realistic for the world. When all parameters have been chosen, and when the initial vector of four stocks is specified, we can define the set Z of feasible paths of amenity vectors over time as paths of variables that solve a set of dynamic equations.

The greatest uncertainty is associated with the production function of biospheric quality, for this entails understanding three relationships—that of commodity production and consequent emissions to the concentration of atmospheric carbon, that of the relationship of that stock to global temperature, and finally that of global temperature to human welfare. As I said, the science on these questions is in flux. LRSa simplifies the problem by taking, as the measure of biospheric quality, the amount of 'clean biosphere,' the difference between a catastrophically high level of carbon particulate concentration and the present level. This resource of clean biosphere is then related to global temperature, which is related to human welfare, using current estimates. We fix one emissions path and optimize over all other variables. It is assumed in these calculations that population size is constant. We take the emissions path from the Intergovernmental Panel on Climate Change (2008), the fourth assessment report of the IPCC. In the tables below, results are presented for this emissions path, which would putatively stabilize CO_2 concentration at 450 ppm. The optimal paths are to be interpreted as ones that US residents would experience, since we have used US data to calibrate.

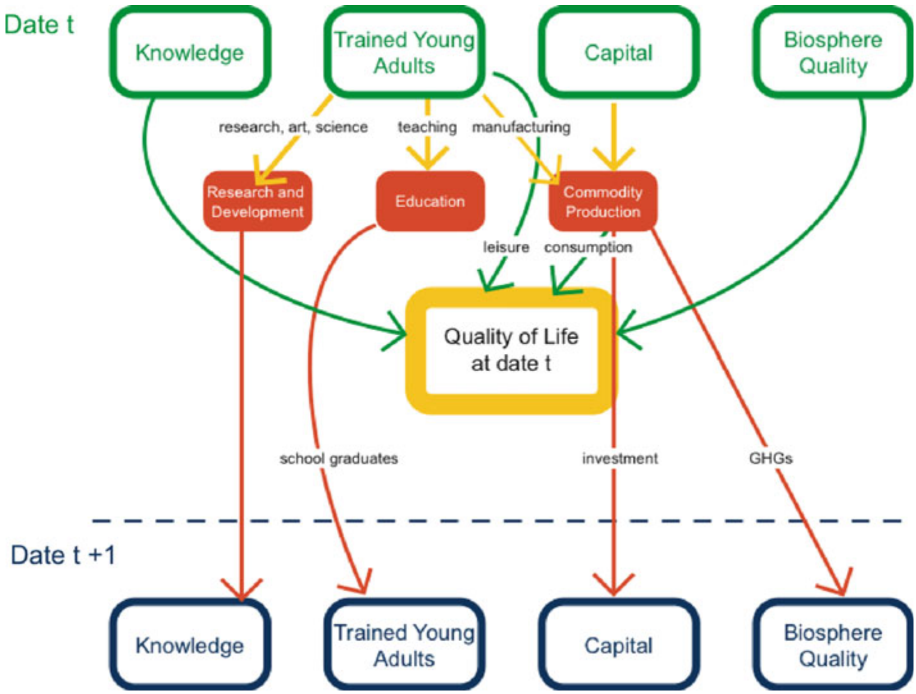

Fig. 1 Flows in the model of Sect. 5

We present two scenarios, one in which the US would continue to emit 24% of global emissions (Scenario 1), and the other about 4% of global emissions, its per capita share of global emissions (Scenario 2). An approximation to the optimal sustainable path (with a growth rate of zero) is presented in Table 1. In Scenario 2, consumption stabilizes in four generations at 23.64, approximately 15% *below* year 2000 levels.[9] Nevertheless, welfare will stabilize at a level 17% higher than its 2000 level. In 100 years, the capital stock stabilizes at 52% above its year-2000 level, which represents an annual rate of growth of 0.42%. The stock of knowledge stabilizes at 356% of its year-2000 level, which represents an annual growth rate of 1.28 %. In Table 2, we see that the level of education of the generation 100 years hence would be 135% of the year-2000 level, which represents an annual growth rate of 0.30%. We note as well that leisure time does not increase—in fact, it decreases a small amount. In 2000, two-thirds (0.6667) of time was spent in leisure, while in 100 years, it would fall to 0.6574 in the optimal scenario. Indeed, a significantly *smaller* fraction of labor would be teaching the next generation, and a significantly larger fraction would be working in the knowledge industry (research and the arts). Note that the educational level of children would be higher, because it is a function of the skill-hours employed in teaching, not just ‘hours,’ and the educational level of teachers increases.

