This version: April 1, 2011

"Sustainability in the presence of global warming: theory and empirics"

by

Humberto Llavador (humberto.llavador@gmail.com)

John E. Roemer (john.roemer@yale.edu)

Joaquim Silvestre* (jbsilvestre@ucdavis.edu)

1. Introduction

For thousands of years prior to 1850, the concentration of carbon dioxide (CO_2) in the earth's atmosphere was approximately 280 parts per million (ppm). Over a longer period, the 800,000 years prior to that date, the average concentration of atmospheric CO_2 was approximately 240 ppm, with small cyclical variations. But since 1850, due primarily to the burning of fossil fuels, atmospheric CO_2 concentration has risen to approximately 390 ppm; the present rate of increase is about 9 gigatons of carbon (GtC) per annum, equivalent to 1-2 ppm per annum. The earth's temperature is a logarithmic function of CO_2 concentration. There is uncertainty concerning its exact parameters; but the most common estimate is that every doubling of atmospheric CO_2 concentration will increase the earth's temperature by 3^0C, in long-run equilibrium[1]. (The long-run will most likely take at least several centuries to be realized.) Since 1850, the earth's average temperature has increased 0.75ºC. If we continue to emit 9 GtC per annum, by 2100, the atmospheric concentration will be approximately 500 ppm, inducing an equilibrium temperature increase of about 2.5ºC (or perhaps an increase as high as 5ºC). The last time that atmospheric concentration was this high was four million years ago, when the sea level was 16 to 20 meters higher than at present, due to melting of the Greenland ice sheet and parts of the Antarctic ice sheet. We can therefore project that, at a long-run equilibrium of 500 ppm CO_2, sea levels would rise 16 to 20 meters.

* The authors teach at the University of Pompeu Fabra, Yale University, and the University of California, Davis, respectively. This paper is prepared for the United Nations Development Program (UNDP).

[1] The number of degrees Celsius associated with a doubling of atmospheric CO_2 concentration is called *climate sensitivity*. Although a value of 3ºC is commonly used, there is substantial uncertainty, with some scientists saying 6ºC, or even higher, is more realistic.

Nicholas Stern (2008) has written that "greenhouse gas (GHG) emissions are externalities and represent the biggest market failure that the world has seen." Mankind must cooperate to reduce greenhouse gas (GHG) emissions to prevent a catastrophic rise in global temperature, with its concomitant effects on sea level, rainfall, drought, storms, agricultural production, and human migration. What is the appropriate way of evaluating how the costs of reducing GHG emissions should be shared across the present and future generations, and within the next few generations, across regions of the world? How should the intergenerational and inter-regional resource allocation be regulated?

These are normative questions: their answers depend upon the theory of distributive justice held. We will argue, with respect to the intergenerational question, for a theory of justice that is motivated by the concept of *sustainability*. With respect to the inter-regional issue, which we discuss in sections 9 and 10, we will not take a fundamentally normative approach, but rather a political approach, where we propose what we believe is a politically acceptable solution to the bargaining problem that is at present taking place between major national actors (think: the US and China) concerning the reductions in GHGs that should be implemented. One reason for the difference in our approaches to these two problems is that future generations cannot bargain with us, and so we should take an ethical posture towards them. Major nations of the world, however, are actively engaged in arguing and bargaining over the second problem, and our role with respect to these negotiations is to behave like an arbitrator and propose what (we believe) is a mutually acceptable solution.

2. The concept of human well-being

We frame our discussion in a conventional formal model. Consider an intergenerational society evolving through a countable number of periods or "dates," indexed by $t \geq 1$. To each date t corresponds a generation, also indexed by t, and we postulate a single representative person at each generation.[2] Generation t's well-being is taken to be a function of five arguments: consumption, leisure, education, the stock of

[2] We think of a date or a generation as spanning 25 years.

human knowledge, and the biospheric stock.[3] The first three are private goods, the last two public goods. In particular, we choose as the utility function

$$(c_t)^{\alpha_c}(x_t^l)^{\alpha_l}(S_t^n)^{\alpha_n}(\hat{S}^m - S_t^m)^{\alpha_m}, \quad (1)$$

where c_t is the consumption of a single produced commodity, x_t^l is leisure time multiplied by the level of education (or human capital), S_t^n is the stock of human knowledge, S_t^m is the atmospheric concentration of carbon (all these variables referring to generation *t*), and $\hat{S}^m$ is a level of atmospheric concentration that we take to be catastrophic. For future reference, the vector of five arguments may be written $x_t \equiv (c_t, x_t^l, S_t^n, \hat{S}^m - S_t^m)$ and we may write the utility function in the abstract form $u(x_t)$.

The utility function (1) is non-traditional in several ways. First, we believe that leisure is more valuable to the individual if it is educated: the idea is that education increases the diversity of uses of one's leisure time, and diversity should, ceteris paribus, increase the quality of life.[4] Secondly, we include the stock of human knowledge: this models the idea that the quest for knowledge is part of human nature, and that our lives go better to the extent that we collectively understand more about the universe. More prosaically, one might also take the level of knowledge to be a proxy for life expectancy. Thirdly, we include the biospheric good directly in the utility function and not only as damage in the production function (a contrast with what many economists do, as we shall report below).[5]

We remark that there are close similarities between our concept of utility and the Human Development Index (HDI), proposed by the first *United Nations Development Report* (United Nations Development Programme, 1990). The HDI aggregates three

---

[3] Many utility functions present in the literature include one or several of these arguments. Environmental amenities, in particular, often appear as arguments in natural resource models, see, e. g., Jeffrey Krautkraemer (1985).

[4] More generally, education also increases the enjoyment of labor time –jobs requiring education are in general more fulfilling and interesting than unskilled jobs. Thus, there are two justifications for including the level of education in the utility function.

[5] However, our calibration of the utility function, $(\alpha_c, \alpha_l, \alpha_n, \alpha_m) = (0.319, 0.637, 0.016, 0.028)$, puts small weights on the non-traditional arguments knowledge and biospheric quality.

dimensions: health, education and living standards, the last one being measured by GNI per capita (see, e. g., United Nations Development Programme, 2010, Figure 1.1). The arguments in our utility function for the representative person explicitly include consumption (a component of GNI per capita), and education. As just mentioned, health can be thought of as at least partially proxied by knowledge, because the improvements in life expectancy, health status and infant survival are to a large extent due to medical discoveries. The more recent Human Development Reports (e. g., United Nations Development Programme, 2010), as well as Eric Neumayer (2001) among others, emphasize three dimensions of human welfare acknowledged by the authors of the 1990 HDI but neglected in its definition, namely environmental and climate sustainability, the equality of distribution within and across countries, and human security, rights and freedoms. Our utility function shares with the 1990 HDI its abstraction from security, rights and freedoms, but it explicitly includes as an argument the quality of the biosphere. Distributional concerns within a country are neglected both by the 1990 HDI and our utility function, but our analysis in sections 9 and 10 below does address the convergence of North and South within two generations and the fair allocation of CO_2 emissions between world regions.

3. Sustainability

Sustainability has been advocated as the right ethic for dealing with environmental externalities by many authors – too many for us to review here. (For references, see Llavador, Roemer and Silvestre, 2011a.) All definitions of the concept fall into one of two categories, called weak and strong sustainability. Strong sustainability obtains when we engage in choices over time which preserve the levels of certain resources, which include natural and biospheric resources. (In the present case, the key resource is a non-carbon-infused atmosphere, with the corollary preservation of species that accompanies it.) Weak sustainability does not require preserving amounts of physical resources, but rather preserving, intertemporally, measures of human well-being – whether that be called utility, welfare, capability, opportunities, standards of living, quality of life, or whatever. Weak sustainability would accept, in principle, the destruction of biospheric resources if human well-being could be maintained through substitution. One important distinction between the two varieties of sustainability is that the weak variety is anthropocentric. Non-human species are valuable only to the degree that they foster human well-being.

A. Pure sustainability

We find strong sustainability to be an attractive idea, but we shall not advocate it in this paper. Rather, we shall advocate and model weak sustainability. Given the technology available at the beginning (date zero), the stocks of resources, capital, and labor available at date zero, and a theory of how technology evolves as a function of investments in education and knowledge, we can define a set of *feasible intergenerational paths of utility arguments* $x \equiv (x_1, x_2, \ldots)$: call the set of such feasible paths X.[6] We define the *problem of pure sustainability* as the problem of choosing the path in X which guarantees the *highest feasible* level of well-being to *every* generation in the infinite future.[7] That is, the *sustainable path* is the solution to:

$$\max \Lambda \qquad \text{subject to:} \quad u(x_1) \geq \Lambda, \; x \equiv (x_1, x_2, \ldots) \in X \; . \quad \text{(SUS)} \qquad (2)$$

Another way of writing program (SUS) is:

$$\max_{x \in X} \min_{t \geq 1} u(x_t). \qquad \text{(SUS*)} \qquad (3)$$

Thus, our concept of sustainability is simply an intergenerational application of the Rawlsian difference principle, or what is called, in social-choice theory, ‘maximin.’

To formulate our approach in the terminology of social-choice theory, we say that a sustainabilitarian Ethical Observer (EO) maximizes the intergenerational social welfare function:

$$\min_{t \geq 1} u(x_t) \, .$$

---

[6] The discussion that follows abstracts from the list of arguments and form of the utility function (1). In particular, x_t can be interpreted as an abstract list of utility arguments.
[7] We refer to “pure sustainability” in contradistinction to “sustainability with growth,” to be studied in Section 2.B below.

If the solution to (SUS) is $\left(x^*, \Lambda^*\right)$, then we say it is possible to sustain human well-being forever at the level Λ^*, and at no higher level. In the application to the climate-change problem, we will be interested in finding the sustainable level of human well-being associated with a suitable choice of the intertemporal path of carbon emissions. Presumably, if our intuition is correct, paths that constrain emissions such as to maintain atmospheric carbon concentration at fairly low levels will sustain higher levels of human welfare than paths which do not.

The ethical justification for sustainability, in our view, is that the date at which a person is born is morally arbitrary. Thus every generation has a *right* to a level of well-being at least as high as that of any other generation. This leads to SUS. Our concept of sustainability is in line with the one in Sudhir Anand and Amartya Sen (2000), who justify it by an appeal to "universalism." But we justify sustainability on the grounds that the date at which a person is born is morally arbitrary, which in turn validates "universalism."