Note that, in Scenario 1, consumption *increases* from its 2000 level to its stable level. We can expect that a realistic scenario for US emissions lies between scenarios 1 and 2.

[9] In the truly optimal solution, utility would be constant across generations. We are unable to compute this solution precisely; the approximation presented in Table 1 converges to constant levels of utility after several generations.

Table 1 Optimal pure sustainability ($g = 0$) paths under two scenarios: in Scenario 1, the US emits 24% of global emissions; in Scenario 2, it emits its per capita share of global emissions

	$\frac{\Lambda_t}{\Lambda_0}$	$\frac{\Lambda_t}{\Lambda_{t-1}}$	c_t	$\frac{c_t}{c_0}$	$\frac{c_t}{c_{t-1}}$	i_t	S_t^k	S_t^n
Gen	Scenario 1 $\left(e^{US} = 0.24 \times e^{World}\right)$							
2000	1.00	1.00	27.78	1	–	6.83	73.65	15.64
1	1.2582	1.2582	41.0709	1.4784	1.4784	14.46	205.32	48.93
2	1.2582	1.0000	38.2724	1.3777	0.9319	7.76	145.55	54.84
3	1.2582	1.0000	30.6556	1.1035	0.8010	8.73	145.55	54.84
4	1.2582	1.0000	30.6556	1.1035	1.0000	8.73	145.55	54.84
Gen	Scenario 2 $\left(e^{US}_{\text{per capita}} = e^{World}_{\text{per capita}}\right)$							
2000	1.00	1.00	27.78	1	–	6.83	73.65	15.64
1	1.1687	1.1687	32.4637	1.1686	1.1686	11.08	161.03	48.82
2	1.1687	1.0000	29.8476	1.0744	0.9194	5.94	112.23	55.61
3	1.1687	1.0000	23.6389	0.8509	0.7920	6.73	112.23	55.61
4	1.1687	1.0000	23.6389	0.8509	1.0000	6.73	112.23	55.61

Key $\Lambda_t =$ utililty at date t
$c_t =$ consumption per capita at date t
$i_t =$ investment per capita at date t
$S_t^k =$ per capita capital stock at date t
$S_t^n =$ per capita stock of knowledge at date t

Table 2 Optimal path of labor allocation and education, zero growth

	x_t	x_t^e	x_t^c	x_t^n	x_t^l	x_t^e (%)	x_t^c(%)	x_t^n(%)	x_t^l (%)
Gen	Scenario 1 $\left(e^{US} = 0.24 \times e^{World}\right)$								
2000	1.396	0.047	0.396	0.023	0.931	0.0333	0.2833	0.0167	0.6667
1	1.648	0.047	0.472	0.070	1.059	0.0282	0.2866	0.0426	0.6426
2	1.650	0.052	0.434	0.068	1.095	0.0318	0.2632	0.0415	0.6635
3	1.858	0.052	0.518	0.066	1.222	0.0282	0.2786	0.0358	0.6574
4	1.858	0.052	0.518	0.066	1.222	0.0282	0.2786	0.0358	0.6574
Gen	Scenario 2 $\left(e^{US}_{\text{per capita}} = e^{World}_{\text{per capita}}\right)$								
2000	1.396	0.047	0.396	0.023	0.931	0.0333	0.2833	0.0167	0.6667
1	1.648	0.047	0.470	0.070	1.062	0.0285	0.2850	0.0425	0.6441
2	1.663	0.053	0.437	0.070	1.104	0.0320	0.2625	0.0419	0.6637
3	1.884	0.053	0.525	0.067	1.239	0.0282	0.2786	0.0358	0.6574
4	1.884	0.053	0.525	0.067	1.239	0.0282	0.2786	0.0358	0.6574

Key $x_t =$ per capita value of educated labor at date t
$x_t^e =$ per capita value of labor allocated to teaching at date t
$x_t^c =$ per capita value of labor allocated to commodity production at date t
$x_t^n =$ per capita value of labor allocated to R & D (knowledge) at date t
$x_t^l =$ per capita value of leisure at date t

We next present results for a path which has the same emission constraints as the previous one, but implements a 1% per annum (28% per generation) increase in the generational utility forever. (This corresponds to choosing $g = 0.28$ in Eq. (8).) Of course, some generation(s) must sacrifice to permit this growth, and we will see it is (only) the first generation. The

Table 3 Optimal paths with $g = 28\%$ per generation (1% per annum)

	$\frac{\Lambda_t}{\Lambda_0}$	$\frac{\Lambda_t}{\Lambda_{t-1}}$	c_t	$\frac{c_t}{c_0}$	$\frac{c_t}{c_{t-1}}$	i_t	S_t^k	S_t^n
Gen	Scenario 1 $\left(e^{US} = 0.24 \times e^{World}\right)$							
2000	1.00	1.00	27.78	1	–	6.83	73.65	15.64
1	1.2476	1.2476	40.7171	1.4657	1.4657	14.32	203.50	48.50
2	1.6000	1.2824	48.8354	1.7579	1.1994	11.11	189.08	70.50
3	2.0518	1.2824	50.3244	1.8115	1.0305	15.46	243.09	90.64
4	2.6313	1.2824	64.7000	2.3290	1.2857	19.88	312.54	116.53
Gen	Scenario 2 $\left(e^{US}_{\text{per capita}} = e^{World}_{\text{per capita}}\right)$							
2000	1.00	1.00	27.78	1	–	6.83	73.65	15.64
1	1.1588	1.1588	32.1824	1.1585	1.1585	10.97	159.58	48.39
2	1.4860	1.2824	38.0823	1.3709	1.1833	8.52	145.79	71.49
3	1.9057	1.2824	38.8037	1.3968	1.0189	11.92	187.44	91.91
4	2.4440	1.2824	49.8883	1.7958	1.2857	15.33	240.99	118.16

Table 4 Labor allocation along optimal paths at $g = 28\%$ per generation

	x_t	x_t^e	x_t^c	x_t^n	x_t^l	x_t^e (%)	x_t^c(%)	x_t^n(%)	x_t^l (%)
Gen	Scenario 1 $\left(e^{US} = 0.24 \times e^{World}\right)$								
2000	1.396	0.047	0.396	0.023	0.931	0.0333	0.2833	0.0167	0.6667
1	1.648	0.060	0.468	0.070	1.050	0.0366	0.2841	0.0422	0.6371
2	2.139	0.087	0.565	0.093	1.394	0.0408	0.2640	0.0433	0.6520
3	3.090	0.112	0.862	0.116	2.000	0.0363	0.2790	0.0377	0.6471
4	3.973	0.144	1.109	0.150	2.571	0.0363	0.2790	0.0377	0.6471
Gen	Scenario 2 $\left(e^{US}_{\text{per capita}} = e^{World}_{\text{per capita}}\right)$								
2000	1.396	0.047	0.396	0.023	0.931	0.0333	0.2833	0.0167	0.6667
1	1.648	0.061	0.466	0.069	1.053	0.0369	0.2824	0.0421	0.6386
2	2.156	0.088	0.568	0.094	1.406	0.0410	0.2632	0.0437	0.6521
3	3.134	0.114	0.874	0.118	2.028	0.0363	0.2790	0.0377	0.6471
4	4.029	0.146	1.124	0.152	2.607	0.0363	0.2790	0.0377	0.6471