Now one might object that the human species will not last infinitely into the future, and so the formulation of SUS is utopian – a point we will address in Section 2.C below.

B. Sustainability with growth

The concept of well-being defined above might be called the quality of life, or even the standard of living. But it omits something that may be of great importance to people, the development of the human species over time. We may be rightfully proud that, as a species, our lives have improved immeasurably over time due to the organization of our societies, and may value that human development continues into the future. Yet the value we place on that future development is not included in our definition of well-being. It is for this reason that we prefer to think of the utility measured by (1) as quality of life/standard of living rather than overall welfare.

If indeed we do put value on human development, we –the present generation– may choose *not to enforce our right* to be as well off as all future generations: we may prefer to allow future generations to become better off than we are, at some (perhaps small) cost to ourselves. A simple way to model this is as follows. The problem of *sustainable growth at rate g* is:

$$\max \Lambda \qquad \text{subject to:} \quad u(x_t) \geq (1+g)^{t-1}\Lambda,\ x \equiv (x_1, x_2, \ldots) \in X \qquad (g\text{-SUS}) \quad (4)$$

In words, the solution of (*g*-SUS) is the feasible path which maximizes the well-being of the first generation ($t=1$) subject to guaranteeing a rate of growth of g in well-being, per generation, forever. Problem g-SUS becomes (SUS) for $g=0$.

We can also formulate (*g*-SUS) as a maximin problem. It is equivalent to:

$$\max_{x \in X} \min_{t \geq 1} \frac{u(x_t)}{(1+g)^{t-1}}. \qquad (5)$$

For the set *X* that describes our own economy, the value of Λ in the solution to (*g*-SUS), if $g=0$, will be smaller than the value of Λ that solves (SUS): in other words, the first generation will be worse off under sustainable positive growth than under pure sustainability. If the value of *g* is sufficiently small, and our generation does value human development, then presumably the trade-off just described will be acceptable to us.

We compute below the solutions to (SUS) and (*g*-SUS), for small values of *g*, for our economy.

C. Sustainability when the existence of future generations is uncertain

It has been traditional in growth theory to study the problem of maximizing inter-temporal social welfare when there is an exogenous probability *p* that each generation will be the last one, assuming that the species has not already ended. Under this simple stochastic process, the probability that generation *T* will be the last one is

$$(1-p)^{T-1}p. \qquad (6)$$

The Ethical Observer (EO) must recommend a feasible infinite path $x \equiv (x_1, x_2, \dots)$ of utility arguments. Given *x*, if Generation *T* were the last one, then the minimum utility reached would be

$$\min_{1 \le t \le T} u(x_t)\,, \tag{7}$$

and this is the function that the ‘sustainabilitarian’ EO would maximize if she knew that the human species would last exactly *T* dates. But in actuality the EO faces uncertainty as to the date of extinction of the species, which may naturally lead her to maximize the expected value of (7), which because of (6) is:

$$\sum_{T=1}^{\infty} (1-p)^{T-1} p \min_{1 \le t \le T} u(x_t)\,. \tag{8}$$

We will report results of maximizing (8) below.

Clearly, the idea of (8) immediately generalizes to sustainability with growth, in which case it becomes

$$\sum_{T=1}^{\infty} (1-p)^{T-1} p \ \min_{1 \le t \le T} \frac{u(x_t)}{(1+g)^{t-1}}\,.$$

4. Utilitarianism

The venerable foil to maximin in social choice theory and political philosophy is *utilitarianism*, which in its original, undiscounted version (Jeremy Bentham, 1789, John Stuart Mill, 1848), proposes the maximization of the sum of the individual utilities, modeled by the social welfare function:

$$\sum\nolimits_{t=1}^{\infty} u(x_t) \tag{9}$$

where the utilities are unweighted because “each individual must count as one, and none as more than one.” (See Jon Elster, 2008.)[8]

---

[8] There is an important mathematical difference between problem SUS (2) and problem of maximizing (9) subject to $(x_1, x_2, \dots) \in X$, a problem for which a solution may not exist: it might be possible to find

As is well-known, John Rawls's (1971) main task was to discredit utilitarianism as a social ethic, and to replace it with maximin. Although his influence has been major among political philosophers, it has been minor among economists.

This is true in particular of environmental and resource economists, who continue, in large part, to base their ethical prescriptions on utilitarianism, more specifically on *discounted utilitarianism*, a modification of (9) where the utilities of future generations are discounted by a factor φ < 1, so that the social welfare function (9) becomes

$$\sum_{t=1}^{\infty} \varphi^{t-1} u(x_t) \,, \tag{10}$$

i. e., the utility of a person born in the future is socially less worthy than that of one born today: a person born in the future "counts for less than one."

Even if one favors the utilitarian approach, why should the utilities of future generations be discounted? One answer is to justify relatively high discount factors by the uncertainty with regard to the lifetime of the species, as in the framework of Section D above: future generations should count less because they may never exist. More precisely, let the EO be utilitarian. If she knew the world would last exactly *T* generations, she would want to maximize

$$\sum_{t=1}^{T} u(x_t) \,. \tag{11}$$

(Note the contrast with the sustainabilitarian EO, who would maximize (7).) Because of the uncertainty with the probabilities expressed by (6), she would wish to maximize the expected value of (11), namely

---

infinite paths for which the utilitarian social-welfare function (9) becomes infinite, without a way of choosing among these paths. A solution to (2) always exists, on the contrary, for the set *X* describing our world. Note that there is a maximum to how large $u(x_1)$ can be: we cannot possible make the utility of the first generation infinite. But this value is an upper bound on the value of Λ in (2). Therefore, the solution to (2) exists. In other words, utilitarianism may not define a complete order on the set of paths *X*, while sustainabilitarianism always does. Note also that the difficulties in the existence of a solution become alleviated when (9) is replaced by the discounted utilitarian function (10), at least for small values of the discount factor φ.

$$p\sum_{t=1}^{T}(1-p)^{T-1}u(x_t)\,. \tag{12}$$

But, for $p > 0$, maximizing (12) is the same as maximizing (10) when $\varphi = 1 - p$.

The most prominent study of climate-change economics which uses the evaluative criterion (10) is that of Stern (2007). The Stern Review takes a value $p = 0.001$ per annum, implying a discount factor of $\varphi = 0.999$. Consequently, Stern discounts the welfare of those living a century from now by about 10% (because $(0.001)^{99} = 0.905$).

It should be also noted that the utility function used by Stern (2007) and the practitioners discussed in what follows (such as William Nordhaus, 2008) includes *only* consumption as an argument (and it is strictly concave in it). There is no reason these authors could not include the arguments that we include in (1) –but they have not. Section 7.C below discusses the implications, in our model, of including only consumption.

There are many environmental economists who use the discounted utilitarian formula (10) but choose smaller values for φ. Most prominent among these, in climate-change work, is Nordhaus (2008). Nordhaus, however, arrives at formula (10) not by the reasoning that leads to (12), but by assuming that the correct intergenerational social-welfare function is of the form:

$$\sum_{t=1}^{\infty}\left(\frac{1}{1+\delta}\right)^{t-1}u(x_t)\,, \tag{13}$$

where δ is a time rate of discount (i. e., the discount factor is $\varphi \equiv 1/(1+\delta)$). The justification that he gives for using this social welfare function is that humans do, indeed, discount their own future welfare. We fail to see why this implies that an Ethical Observer should discount the welfare of future generations. We believe Nordhaus commits the error of making a false analogy between the self-interested behavior of an individual who has preferences concerning her own consumption over time, and an ethical treatment of many equally deserving generations of humans.

Consistent with this justification, Nordhaus calibrates δ by looking at actual market interest rates.[9] The famous 'Ramsey equation' (see Roemer, 2011) expresses the relationship between the discount rate δ of a representative consumer who maximizes (13) subject to capital and labor constraints as a function of equilibrium interest and growth rates. Nordhaus uses *observed* real interest rates and growth rates to deduce a value of δ via the Ramsey equation: he gets δ = 0.015 per annum. Consequently, in Nordhaus (2008), the utility of those living a century from now is discounted by a factor of

$$\left(\frac{1}{1.015}\right)^{99} = 0.229\,;$$

their utility counts only 23% of ours.

The justifications that economists give for using discounted utilitarianism are often confused and circular. Partha Dasgupta (2008) writes:

> There are two reasons why it may be reasonable [to discount future consumption at a positive rate]. First, an additional unit of consumption tomorrow would be of less value than an additional unit of consumption today *if society is impatient to enjoy that additional unit now* (our italics). Therefore, impatience is a reason for discounting future costs and benefits at a positive rate. Second, considerations of justice and equality demand that consumption should be evenly spread across the generations. So, if future generations are likely to be richer than us [sic], there is a case for valuing an extra unit of their consumption less than an extra unit of our consumption, other things being equal. Rising consumption provides a second justification for discounting future consumption costs and benefits at a positive rate.

Consider the italicized phrase: who is society in that phrase? Surely, not future generations –*they* cannot be impatient to enjoy consumption *now*. Evidently, 'society' is the present generation –but why, we say, should their impatience justify depriving future

[9] Nordhaus also inserts a parameter η into the utility function which captures "the aversion to inequality of different generations." (Nordhaus 2008, p. 33, 60). Informally speaking, if the rates of growth turn out to be negative, then δ and η pull in opposite directions, a high δ favoring the earlier generations, and a high η favoring the later, less well-off ones. But for positive rates of growth, which makes later generations better off, high values of either δ or η favor the earlier generations: this is the case in the paths proposed by Nordhaus (2008). For a full discussion, see Llavador, Roemer and Silvestre (2011a, Section 6).

generations of welfare? The second reason Dasgupta provides is circular: *whether or not* future generations are richer than we are depends upon the path of resource allocation that is implemented –that path we are trying to solve for! If, for example, we decide to saturate the atmosphere with carbon, future generations will not be richer than we are. Dasgupta implies that he would increase the discount rate in (13) until the solution to the program spread consumption 'evenly across generations.' What, then, is the purpose of the optimization? If 'evenness of consumption' is the desideratum, why not just equalize consumption at the highest possible level over time –sustainable consumption?