results are presented in Tables 3 and 4. Indeed, this growth rate is feasible, while achieving carbon stabilization at 450 ppm in two generations. The trade-off is seen by looking at the utility of generation 1. In Table 1, this welfare level was 1.1687, and in Table 2 it is 1.1588—a bit less than 1% lower. This sacrifice by Generation 1, however, entails significant growth in utility for all generations after that: indeed, in a century, utility is 244% as high as in year 2000. Educational levels are 289% as high as the year 2000 level. The small sacrifice of the first generation might well be ethically appealing, in the light of future welfare gains that it triggers. Note that, with this rate of growth, we now devote a somewhat larger fraction of labor time to education than in the benchmark year 2000 (Table 4). Because the sacrifice of the first generation appears to be so small, it is therefore not obvious that pure sustainability ethically dominates a path that allows for some human development.

The reader is referred to LRSa for a scenario where growth of welfare is 2% per annum. Again, only the first generation has a lower welfare level than in the zero-growth scenario, and the reduction appears to be small. As I said, we do not propose a precise rule for trading off the welfare of the first generation against future growth.

What would the discounted utilitarian solution be on the set of optimal paths that we postulate? If we take the discount factor, following Stern, to be 0.001 per annum, then the discounted utilitarian program *diverges*.[10] Nordhaus (2008) computes approximately optimal paths (which do *not* diverge), with his smaller discount factor, but his set of feasible paths is so different in its details from ours that a comparison of the results would not be terribly meaningful. The fact that, in the model of LRSa, the discounted-utilitarian program diverges, has important implications when we introduce uncertainty in the next section.

I conclude this section with a comment on emissions abatement. The production function for commodities in the model of LRSa has as inputs skilled labor, the stocks of capital and knowledge, the flow of emissions, and the stock of biospheric quality. It is a Cobb-Douglas function of these inputs. Emissions and biospheric quality enter positively into production. The presence of knowledge models technological change: since the level of knowledge is endogenous in the model, so is technological change. Other inputs (labor, capital) can be substituted for emissions: thus abatement possibilities are embedded in the production function. Since our program finds optimal paths, the abatement that is thus implicitly undertaken is optimal as well. Since emissions fall sharply on our assumed path, and output rises (at least in the 1% growth scenario), the emissions-output ratio falls sharply. This can be viewed as the consequence of substituting capital, educated labor, and knowledge for emissions in the production of commodities, since these three inputs all increase sharply.

7 Uncertainty

Now suppose we impose uncertainty of the kind discussed in Approach B of Sect. 2 on this problem: there is an exogenously given probability p that, at each date, the human species will become extinct. The Ethical Observer is now a sustainabilitarian, with von Neumann-Morgenstern utility function derived from W^S. That is, the von Neumann Morgenstern utility that the Ethical Observer enjoys if she has chosen the infinite amenity path $\mathbf{z}$ and the world, in the event, lasts for exactly T dates, is

$$W^S(u(z_1), u(z_2), \ldots, u(z_T)) = \min[u(z_1), u(z_2), \ldots, u(z_T)]. \tag{9a}$$

Then she should choose the consumption path $\mathbf{z}$ that maximizes her expected utility, which is now:

$$\pi(1)u(z_1) + \pi(2)\min[u(z_1), u(z_2)] + \pi(3)\min[u(z_1), u(z_2), u(z_3)] + \cdots$$

This is a well-defined mathematical problem, which (it is easy to show) we can write as:

$$\sum_{t=1}^{\infty} \rho^{t-1} \min[u(x_1), \ldots, u(x_t)] \text{ where } \rho = 1 - p. \tag{9b}$$

Thus, the 'discounted sustainabilitarian' uses the same discount rate as the discounted utilitarian of Approach B, but with a different von Neumann-Moregenstern utilitiy function.