5. Sustainability versus discounted utilitarianism

There are, in sum, two justifications for using discounted utilitarianism in the climate-change literature. The first assumes a utilitarian Ethical Observer who discounts the well-being of future generations only because they might, in the event, not exist. This is the approach of the Stern Review, and it is logical and consistent. The discount factor φ is chosen very close to one to reflect the small probability of human extinction in any given year. The second approach – of Nordhaus, Dasgupta, and many others— adopts (13), but uses a much smaller discount factor, associated with rates of time preference of living consumers. *There is no clear, logical justification for this practice*. Since small discount factors are used by these practitioners, they typically recommend less stringent constraints on GHG emissions.

Neither of the discounted-utilitarian approaches we have described has any clear relationship to the ethic of sustainability. As we said, utilitarianism was, until around 1970, the unchallenged theory of distributive justice in political philosophy, and it continues to exert its influence upon economists, despite the many criticisms that have been waged against it. Second, there is a long tradition in growth theory of postulating an infinitely lived consumer, whose utility function for consumption over time is given by expression (13). Many economists have insisted upon a poor analogy between the decisions of such an infinitely lived and self-interested consumer, and *social* decisions concerning future generations. It is noteworthy that Frank Ramsey (1928), the originator of formal growth theory, insisted that discounting the welfare of future generations was a practice due only to the 'failure of the imagination.'

6. The set of feasible paths

The society in the first model that we describe consists of an infinite sequence of generations, as described in Section 2 above. Recall in particular that (1) expresses the utility –welfare, standard of living or quality-of-life– of the representative person in Generation t. In this section, we describe the set of feasible paths of resource use, X of this intergenerational society. In the next section, we discuss optimization.

The set of feasible paths of resource allocation, are specified as follows.

- There are three production sectors: *commodity production* uses as inputs skilled labor, capital, accumulated human knowledge, biospheric quality, and the level of GHG emissions permitted. The *production of knowledge* is purely labor-intensive using only skilled labor and past knowledge (think corporate research and development, and university research). *Education* is purely labor intensive, using only the skilled labor of the preceding generation.
- There are four conduits of intergenerational transmission: capital passes from one generation to the next, after investment and depreciation; knowledge passes in like manner, with depreciation; the stock of biospheric quality diminished by emissions passes to the next generation. The fourth conduit is education: the education effort of one generation increases the efficiency of the labor time of the next one. Even though the total time, in hours, available to a generation for work and leisure is constant, the number of *efficiency units* of labor-leisure time available to a generations increases with the accumulated investment in education.
- One very important production function is not explicitly modeled: the evolution of biospheric quality from emissions. One might postulate a law of motion for the process by which biospheric quality at date t+1 consists of biospheric quality at date t, partially rejuvenated by natural processes that absorb carbon dioxide, plus the impact of new emissions of GHGs. However, the scientific view on the nature of this law of motion is very

much in flux, and so we have elected *not* to imply a false precision by inserting such a law into our model. In place of doing so, we simply take a path of emissions and concomitant atmospheric concentration of carbon dioxide computed from the popular Model for the Assessment of Greenhouse-gas Induced Climate Change (MAGICC; a previous version was used by the IPCC AR 4, Working Group I, see Meehl *et al*., 2007) which stabilizes the atmospheric concentration at 450 ppm CO_2, and we constrain our production sector not to emit more than is allowed on this path. That is to say: *we do not* optimize over possible paths of future emissions.

- The set of feasible paths consists of all paths of resource allocation beginning at date zero (taken to be 2000). A feasible path specifies the allocation of labor into four uses at each date (commodity production, education, research and development, or knowledge, and leisure), and the allocation of commodity output between investment in capital and consumption. At each date a pre-specified level of GHG emissions constrains the output of the commodity-producing sector. The inter-generational links described determine the social endowments of educated labor, knowledge, capital, and biospheric quality at the next date. At each date, the utility of the representative agent is computed along the path, as a function of the arguments described above. Figure 1 summarizes the stocks and flows that exist at a given date in the model. The set of all feasible paths, beginning with endowments at 2000 is the set *X* described above.

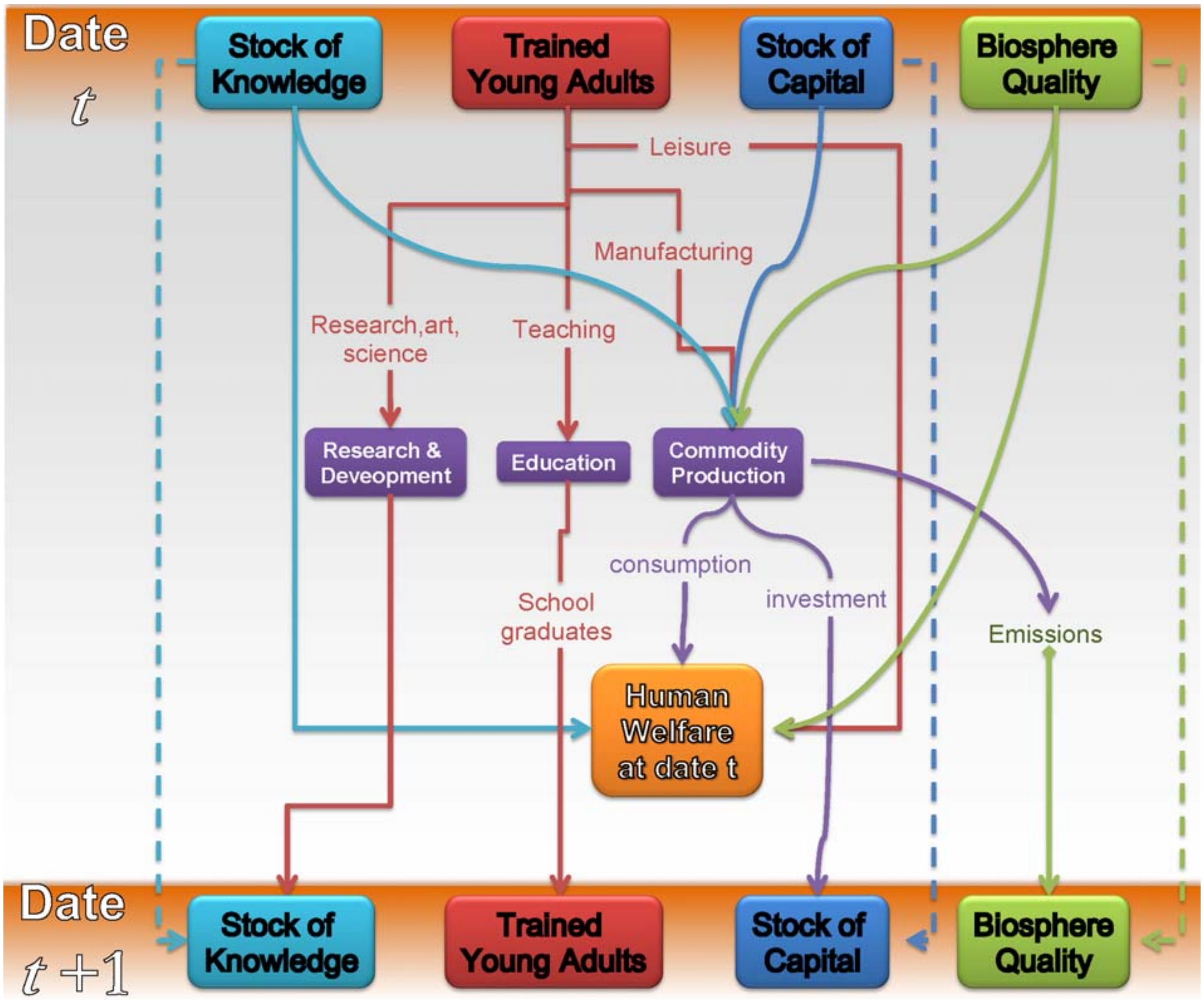

Figure 1. The stocks and flows in the model

Figure 1 illustrates the stocks and flows in the model.

It is important to remark that technological change is modeled by the presence of knowledge, accumulated through investment in R&D, as an input in commodity production. Thus knowledge can substitute for capital, labor, and emissions through the process of technological change.

The three production functions, the utility function, and the initial aggregate endowment vector in this model are calibrated using US data. Of course, it would be preferable to be able to calculate 'global' production functions. Therefore, our simulations in this model must be interpreted as referring to paths of resource allocation for a representative US household across time.

In Section 10 we present a global model with two world regions: North and South.

7. Optimal sustainable paths

We choose the path to maximize the utility of the first generation, subject to guaranteeing a rate of growth of utility of *g* for all future generations as in program (*g*-SUS) above. We compute this path for various values of *g*. We do not propose a rule for adjudicating among various values of *g*: but our calculations suggest that values of *g* of 2% per annum (64% per generation) are more ethically attractive than the optimal path at $g = 0$.

What are the lessons of the model which interest us? First, we seek to understand what rates of growth of human welfare can be sustained, given the postulated constraints on emissions. Second, we wish to understand the trade-offs implied by choosing to grow at higher rates: for instance, it turns out to be feasible to support welfare growth of 64% per generation with our calibration (2% per annum), but the cost will be lower welfare for the first generation than it would enjoy under a 0% growth scenario. What is the magnitude of this trade-off? Third, we wish to understand how labor is allocated among its four uses on the optimal path for various values of *g*: labor allocated to commodity production, to educating children, to research and knowledge production, and to leisure. Should we radically re-allocate labor from its present allocation?

For a mathematical statement of the set of feasible paths, using the explicit production functions that we calibrate, and an explanation of how these optimal paths are computed, see Llavador, Roemer and Silvestre (2011a).

As noted above, we *fix* a path of global GHG emissions, which will approximately converge to an atmospheric CO_2 concentration of 450 ppm by 2075. Emissions are modeled as an *input* into commodity production, and this amounts to a constraint on commodity production. Since the model must be interpreted as referring to the US economy, but our emissions path specifies global emissions, we must decide what fraction of global emissions to allocate to the US. We study two scenarios: in the first, the US is allocated 24% of global emissions, its recent de facto share; in the second, it is allocated emissions in proportion to its population –approximately 4% of global emissions. Table 1 summarizes these two emissions scenarios.