[10] A program diverges when there are feasible paths upon which it achieves an infinite value.

At every date, she is concerned with the minimum utility of all those who have thus far lived, rather than with the sum of those utilities.[11]

There is no explicit treatment of uncertainty in either Nordhaus (2008) or LRSa. In this section, we present some results comparing discounted utilitarianism (DU) to sustainabilitarianism (SUS) in the presence of the kind of certainty discussed in Sect. 2, what we called the uncertain existence of future generations.

Ideally, we would like to analyze the optimal discounted-utilitarian and sustainabilitarian paths for the model of LRSa, described in Sect. 4. That model, however, is complex, and characterizing the optimal paths for general vectors of parameters is very difficult. In Llavador et al. (in press), hereafter LRSb, we study these solutions for a much simpler model, which, we hope, will suggest what we can expect for a more complex and realistic model.

The model of LRSb has only two sectors: commodity production and education. The initial social endowment has two elements: the stock of capital, and the stock of educated labor at the beginning date. Capital and labor produce a commodity, which is used for investment and consumption. Labor is partitioned into teaching, commodity production, and leisure. Utility is a function of consumption and educated leisure. There is no issue of biospheric quality, and no knowledge sector.

Define:

c_t as consumption of generation t
x_t^e as the amount of educated labor allocated to teaching at generation t
x_t^c as the amount of educated labor in commodity production at generation t
x_t^l as the amount of skill-indexed leisure time at generation t
$x_t = x_t^e + x_t^c + x_t^l$ as the total value of skilled labor at generation t

i_t as investment at generation t
s_t^k as the capital stock at date t
d rate of depreciation of capital per generation.

The production function which describes the amount of the produced commodity forthcoming from a vector of capital and skilled labor is $f(s^k, x^c)$. A *feasible path* for the economy consists of a sequence of choices concerning the labor allocation among teaching the next generation, production of the commodity, and leisure, and the allocation of the produced commodity between consumption and investment, which can be achieved, beginning with the initial social endowment of capital and labor in the education sector, (s_0^k, x_0^e). It is a path of choices $\{(s_t^k, i_t, x_t^c, x_t^e, x_t^l, c_t) | t = 1, 2, 3, \ldots\}$ satisfying the following equations:

$$s_t^k = (1-d)s_{t-1}^k + i_t \quad \text{for } t = 1, 2, \ldots \qquad \text{(P1)}$$
$$c_t + i_t = f(s_t^k, x_t^c) \quad \text{for } t = 1, 2, \ldots \qquad \text{(P2)}$$
$$kx_{t-1}^e = x_t \equiv x_t^c + x_t^e + x_t^l \quad \text{for } t = 1, 2, \ldots \qquad \text{(P3)}$$

[11] There is an important criticism of the sustainabilitarian ethic under uncertainty, as modeled in (9). Suppose, in one situation, the world lasts for four generations, and the utility levels are given by (1, 1, 1, 1). Suppose in a second situation, the world lasts for 10 generations, and the utility levels are (1, 1, ..., 1, 0.5). Then the vNM Ethical Observer prefers the first situation, because the minimum utility is larger! In our paper Llavador et al. (in press), hereafter LRSb, we therefore introduce a generalization of W^S:

$$W^{S,\theta}(u_1, \ldots, u_T; T) = (1 + \theta(T-1)) \min[u_1, \ldots, u_T].$$

If $\theta = 0$, this reduces to W^S; if $\theta = 1$, this vNM utility function gives weight to the length of time the species exists. Result 2, reported below, holds for the formulation $W^{S,\theta}$ for all $\theta \in [0, 1]$.

Equations (P1) state that the capital stock at date t equals the depreciated capital stock from date $t-1$ plus new investment; equations (P2) state that the commodity, produced from capital and labor, is partitioned at every date between consumption and investment; equations (P3) state that the amount of skill units of labor at date t is proportional to the skill units of labor allocated to teaching at date $t-1$. The set of all feasible paths emanating from the endowment (s_0^k, x_0^e) will be called $\hat{Z}(s_0^k, x_0^e)$. A path of feasible amenity vectors is a path of consumption—leisure pairs, that is of vectors (c_t, x_t^l), that is part of some feasible path. We write such a path as $\mathbf{z} = ((c_1, x_1^l), (c_2, x_2^l), \ldots)$. What humans care about is consumption and educated leisure, and the other variables in a feasible path are subordinate to producing an amenity path. The utility function for every generation is $u(c, x^l)$, which we take to be a Cobb-Douglas function of these two arguments.

We now state three problems of interest. The first is the problem of the sustainabilitarian ethical observer, in an environment with no uncertainty about the existence of future generations. This problem is to find the feasible path that maximizes the utility that can be sustained forever, from an initial endowment vector (s_0^k, x_0^e). It is written:

$$\max \Lambda \text{ subject to} \quad \left.\begin{array}{ll} u(c_t, x_t^l) \geq \Lambda & \text{(P0)} \\ s_t^k = (1-d)s_{t-1}^k + i_t, \quad t \geq 1 & \text{(P1)} \\ c_t + i_t = f(s_t^k, x_t^c), \quad t \geq 1 & \text{(P2)} \\ kx_{t-1}^e = x_t \equiv x_t^c + x_t^e + x_t^l, \quad t \geq 1 & \text{(P3)} \end{array}\right\} \text{(program SUS)}$$

The next two programs are relevant in a world where the existence of future generations is uncertain. The first is the program of the utilitarian ethical observer who faces uncertainty:

$$\max \sum_{t=1}^{\infty} \rho^{t-1} u(x_t) \text{ subject to} \quad \left.\begin{array}{ll} s_t^k = (1-d)s_{t-1}^k + i_t, \quad t \geq 1 & \text{(P1)} \\ c_t + i_t = f(s_t^k, x_t^c), \quad t \geq 1 & \text{(P2)} \\ kx_{t-1}^e = x_t \equiv x_t^c + x_t^e + x_t^l, \quad t \geq 1 & \text{(P3)} \\ \text{where } \rho = 1 - p. & \end{array}\right\} \text{(program DU)}$$

This is the *discounted utilitarian* program. The next program is the one of the sustainabilitarian who faces uncertainty, which is:

$$\max \sum_{t=1}^{\infty} \rho^{t-1} \min[u(x_1), \ldots, u(x_t)] \text{ subject to} \quad \left.\begin{array}{ll} s_t^k = (1-d)s_{t-1}^k + i_t, \quad t \geq 1 & \text{(P1)} \\ c_t + i_t = f(s_t^k, x_t^c), \quad t \geq 1 & \text{(P2)} \\ kx_{t-1}^e = x_t \equiv x_t^c + x_t^e + x_t^l, \quad t \geq 1 & \text{(P3)} \\ \text{where } \rho = 1 - p. & \end{array}\right\} \text{(program R)}$$

The two key parameters that appear in the analysis of these programs are k and ρ. As we have discussed, and following Stern, an appropriate value for ρ is gotten by using the discount factor of 0.001 per annum and compounding this to give a generational discount *factor* of $\rho = (0.999)^{25} = 0.975$. An appropriate value for k is 20; about 5% of the labor force is involved in teaching the next generation, and so we may say that one unit of skilled teaching labor produces twenty units of skill in the next generation (this assumes that skill is not increasing with time).

These programs are simpler to analyze than the fully articulated program with emissions and global warming, described in Sect. 4, but we hope, by studying them, to gain some intuition for what may be the case with the three analogues of these programs, when the set of feasible paths involves the full complexity of emissions, the quality of the biosphere, and the production of knowledge.

I now summarize some of the results of LRSb, which studies programs SUS, DU, and R. The first question to answer is: When does the DU program *diverge?* If this is the case, there will be many feasible paths for which the infinite value occurs, and we then must adopt some other method for assessing the relative desirability of these various paths. Clearly the most desirable path must be one of these (since any one dominates any path giving a finite value to the objective), but how do we choose among them? More on this below.