	World CO_2 Emissions (GtC)	US CO_2 Emissions (GtC) Scenario 1 $\left(e^{US} = 0.24 \times e^{World}\right)$	US CO_2 Emissions (GtC) Scenario 2 $\left(e^{US}_{\text{per capita}} = e^{World}_{\text{per capita}}\right)$	Stock of CO_2 in (World) Atmosphere (GtC)
Year 2000	$\overline{e}^{World}_{2000} = 6.58$	$\overline{e}^{US}_{2000} = 1.6$		$\overline{S}^{m}_{2000} = 772.6$
Generation 1	$\overline{e}^{World}_{1} = 7.69$	$e^{US1}_{1} = 1.85$	$e^{US2}_{1} = 0.27$	$S^{m}_{1} = 882$
Generation 2	$e^{World}_{2} = 6.05$	$e^{US1}_{2} = 1.45$	$e^{US2}_{2} = 0.19$	$S^{m}_{2} = 936.1$
Generation t, $t \geq 3$	$\overline{e}^{World*} = 4.14$	$e^{US1*} = 0.99$	$e^{US2*} = 0.13$	$S^{m*} = 954.1$

Table 1: Paths of environmental variables.

A. Pure sustainability path

Tables 2 and 3 summarize the characteristics of the optimal *pure sustainable path* –the solution to (g-SUS) for $g = 0$. Table 2 presents the paths for economic variables, relative to the base year 2000, and shows the following. In Scenario 1 (Scenario 2), utility is sustainable forever at a level 24% (15%) higher than year 2000 utility in the United States. Consumption per capita in Scenario 1 (Scenario 2) stabilizes at level 40.8 % (9.6%) higher than in 2000. The stock of capital stabilizes at about twice its 2000 level in Scenario 2, while the stock of knowledge stabilizes at *close to three times* its 2000 level in Scenario 2.

The first column of Table 3 indicates the quality of labor, i. e., number of efficiency units of labor, available to the generation for work and leisure, also relative to the year-2000 baseline. Note that in both scenarios there is growth of approximately 18% in the quality of labor. The last five columns of Table 3 convey the optimal fractions of the available labor-leisure efficiency units (or available labor-leisure clock time, it does not matter) devoted to the five activities: production of commodities for consumption, production of commodities for investment in physical capital, production of knowledge,

Gen.	*Utility*	*Generational utility growth factor*	*Consumption*	*Generational consumption growth factor*	*Investment*	*Stock of capital*	*Stock of knowledge*
Gen.	Scenario 1 $\left(e^{US} = 0.24 \times e^{World}\right)$						
2000	1.00	-	1.000	-	1.000	1.000	1.000
1	1.24	1.24	1.484	1.484	2.098	2.766	2.747
2	1.24	1.00	1.450	0.977	1.568	2.496	2.743
3	1.24	1.00	1.408	0.971	1.615	2.496	2.743
4	1.24	1.00	1.408	1.000	1.615	2.496	2.743
Gen.	Scenario 2 $\left(e^{US}_{\text{per capita}} = e^{World}_{\text{per capita}}\right)$						
2000	1.00	-	1.000	-	1.000	1.000	1.000
1	1.15	1.15	1.169	1.169	1.602	2.162	2.740
2	1.15	1.00	1.129	0.966	1.219	1.944	2.765
3	1.15	1.00	1.096	0.971	1.257	1.944	2.765
4	1.15	1.00	1.096	1.000	1.257	1.944	2.765

Table 2. Pure sustainability (g = 0). Paths for economic variables.

	Efficiency units of labor	Labor allocation (% of total efficiency units or of total time)				
		% in consumption	% in investment	% in knowledge	% in education	% in leisure
Gen.	Scenario 1 $\left(e^{US} = 0.24 \times e^{World}\right)$					
2000	1.0000	0.2251	0.0554	0.0165	0.0330	0.6700
1	1.1699	0.2150	0.0748	0.0374	0.0279	0.6444
2	1.1573	0.2201	0.0585	0.0322	0.0287	0.6604
3	1.1781	0.2196	0.0619	0.0316	0.0282	0.6587
4	1.1781	0.2196	0.0619	0.0316	0.0282	0.6587
Gen.	Scenario 2 $\left(e^{US}_{\text{per capita}} = e^{World}_{\text{per capita}}\right)$					
2000	1.0000	0.2251	0.0554	0.0165	0.0330	0.6700
1	1.1699	0.2155	0.0726	0.0373	0.0281	0.6460
2	1.1667	0.2201	0.0584	0.0323	0.0287	0.6604
3	1.1876	0.2196	0.0619	0.0316	0.0282	0.6587
4	1.1876	0.2196	0.0619	0.0316	0.0282	0.6587

Table 3. Pure sustainability (g = 0). The allocation of labor.

education, and finally leisure. The fraction of the labor-leisure total devoted to production for consumption, as well as the fraction taken in leisure time fall somewhat (of the order of 2%) compared to the 2000 benchmark. The fraction devoted to education experiences a decrease of about 15% in the stationary state, while the fraction devoted to research and development *almost doubles.*

The interpretation of these results is that we can sustain a level of utility higher than present levels, *while constraining emissions to keep atmospheric* CO_2 *concentrations at* 450 *ppm*, but doing so requires a re-allocation of labor from other uses to knowledge production. It is perhaps notable that the stationary-state labor allocations are very similar in the two scenarios.

As we said, these are computations of feasible paths, in order to maximize an intergenerational social welfare function. There are no markets or prices here. Because these paths are Pareto efficient, there are prices and Pigouvian taxes which can implement them in a market economy. For example, the salaries of knowledge workers would be higher, relatively, than at present, to encourage more workers to enter the knowledge industry.

Much has been written about the pricing of carbon. What should firms and consumers be paying to emit carbon into the biosphere? The answer, in our model, is complicated, because emissions enter directly as an input in production, but also as a negative externality in both production and welfare (since the level of carbon concentration enters negatively into both our production and utility functions). The social price of carbon should be *at least* the marginal value product of emissions in commodity production. For the first generation on the optimal path in Scenario 1, this is \$819 per (metric) ton of carbon or \$0.37 per pound. Since a gallon of gasoline contains 5.48 pounds of carbon, this implies a carbon tax of at least \$2 per gallon. In Germany, current gasoline taxes are approximately \$3.50, considerably above what we have calculated as a lower bound on the optimal Pigouvian tax.[10]

---

[10] Here is the calculation. In Scenario 1, date 1, total production from table 2 (consumption plus investment) is $\$55.5\times10^3$ per capita, taking as the per capita values for the year 2000 consumption equal to 27.78 and investment 6.83. From Table 1, emissions are 1.85 GtC. We have calibrated the elasticity of emissions in total output as $\theta_e = 0.091$ (see Llavador, Roemer and Silvestre, 2011a). Taking the US population to be 3×10^8, the marginal value product of emissions in dollars per metric ton is

B. Sustainable growth path

Tables 4 and 5 present characteristics of the optimal path for sustainable growth of welfare at g = 2% per annum (or 64% per generation). First, note that by Generation 2 in both scenarios, utility is already much higher than on the pure sustainable path of Table 2. The cost of this growth is a diminution of utility in Generation 1: for instance, utility in Scenario 1 is 22% higher than in 2000, rather than, as in Table 2, 24% higher: a rather small cost to Generation 1 permits welfare growth that becomes huge over time.

Perhaps the most interesting contrast with the zero-growth optimal paths concerns the allocation of labor. There is a 5% diminution both in the fraction of total time spent in leisure as in the fraction of the total time devoted to the production of consumption commodities compared to 2000, vs. a 2% diminution in the purely sustainable, zero-growth path. Labor in the knowledge industry doubles, a somewhat higher increase than in the purely sustainable solution. But now the fraction of labor in education rises significantly, a 42% increase over the reference year 2000. Indeed, 22.5% of the labor force (which does not include leisure time) works in the knowledge and teaching sectors, compared to 17.5% in the purely sustainable solution, and to 15% in 2000.

Which optimal path do we prefer – the zero-growth-rate path, or the one which sustains 2% growth forever? The trade-off involves the utility of generation one. As we wrote earlier, although generation one has a right to insist on the zero-growth path, it may well itself prefer to purchase the future growth of human welfare, for the apparently small sacrifice in welfare which it must sustain to support it. On the other hand, one must recall that the present model assumes a representative household at each date, and thus it is incapable of dealing with intra-generational inequality. In reality, the diminution of utility of the first generation associated with the 2% annual growth rate would hit the very poor the hardest: taking this into account, the trade-off might not be worth it.

$\theta_e Y/e = \left(0.091\times 55.5\times 3\times 10^{11}\right)\big/\left(1.85\times 10^{9}\right) = \$8.19$. Since there are 2200 pounds in a metric ton, this is equivalent to \$0.37 per pound of carbon, or \$2.03 per gallon of gasoline.

Gen	*Utility*	*Generational utility growth factor*	*Consumption*	*Generational consumption growth factor*	*Investment*	*Stock of capital*	*Stock of knowledge*
	Scenario 1 $\left(e^{US} = 0.24 \times e^{World}\right)$						
2000	1.00	1.00	1.000	-	1.000	1.000	1.000
1	1.22	1.22	1.455	1.455	2.053	2.710	2.692
2	2.00	1.64	2.362	1.624	2.976	4.197	4.558
3	3.28	1.64	3.821	1.618	5.008	6.986	7.587
4	5.38	1.64	6.359	1.664	8.335	11.627	12.628
Gen	Scenario 2 $\left(e^{US}_{\text{per capita}} = e^{World}_{\text{per capita}}\right)$						
2000	1.00	1.00	1.000	-	1.000	1.000	1.000
1	1.13	1.13	1.146	1.146	1.566	2.118	2.685
2	1.85	1.64	1.839	1.604	2.315	3.268	4.595
3	3.04	1.64	2.975	1.618	3.899	5.439	7.648
4	4.99	1.64	4.951	1.664	6.490	9.053	12.729

Table 4. 2% annual growth or 64% generational growth: Paths for economic variables.