We have:

Result 1 *The DU program diverges if and only if* $\rho k > 1$

If we choose parameters $\rho = 0.975$ and $k \approx 20$, which I have argued are reasonable values, then the DU program diverges.

The second result is quite remarkable; it states

Result 2 *If the DU program diverges, then the feasible path that solves SUS is also the feasible path that solves R.*

In particular, although the DU program may diverge, the SUS program never diverges: regardless of the value of the parameters (ρ, k) there is a well-defined solution of SUS. Result 2 states that if $\rho k > 1$ then the sustainabilitarian ethical observer, who accepts the uncertainty of the existence of future generations, can nevertheless *ignore this uncertainty,* in the sense that the solution to the program R with which she is concerned is exactly the same as the solution the program SUS, which ignores the uncertainty!

What is the intuition behind this result? If the DU program diverges—that is, if $\rho k > 1$— it is, intuitively, because the educational technology is highly productive, that is, because k is 'large.' Now when the sustainabilitarian takes account of uncertainty, she wants, at first blush, possibly to reduce the amount of resources allocated to generations very far in the future, because there is a substantial probability that these generations will not exist (and so it would be a waste, perhaps, to provide for them). But if the educational technology is sufficiently productive, she need not economize in this way. Put another way, the only time she *would* rationally reduce resources allocated to generations far in the future is if the educational technology were not very productive. If you will, a large value of k trumps the uncertainty involved in the possible non-existence of future generations: a large k makes society so 'wealthy' that it can afford to ignore the possible waste of resources on future generations who turn out not to exist.

We finally confront the issue of what the utilitarian Ethical Observer should recommend in the case that the DU program diverges. We must adopt some rule for comparing feasible paths which give an infinite value to the DU program—for surely, as we said, it is among these paths that the EO must choose. Economists have proposed choosing among these paths by employing an *overtaking criterion*. There are a number of overtaking criteria proposed in the literature. The most recent such proposal is by Basu and Mitra (2007), who propose that a feasible path of amenity vectors, which we will denote $\{z(t)|t = 1, 2, \ldots\}$ is *at least as desirable* as a feasible path of amenity vectors $\{\hat{z}(t)|t = 1, 2, \ldots\}$ if and only if:

$$(\exists T)\left(\sum_{t=1}^{T} \rho^{T-1} u(z(t)) \geq \sum_{t=1}^{T} \rho^{T-1} u(\hat{z}(t))\right) \text{ and}$$
$$(t > T \Rightarrow u(z(t)) \geq u(\hat{z}(t))).$$

In English, this says that the sum of discounted utilities up to some date T on the first path is at least as large as that sum for the second path, and for dates later than T, the utility of every generation on the first path is at least as large as the utility on the second path.

The Basu-Mitra criterion induces a partial ordering of paths for which DU diverges, not a complete ordering. Nevertheless, it is an appealing proposal. What are its consequences for our problem? One can show:

Result 3 *If $\{z(t)\}$ is a path upon which utility eventually grows at a rate g, and if $\{\hat{z}(t)\}$ is a path at which utility eventually grows at rate $\hat{g}$, and $g > \hat{g}$, then the path $\{z(t)\}$ is more desirable than the path $\{\hat{z}(t)\}$ according to the Basu-Mitra overtaking criterion.*

Now if path $\{z(t)\}$ eventually grows faster than path $\{\hat{z}(t)\}$, that must be because resources have been transferred *away* from early generations *towards* future generations in the path $\{z(t)\}$ compared to the path$\{\hat{z}(t)\}$—that is the early generations consume less and work harder to accumulate capital that benefits the future generations. In other words, if the DU program diverges, then the utilitarian ethical observer recommends paths that reduce utility to the lowest possible levels for early generations, entailing the highest possible rate of growth of utility for later generations. (Indeed, there is no 'best' path according to the Basu-Mitra criterion, but we can say that some paths are more desirable than others, and ones which are high on desirability almost starve the early generations.) In contrast, what does the sustainabilitarian do when DU diverges? She recommends the path that equalizes the utility for all future generations (even if, ex post, at some point, they will turn out not to exist) at the highest possible level.

Discounted utilitarianism is usually associated with an ethical view that awards lower utility to generations far in the future than to generations near the present. But this intuition is only valid if the DU program converges. When it diverges—and if we extend the DU ordering using an overtaking criterion such as Basu and Mitra's—in fact *higher* utility is awarded to generations far in the future than near the present. Indeed, the recommendation is to impoverish the early generations.

Now let us return to the problem of global warming, which interests us. We have shown in LRSa that, even if we constrain emissions not to be so great as to engender unacceptably high global temperatures, the process of technological innovation and the productivity of the education technology will allow us to construct feasible paths upon which the discounted sum of utilities will be infinite—and this claim depends, as well, on our view that the value of ρ should be quite close to one. Assuming that Result 2 extends to that model, then the sustainabilitarian who takes account of uncertainty can ignore the uncertainty. The discounted utilitarian, if she invokes a Basu-Mitra overtaking criterion to compare feasible paths, must recommend starving early generations in favor of generating high rates of growth of the utility for the distant future. In contrast, the sustainabilitiarian, by Result 2, even acknowledging uncertainty, will recommend the path which equalizes the quality-of-life for all generations at the highest possible level, even though, at some date, those generations will, with probability approaching one, not exist.

8 Conclusion

Our main conclusions are these:

1. Solving the problem of the just intertemporal amenity path for a society consisting of many generations requires the maximization of a social welfare function[12] provided by a theory of justice, subject to constraints provided by facts about the world.
2. Two major weaknesses exist in current economic analysis of this problem: first, the analyses almost always assume that the correct intergenerational ethic is utilitarianism, and second, the modification to discounted utilitarianism is often based on the ethical view that the decision problem for a society with many generations is ethically equivalent to the decision problem of an infinitely-lived consumer.
3. The two main approaches for determining what the discount rate should be are that of maximizing the utility of the present generation as a proxy for society (Approach A), and that of the uncertainty of the existence of future generations (Approach B). We argue that only method B is ethically valid. Consequently, the discount factor (discount rate) that most analyses have adopted is too small (respectively, large).
4. Intergenerational maximin, or sustainability of welfare, is arguably a more attractive ethic than utilitarianism. Maximizing a sustainabilitarian objective will give quite different policy recommendations from maximizing a discounted utilitarian one.
5. If Approach B is adopted as the justification of discounting, and if the technology is sufficiently productive, then the discounted utilitarian program diverges. In this case, the solution of the discounted sustainabilitarian program is exactly the same as the solution of the undiscounted sustainabilitarian program. In other words, the sustainabilitarian ethical observer may ignore the kind of uncertainty discussed in approach B.
6. A consumptionist fallacy is prevalent in intergenerational welfare analysis, which conceptualizes human welfare as a function only of commodity consumption. This limits seriously the avenues for maintaining or increasing welfare.
7. Based upon parameterization of a model in which the consumptionist fallacy is avoided, by recognizing that educated leisure, the quality of the biosphere, and knowledge are direct inputs in human welfare, we estimate that to achieve welfare sustainability at the highest possible levels, we should be investing at a considerably higher rate in capital and knowledge than we currently are. If our conjectures about high productivity are correct, this policy recommendation holds *even if we are uncertain* about the existence of future generations (see point 5).
8. If the discounted utilitarian program diverges, and the utilitarian ethical observer adopts an overtaking criterion to order amenity paths, she will recommend virtual starvation of early generations in order to maximize the long-term rate of growth of welfare for later generations—even though very late generations may not exist.

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[12] More generally, choosing among paths of infinite utility streams.

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