Gen	Efficiency units of labor	Labor allocation (% of total efficiency units or of total time)				
		% in consumption	% in investment	% in knowledge	% in education	% in leisure
	Scenario 1 $\left(e^{US} = 0.24 \times e^{World}\right)$					
2000	1.0000	0.2251	0.0554	0.0165	0.0330	0.6700
1	1.1699	0.2108	0.0731	0.0366	0.0471	0.6318
2	1.9547	0.2128	0.0659	0.0352	0.0477	0.6384
3	3.3063	0.2125	0.0685	0.0345	0.0470	0.6375
4	5.5031	0.2125	0.0685	0.0345	0.0470	0.6375
Gen	Scenario 2 $\left(e^{US}_{\text{per capita}} = e^{World}_{\text{per capita}}\right)$					
2000	1.0000	0.2251	0.0554	0.0165	0.0330	0.6700
1	1.1699	0.2113	0.0710	0.0365	0.0475	0.6333
2	1.9703	0.2128	0.0659	0.0352	0.0477	0.6384
3	3.3327	0.2125	0.0685	0.0345	0.0470	0.6375
4	5.5470	0.2125	0.0685	0.0345	0.0470	0.6375

Table 5. 2% annual growth or 64% generational growth: The allocation of labor.

C. Sustaining consumption

We turn now to focus upon our choice of utility function, with its many arguments. As we noted, Nordhaus (2008) and Stern (2007) both use a utility function with a single argument –consumption. We can ask: what would the optimal path for our set X look like, if instead of seeking the highest sustainable level of utility, we sought only the highest sustainable level of *consumption?* That is –if we replaced program (3) with:

$$\max_{x \in X} \min_{t \geq 1} c_t \text{ ?}$$

We performed this computation. Of course there will still be, on the optimal path, levels of knowledge and education –but those factors will have no direct effect on welfare, only on production. Knowledge's unique role will be technological development, and education's will be improving the skill of the labor force, for productive purposes. Figure 2 compares consumption, knowledge, and education on the optimal paths for utility-sustainability (at $g = 0$) and consumption-sustainability.

The result is that, if we sustain only consumption at the highest possible level, consumption is only slightly greater than it is on optimal *utility*-sustaining path: but knowledge and education are a great deal less on the consumption-only path. This shows quite dramatically, we believe, the importance of including knowledge and education in the objective of the sustainability program – assuming that we think knowledge and education, as well as consumption, *augment human welfare directly*. Indeed, this point also motivates the calculation of the human development index (HDI). If we agree that literacy, health and longevity are important for human welfare, then we should track something like the HDI rather than, simply, GDP or GNI per capita. Here, we say that, if we only try to sustain consumption at the highest possible level, we will forfeit important gains that could –and should– be had in knowledge and education.

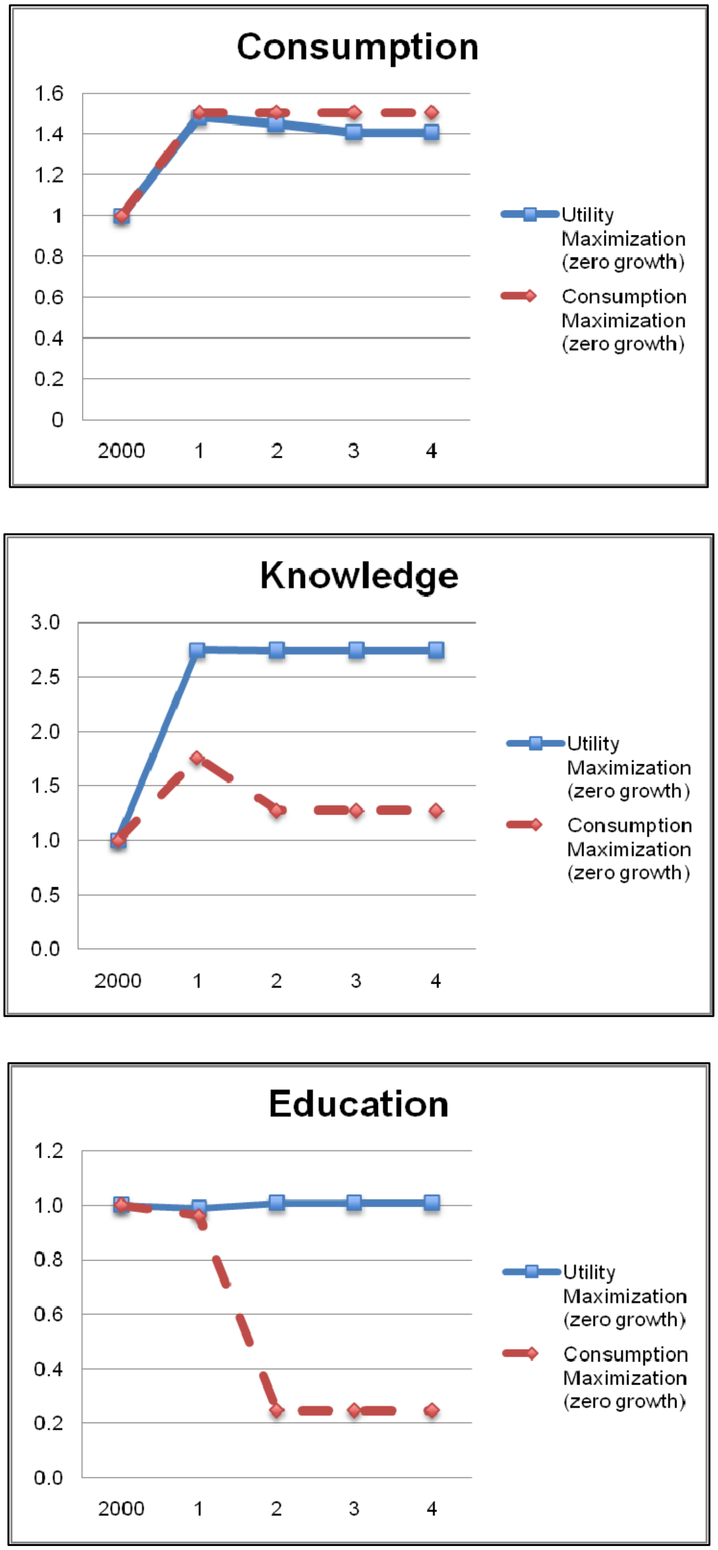

Figure 2. Paths of consumption, knowledge, and education on two alternative sustainable paths.

8. Uncertainty

As we wrote, the only justification that we believe holds water for discounting the welfare of future generations is due to the possibility that the human species will at some point come to an end, so we should rationally economize on what we allocate to generations in the distant future.

The uncertainty approach presented in Section 3.C above has interesting implications for the formal connection between the pure sustainability without discounting program (3) and pure sustainability with discounting (8). Consider the emissions path that converges to 450 ppm CO_2 by 2075 that we used in Section 7. In Llavador, Roemer and Silvestre (2010), we prove that, if p is sufficiently small – and the Stern Review's choice of $p = .0247$ per generation, corresponding to 0.001 per year is sufficiently small –then *the solution to* (8) *is identical to the solution to program* (3). That is, although discounting is conceptually necessary, the optimal path sustainable path *with* discounting turns out to be identical to the optimal sustainable path *without* discounting. The upshot is that Section 7 has solved for the optimal *discounted* sustainabilitarian path as well, as long as the probability of extinction associated with our choice of emissions path is small. It is beyond our scope in this paper to explain the intuition for this remarkable result, let alone to present the proof of it: the interested reader is referred to Llavador, Roemer and Silvestre (2010).

9. How should the emissions constraint be shared across regions of the world?

The approach we have outlined to the *intergenerational* problem is normative. Next, we will generalize the model discussed thus far to a global one, which addresses the issue of intergenerational and *inter-regional* allocation of resources to meet the climate-change challenge. In contrast, to the purely normative approach above, we here present an approach which combines normative and political assumptions. The normative assumption continues to be sustainability across generations; the political assumption is that the issue of inter-regional allocation, within each generation, will be guided by what we believe to be a reasonable solution to the bargaining problem between developed and developing countries.

We wish to propose a way that regions of the world can share the responsibility of reducing GHG emissions which, we think, is politically feasible. Here, too, we differ from what many writers have done, those who begin the study of the inter-regional allocation of emission rights with an ethical premise.[11] Two possible ethical premises, which are often mentioned are these: (1) that each nation of the world receive an endowment of carbon emission rights proportional to its population, and/or that (2) since most emissions in the past have come from the currently developed countries, the developing countries should now be allocated the lion's share of emission rights. One may hold both of these premises simultaneously. Think of each country's having been (implicitly) allocated carbon emission rights for all time according to premise (1); since the developed countries have been the major polluters thus far, they have used up most (or perhaps even more than) their carbon rights endowments. Thus for the global North to continue emitting carbon, it must pay the global South for the carbon rights that it may already have over-used, in addition to further payments for the purchase of carbon rights to allow it to continue polluting in the future.

We believe that neither of the premises (1) or (2) –attractive as they may be from a purely ethical viewpoint— is politically realistic. Thus, we depart here from a purely ethical stance, and propose what we believe is a realistic solution to the bargaining problem.

The biggest issue at present, we think, is constructing an agreement between China and the United States for restriction of GHG emissions. Together, these two countries at present emit over 40% of global emissions. Let us suppose, for the moment, that the world consists of a developing region with Chinese characteristics and a developed one with US characteristics. How should these giants solve the emissions problem? [12]

We suppose that there is agreement on the *total global emissions* that should be allowed for the coming period – perhaps a decade, or 25 years – and this would be derived from a calculation of how carbon emissions should be constrained in the long-

[11] The most thoughtful work we know written by a political philosopher on the ethics of global resource allocation to meet the climate-change challenge is Richard Miller (2010).
[12] Our assumption appears to leave out the poor part of the developing world which is not like China. See below for how we would incorporate these regions.

run. The issue is how to allocate these permissible emissions between the South and the North. These regions will bargain with each other to solve the problem, and for present purposes we think of this as bargaining between the US and China. (Indeed, this bargaining is going on at present – in its most public form, at the COP meetings in Copenhagen and Cancun, but also behind the scenes.)

A traditional economist's approach would be to apply something like the Nash bargaining solution to this problem. Rather than pursuing this tack, we follow Thomas Schelling's idea: that there may well be a *focal point* in the bargaining game. Indeed, we propose that there is a focal point and it is *the date at which China and the US are predicted to converge in GDP per capita.*

If China's GDP per capita grows at an average rate of 5% in the next century or so, and the US's grows at an average rate of 2%, the stated convergence will occur in approximately 75 years (see below). This date of convergence, we propose, is a focal point for both countries, because of the political importance of the event. If one believes the focal point will be the year at which Chinese GDP per capita reaches —say one-half, or two-thirds, of the US GDP per capita -- our argument will be unchanged.

We claim that the bargaining solution will be, or should be, the following: that a global South like China and a global North like the US share the carbon budget in such a way as to *preserve the date of convergence of their GDPs per capita.* For suppose, to the contrary, a sharing of emissions were proposed which would have the effect of advancing this date of convergence: the US would strenuously object! Why should China benefit in this way because of the climate-change problem? And conversely, suppose a sharing of emissions were proposed which would set back the date of convergence? Likewise, China would have grounds for a strenuous objection.

The idea behind the claim that preserving the date of convergence is the bargaining focal point is that a certain kind of neutrality should apply to solving the problem of carbon pollution. The neutrality here proposed is that neither China nor the US should gain relatively by virtue of the necessity of meeting the carbon challenge. We have taken 'relatively' to mean that the date of their convergence in GDP per capita (or more properly, as we will model it, welfare per capita) should be preserved. But as we next show, this implies a more general kind of neutrality –that both regions constrain their growth factors to the same degree.

Let y^{US} and y^{C} be the present GDPs per capita of the two countries, and let g^{US} and g^{C} be their projected growth rates in the future (or their average growth rates over the next long period), were there to be no climate-change problem. Then the date of convergence is the solution *T* to the following equation:

$$\frac{\left(1+g^{c}\right)^{T} y^{C}}{\left(1+g^{US}\right)^{T} y^{US}}=1. \tag{14}$$

Given that $y^{C}=\$5,970$ and $y^{US}=\$47,400$ (2008 figures) and for the growth rates stated above, the solution to equation (14) is $T=71.5$ years. Now multiply both *growth factors* by the same positive constant *r*: then we have the equation

$$\frac{\left(r\left(1+g^{C}\right)\right)^{T} y^{C}}{\left(r\left(1+g^{US}\right)\right)^{T} y^{US}}=1,$$

which is solved by the same value *T*. Thus, to preserve the date of converge, the two countries should share the carbon budget so as *to cut back their growth factors by the same fraction.* [13]

It obviously does not matter what the date of convergence would be for this argument to hold. If China and the US meet to negotiate sharing the carbon budget once every five years, they need only reduce their growth factors, from what they would have been, by the same fraction, in order to preserve the date of convergence.

We do not claim it will be a trivial matter to arrive at a sharing of the carbon budget, even if the countries agree in principle to reduce their growth factors by the same fraction. There will be many issues of verifiability, involved in calculating what growth rates are reasonable to project for the two countries, how cutting back on emissions will affect growth rates, and so on. But we think the stated principle should emerge, and will vastly simplify the bargaining process, if it does. To put this slightly differently, a neutral

[13] The growth factor is one plus the growth rate.

arbitrator could recommend the italicized formula stated two paragraphs above as a fair solution to the bargaining problem.

We now relax the assumption that the world consists of a region like China and a region like the US. The general principle that we advocate is: All sufficiently rich countries should share the responsibility for reducing GHG emissions by reducing their growth factors by the same fraction. Countries which are poor would be exempted from this restriction. The necessary amount of the reduction can be computed given the target carbon budget, and knowledge of the relationship between emissions and GDP. The analysis can be generalized to include investments in research and development to further reduce emissions.

We wish to emphasize an important aspect of the approach we have outlined here. In most discussions of the problem of how to share the burden of restricting emissions across countries, scholars have proceeded in this way:

Rights → carbon permits → trading permits → production.

In this sequence, there is an initial determination of the carbon permits each country should have rights to, based upon an ethical argument. These rights might be a simple formula of distribution in proportion to populations, or they might take account of the fact that the developed countries have already used up a good portion of their carbon rights in the past, as we have said. Then permits are issued to each country, followed by the opening of a carbon permit market, internationally. The carbon permit market acts as a mechanism, presumably, to transfer wealth from the rich to the poor world. Finally production takes place.

Our approach, which we emphasize is *political*, never discusses rights. It is summarized by

Bargaining focal point → equiproportional reduction of growth factors → optimal emissions assignment → production.

We do not say that the ‘ethical’ approach of beginning with rights is fundamentally unattractive: but we do not believe that it would or will produce an agreement.

10. Optimal sharing of emissions between the US and China: Calculation

Motivated by our view that a politically feasible solution to the bargaining problem entails that the US and China converge in *welfare per capita* in approximately three generations, we have computed an optimal path of resource use in which this occurs. The analysis is presented in Llavador, Roemer and Silvestre (2011b). We amend the model described in sections 6 and 7 above so that, at each date, there are two societies: one, the South, has the per capita endowments of China in the year 2005, and the other, the North, the per capita endowments of the US in that year. The populations of these two regions over next 50 years are assumed to follow the path stipulated in United Nations (2008) for the developed and developing world: these are presented in Table 6.

We fix the global path of emissions to be the one described in Table 1, which converges to an atmospheric CO_2 concentration of approximately 450 ppm in 75 years, and stays approximately fixed thereafter. There are now, however, two kinds of interaction between the South and the North, which did not exist in the model with only one representative household: there is diffusion of knowledge from the North to the South at each date, which is stipulated to exist, independently of any agreements. (See Jonathan Eaton and Samuel Kortum, 1999, and Wolfgang Keller, 2004, for a discussion of international technological diffusion.) Secondly, we allow for transfers of the produced commodity at each date between the two regions. The transfers are not to be thought of as trade – since there is only one good – but rather to be viewed as grants. (More on this below.) Finally, there is an additional new variable at each date that specifies the division of global emissions between the North and South at that date – and these variables are endogenous in the optimization problem.

$t=0$ Year 2005	$t=1$ Year 2030	$t=2$ Year 2055	**Units**
N_0^N	N_1^N	N_2^N	
1,216,550	1,281,630	1,27,240	thousands.
N_0^S	N_1^S	N_2^S	
5,295,730	7,027,630	7,874,740	thousands.
n_0^N	n_1^N	n_2^N	
0.187	0.154	0.139	proportion of total pop.
n_0^S	n_1^S	n_2^S	
0.813	0.846	0.861	proportion of total pop.
N_0	N_1	N_2	
6,512,280	8,309,260	9,149,990	thousands.

Table 6. Projected populations, global North (*N*) and South (*S)* for 2005, 2030, and 2055.

To summarize, the set of feasible paths now begins with initial conditions which are the per capita endowments of knowledge, skilled labor, capital in China and the US, and global carbon concentrations. A feasible path specifies:

- o the allocation of labor to its four uses in both the North and South at each of dates 1, 2, and 3;
- o the allocation of commodity output between native consumption, native investment, and transfers to the other region (which occur, net, in one direction only at each date);
- o diffusion of knowledge from the North to the South, which is taken to be at a constant rate –a model parameter;
- o an allocation of total emissions at each date between the North and the South (where emissions, recall, are an input in commodity production);
- o each path is constrained according to the technological constraints and as well by these requirements:
    - that at each date, global emissions do not exceed those specified on the path of Table 1,

- that welfare per capita converge to the same value in 75 years in the two regions,
- that at the beginning of the third generation the endowments per capita of the two regions are identical, and lie on the steady-state ray where growth can continue ad infinitum at 28% per generation, and
- that along the path, each region's welfare grow by at least 28% per generation (equivalent to 1% per annum) until Generation 3.

What do we optimize on this set of feasible paths? [14] We tried several different objective functions –for discussion, see Llavador, Roemer and Silvestre (2011b). What we finally adopted was the objective of maximizing the utility of the South in Generation 2. This produces a path that converges at date 3 in welfare per capita, illustrated in Figure 3.

As we said, the allocation of global emissions to the two regions is endogenous: it comes as part of the solution of the optimization problem. We present two figures below which show the optimal allocation of emissions between North and South. Figure 4 shows the regional distribution of total emissions in GtC, while Figure 5 presents the same information in emissions per capita in each region.

Before summarizing some of the main characteristics of the optimal path, we wish to emphasize a key point. In the model of the US alone that we described in sections 6 and 7, we were able to compute that US residents could continue to live quite well, under two possible scenarios of the share of global emissions allocated to the US. But we had, at that time, no guarantee that the entire world, with its population projected to rise to about 9 billion people by 2050, could continue to sustain welfare while constraining carbon emissions to an acceptable level. The present model shows that this is, indeed, the case. Note, from Table 6, that in 2050, the global population is indeed taken to be 9.1 billion, and the per capita welfares shown in Figure 3 for citizens of the North and South are calculated under that assumption, and under the assumption of global emissions stated in Table 1. Even if others may wish to quibble with our choice of what to optimize on the set of feasible paths, it is at least important that the path we illustrate in Figure 3 is even feasible.

[14] The set is non-empty, something that must be checked.

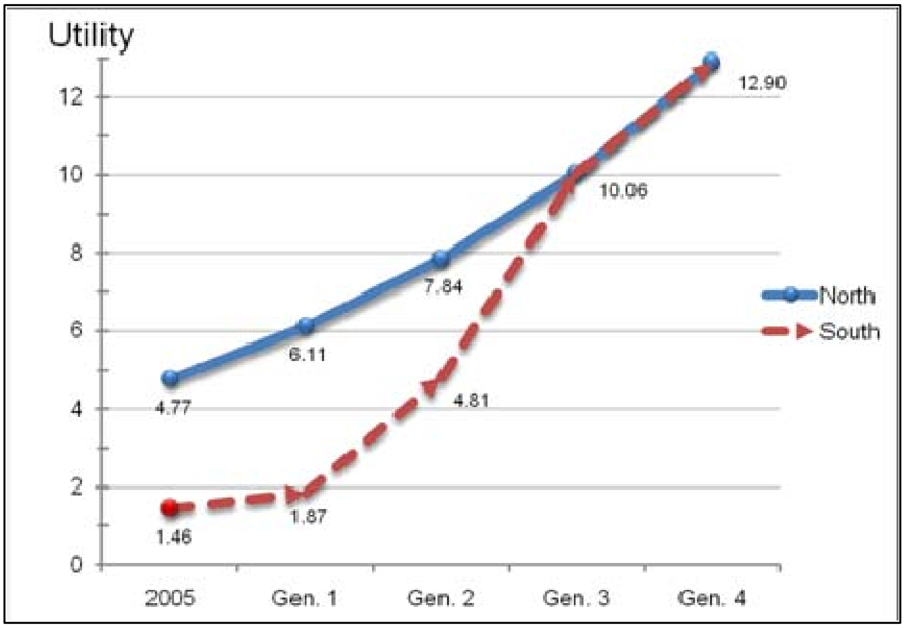

Figure 3. Optimal convergence to a 1% per annum welfare growth path, South and North, in three generations. The level of welfare per capita ("Utility") at each date are noted on the vertical axis.

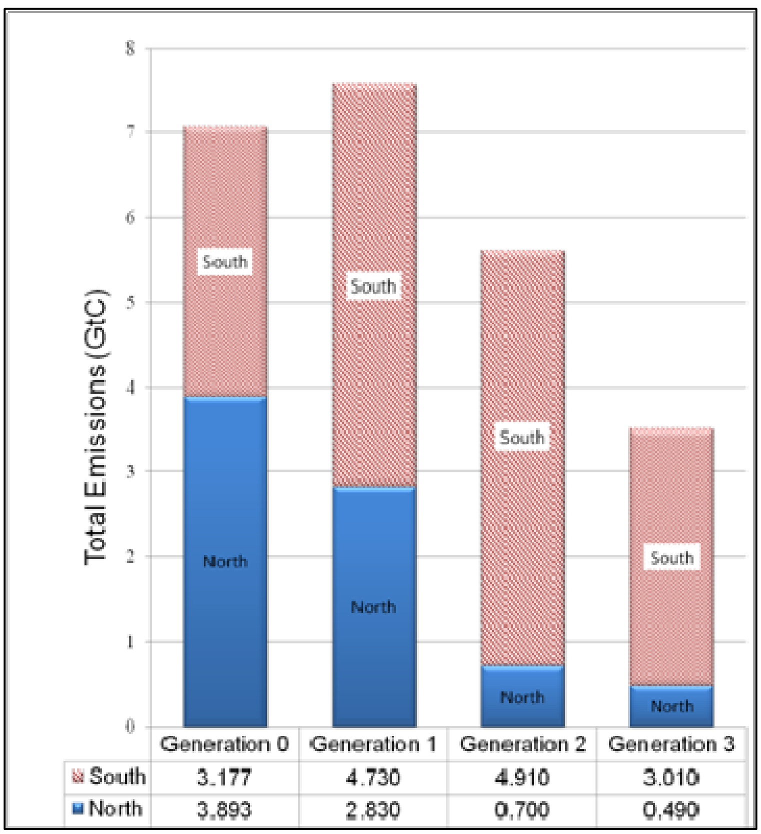

Figure 4. Total emissions, by region and generation, on the optimal path

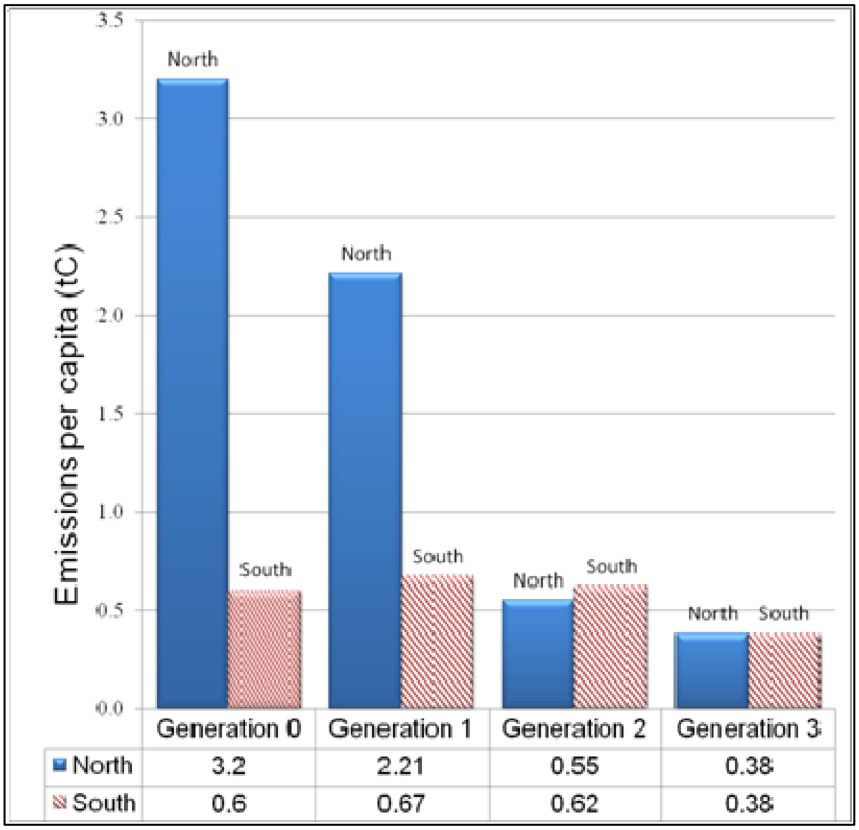

Figure 5. Emissions per capita, by region and generation, on the optimal path

We summarize some important characteristics of the optimal path (the values are presented in tables 7 and 8, in addition to figures 4 and 5).

- At the year-2005 reference values, emissions per capita in the South are substantially lower than in the North: see Figure 5. At the steady state, however, they must be equalized. In fact, the ratio of emissions per capita in the South to that in the North increases from the reference value of 18.8% to 112.7% at $t = 2$, eventually falling down to the steady state 100% . North's emissions per capita decrease almost monotonically. For the South, they first rise moderately, and then they fall.
- In order to catch up with the North, the South's consumption of output has to grow quite fast. Specifically, South's consumption at the convergence

point $t = 3$ is 41 times its reference level, while North more than doubles its consumption with respect to the reference level.

- At the steady state, the shares of labor resources that both North and South devote to leisure do not significantly differ from 2005 values. During the transition, North (resp. South) moderately increases (resp. reduces) its share of time devoted to leisure.
- The optimal path requires substantial changes in the creation of knowledge in both North and South, and in education in the South. The two last columns of Table 7 below show that the fraction of labor devoted to the creation of knowledge jumps to the steady state value of 2.8 % from the reference values of 1.6% in North and 0.03% in South. South's education at the steady state absorbs 3.1% of the labor resource, versus 2% in the reference year. During the transition, the most dramatic changes are the tripling of the fraction of the labor resource devoted to knowledge in North at t = 1 relative to the 2005 value, and the almost quadrupling of the fraction of the labor resource devoted to education in South at t = 1 relative to the 2005 value.
- At the steady state, both North and South devote to investment in physical capital 4.6% of their labor resources, a figure in line with the reference value of 5.2% in North, but substantially lower than the reference 15.2% in South.

We next discuss the *transfers* between the two regions that occur along the optimal path. Here we have a surprise: see the last column of Table 7. The South transfers commodities to the North valued at $957 per capita (North) in Generation 1 and $23,483 in Generation 2 to the North! Since the North's consumption at date 2 is about $54,000 per capita, about 43% of it comes from Southern transfers.

	Utility	Generational utility growth factor	Consumption	Consumption relative to 2005	Generational consumption growth factor	Investment	Stock of Capital	Stock of knowledge	Transfers
Gen	**NORTH**								
0	1.00	-	34.052	1.00	-	7.739	92.999	21.582	0
1	1.282	1.282	42.051	1.235	1.235	8.669	132.516	98.362	-0.957
2	1.645	1.282	54.457	1.599	1.295	14.683	220.963	74.479	-23.483
3	2.109	1.282	89.018	2.614	1.635	18.172	285.430	96.209	0
4	2.705	1.282	114.990	3.377	1.292	23.474	368.704	124.278	0
Gen	**SOUTH**								
0	1	-	2.177	1	-	1.8994	14.739	2.472	0
1	1.282	1.282	12.183	5.597	5.597	2.921	40.688	14.231	-0.175
2	3.304	2.576	31.699	14.564	2.602	16.255	220.963	74.479	-3.803
3	6.904	2.089	89.018	40.898	1.635	18.172	285.430	96.209	0
4	8.854	1.282	114.990	52.830	1.292	23.474	368.704	124.278	0

Table 7. Values along the optimal path sustaining a 1% annual growth.

	Total efficiency units	Efficiency units in leisure	Efficiency units in the production of commodities	Efficiency units in the production of knowledge	Efficiency units in education commodities	% Efficiency units in leisure	% Efficiency units in the production of commodities	% Efficiency units in the production of knowledge	% Efficiency units in education
Gen.	**NORTH**								
0	2.035	1.363	0.571	0.034	0.067	0.670	0.280	0.016	0.033
1	2.641	1.749	0.690	0.127	0.074	0.662	0.261	0.048	0.028
2	3.096	2.290	0.640	0.072	0.095	0.739	0.207	0.023	0.031
3	3.917	2.630	1.056	0.109	0.122	0.671	0.270	0.028	0.031
4	5.060	3.398	1.364	0.141	0.158	0.671	0.270	0.028	0.031
Gen.	**SOUTH**								
0	1.362	0.885	0.445	0.005	0.027	0.650	0.327	0.003	0.020
1	0.834	0.530	0.222	0.017	0.064	0.636	0.266	0.020	0.077
2	2.378	1.395	0.759	0.129	0.095	0.587	0.319	0.054	0.040
3	3.917	2.630	1.056	0.109	0.122	0.671	0.270	0.028	0.031
4	5.060	3.398	1.364	0.141	0.158	0.671	0.270	0.028	0.031

Table 8. Labor allocation along the optimal path sustaining a 1% annual growth rate.

Why does this occur? During the transition the stock of knowledge of the North spills over to the South, so that at the optimal solution, North must devote to the creation of knowledge a relatively large fraction of its labor resource. Second, as noted, the optimal solution imposes a relatively small allocation of emissions to North during the transition. It turns out that these sacrifices by North are counterbalanced by the South-to-North transfers, so that the constraint, which we impose, that North's utility grow at rate of at least 28% per generation starting from the reference level, is met. In a sense, South has comparative advantage in the production of output during the transition, while North has comparative advantage in the production of knowledge. We conjecture that letting North grow at a slower rate during the transition would reduce or eliminate these commodity transfers, but we feel that such a reduction in North's growth rate would be politically infeasible. Moreover, it would force a lower utility to both North and South for $t \geq 3$.

We can put this result in a more general context as follows. Think about the bargaining problem between China and the US concerning the allocation of global emissions, which motivates our calculation of the optimal path upon which North and South converge in welfare per capita in 75 years. Once it has been agreed that that convergence is the *sine qua non* of the bargaining solution, the two regions should behave as an actor with a common goal of finding the *best possible way* of reaching convergence in 75 years. In our specification of the optimization problem, it is taken as a constraint that North must maintain at least a growth rate of 1% per annum along this path, a constraint we have suggested is politically necessary. This, however, implies a certain amount of transfer from the South to the North along the path, an illustration of the cooperative behavior necessary to implement the bargaining solution. Now if one objects that these South-to-North transfers are not politically feasible from South's viewpoint,[15] then we must return to the drawing board, and impose a further constraint –some upper bound– on South-to-North transfers. If there is a feasible path satisfying this constraint, and all the others, the consequence will be that at date 3, when welfare per capita converges, it will do so at a lower level than the one we have calculated.

[15] Political scientists speak of 'audience costs,' the difficulty governments may have of convincing citizens of policies it wishes to enact.

11. Summary and conclusions

1. We advocate the normative approach of *sustaining human welfare for as long as the human species lasts at its highest possible level* or alternatively *sustaining a given growth rate of human welfare for as long as the human species lasts.* These goals can be precisely formulated as intergenerational welfare optimization problems.

2. We model human welfare as depending on three private goods –consumption, education, and leisure time– and two public goods –the level of human knowledge and biospheric quality. Our motivation is similar to that which motivates Sen's (1985) ideas about functioning and capability. The traditional neoclassical view is that education is only valuable in so far as it enables the person to earn a higher wage and consume more. And in the global-warming literature (e. g., Nordhaus, 2008, and Stern, 2007) the costs of increasing global temperature are measured in terms of consumption foregone in order to adapt to the warmer planet. On the contrary, we declare that education, the stock of human knowledge, and the quality of the biosphere enter directly into utility, as well as being inputs in production.

3. We show that human welfare can be sustained forever at levels higher than the present level, while restricting GHG emissions to a path that would imply a limit value of 450 ppm atmospheric concentration of carbon dioxide. Our first model is calibrated with US data, and thus refers to the welfare of the representative American resident. Even with a sharp restriction of US emissions to 4% of global emissions (at present, they are about 25%), this is true.
   Positive growth rates can be sustained forever as well. At a relatively small welfare cost to generation one, a growth rate of 2% per annum (64% per generation) can be sustained on the given emissions path.

4. Achieving these sustainable paths requires a re-allocation of labor. The 2% growth path requires a substantial transfer of workers from commodity production to the education and knowledge-producing sectors of the economy. These re-allocations

could be implemented with subsidies and taxes, although we do not study the details of how to do so.

5. We believe the only justification for discounting the utility of future generations is because they may not exist. We therefore find the calculations of Nordhaus (2008) to be ethically unsupportable, because he uses discount rates reflecting the rates of time impatience of present consumers. Arguments given by many economists to support this practice are either circular or give the present generation an hegemony over decisions concerning the welfare of future generations which is morally unacceptable. *Equality* across generations is the only acceptable morality, for each generation which exists has a right to be as well off as any other.

6. A holder of a right may, however, choose not to enforce it: thus the present generation may choose to forfeit its right to equal welfare with those in the future, if it so desires, because it values *human development*. In contrast, it goes without saying that a future generation may not decide to be worse off than the present generation, because time's arrow points in only one direction.

7. Discounted utilitarianism, with a discount rate that reflects only the possibility of the extinction of the species, is a permissible ethic, *if* the utilitarian ethic is acceptable. But along with many others, we believe utilitarianism is a poor theory of distributive justice across generations –and within a generation. Hence, we do not agree with the conceptual underpinnings of the Stern (2007) Review.

8. Concerning the intra-temporal problem of allocating the allowable carbon budget across countries, we argue that all sufficiently developed countries should reduce their growth factors by the same fraction. Our view flows from a focal-point approach to the bargaining problem. We have calculated an optimal path in which the relative growth factors of the global South and global North are preserved, so that the date at which a South like China converges to a North like the United States in welfare per capita would be unchanged by the necessity of restricting GHG

emissions to maintain an acceptable level of 450 ppm carbon dioxide in the atmosphere. It is very important to note that even with a global population projected to be over 9 billion in 2050, welfare in both South and North grows substantially along the optimal path.

9. While *global* emissions are fixed along this path, the allocation of emissions to the two regions of the world is endogenous and follows from the optimization. More than the exact results of our calculation, we wish to emphasize the underlying approach, which is to derive the allocation of emission rights to regions not from a pre-conceived ethical postulate, such as equal emissions rights per person globally, but rather as the consequence of how regions of the world might well bargain to a solution. Thus, our solution has the attractiveness of being politically feasible –or so we claim.

10. While we have modeled the international allocation problem as one between two regions, one with the characteristics of China and the other of the United States, we advocate, more specifically, that carbon emissions be allocated among all sufficiently rich regions of the world so as to maintain their relative growth factors. Poor regions would be exempt, and would not enter this compact until their welfares per capita reached a prescribed level.

11. A general conclusion which follows from our analysis is that the problem of sustaining human welfare in the face of the carbon challenge is solvable, from the technological and physical viewpoints. Our assumptions concerning technologies and technological change are standard, and our parameterization of the models is derived from historical data in a conventional manner. The difficult problem is to achieve two kinds of commitment: within each region, a commitment to future generations, and across regions, a commitment to fairness, as encapsulated in our focal-point approach to the inter-regional bargaining problem.

References

Anand, Sudhir, and Amartya Sen (2000). “Human development and economic sustainability,” *World Development* 28 (12): 2029-2049.

Bentham, Jeremy (1789). *An introduction to the Principles of Morals and Legislation*, London: T. Payne & Son.

Dasgupta, Partha (2008). “Discounting climate change,” *Journal of Risk and Uncertainty* 37: 141-169.

Eaton, Jonathan, and Samuel Kortum (1999). “International technology diffusion: Theory and measurement,” *International Economic Review* 40(3):537-70.

Elster, Jon (2008). *Reason and Rationality*, Princeton: Princeton University Press.

Keller, Wolfgang (2004). “International technology diffusion,” *Journal of Economic Literature* 42(3):752-782.

Krautkraemer, Jeffrey (1985). “Optimal growth, resource amenities and the preservation of natural environments,” *Review of Economic Studies* 52: 153-179.

Llavador, Humberto, John E. Roemer and Joaquim Silvestre (2010). “Intergenerational justice when future worlds are uncertain,” *Journal of Mathematical Economics* 46: 728-761.

Llavador, Humberto, John E. Roemer and Joaquim Silvestre (2011a). “A dynamic analysis of human welfare in a warming planet,” http://pantheon.yale.edu/~jer39/climatechange.html.

Llavador, Humberto, John E. Roemer and Joaquim Silvestre (2011b). “North-South convergence and the allocation of CO_2 emissions,” http://pantheon.yale.edu/~jer39/climatechange.html.

Meehl, Gerard A., T. F. Stocker, W. D. Collins, P. Friedlingstein, A. T. Gaye, J. M. Gregory, A. Kitoh, R. Knutti, J. M. Murphy, A. Nodah, S. C. B. Raper, I. G. Watterson, A. J. Weaver and Z. –C. Zhao (2007). “Global climate projections,” Chapter 10 in S. Solomon, D. Qin, M. Manning, Z. Chen, M. Marquis, K. B. Averyt, M. Tignor and H. L. Miller, eds. *Climate Change 2007: The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change*, Cambridge, UK: Cambridge University Press.

Mill, John Stuart (1848). *Principles of Political Economy, with Some of Their Applications to Social Philosophy*, London: John W. Parker.

Miller, Richard W. (2010). *Globalizing Justice: The Ethics of Poverty and Power*, Oxford: Oxford University Press.

Neumayer, Eric (2001). “The human development index and sustainability: A constructive proposal,” *Ecological Economics* 39: 101:114.

Nordhaus, William D. (2008). *A Question of Balance: Weighing the Options on Global Warming Policies*, New Haven: Yale University Press.

Ramsey, Frank (1928). “A mathematical theory of saving,” *Economic Journal* 38: 543-559.

Rawls, John (1971). *A Theory of Justice* , Cambridge, MA: Harvard University Press

Roemer, John E. (2011). “The ethics of intertemporal distribution in a warming planet,” *Environmental and Resource Economics, http://www.springerlink.com/content/l1105m2311182267/*

Sen, Amartya (1985). *Commodities and Capabilities,* Amsterdam: North Holland.

Stern, Nicholas (2007). *The Economics of Climate Change: The Stern Review*, Cambridge, UK: Cambridge University Press.

Stern, Nicholas (2008) “The Economics of Climate Change,” *American Economic Review: Papers & Proceedings* 98 (2), 1-37.

United Nations (2008). Population Division of the Department of Economic and Social Affairs of the United Nations Secretariat. *World Population Prospects: The 2006 Revision* and *World Urbanization Prospects: The 2005 Revision.* http://esa.un.org/unpp.

United Nations Development Programme (1990). *Human Development Report* 1990. http://hdr.undp.org/en/reports/global/hdr1990/chapters/

United Nations Development Programme (2010). *Human Development Report* 2010. http://hdr.undp.org/en/reports/global/hdr 2010.