APEC 5152
Utility, Indirect Utility and Expenditure

Consumption

We'll use Cobb-Douglas preferences, $u = q1^{\gamma 1}\, q2^{\gamma 2}\, q3^{\gamma 3}$, where $\gamma 1 + \gamma 2 + \gamma 3 = 1$. It turns out that using the Cobb-Douglas function doesn't work with the Solve function. Taking a monotonic transformation of the function - a log transformation - however, yields a solution.

The Indirect Utility Function

The lagrangian

In[62]:=
```
U = Log[q1^γ1 q2^γ2 q3^γ3] // PowerExpand
L0 = U + λ (wv - p1 q1 - p2 q2 - p3 q3)
```

Out[62]= $\gamma 1\, \text{Log}[q1] + \gamma 2\, \text{Log}[q2] + \gamma 3\, \text{Log}[q3]$

Out[63]= $(-p1\, q1 - p2\, q2 - p3\, q3 + wv)\, \lambda + \gamma 1\, \text{Log}[q1] + \gamma 2\, \text{Log}[q2] + \gamma 3\, \text{Log}[q3]$

The first-order conditions

In[3]:=
```
FOC10 = ∂q1 L0
FOC20 = ∂q2 L0
FOC30 = ∂q3 L0
FOCλ0 = ∂λ L0
```

Out[3]= $\frac{\gamma 1}{q1} - p1\, \lambda$

Out[4]= $\frac{\gamma 2}{q2} - p2\, \lambda$

Out[5]= $\frac{\gamma 3}{q3} - p3\, \lambda$

Out[6]= $-p1\, q1 - p2\, q2 - p3\, q3 + wv$

In[7]:=
```
MarshSol =
 Solve[{FOC10 == 0, FOC20 == 0, FOC30 == 0, FOCλ0 == 0}, {q1, q2, q3, λ}] /. γ1 + γ2 + γ3 → 1 //
   ExpandAll // FullSimplify
```

Out[7]= $\{\{q1 \to \frac{wv\,\gamma1}{p1},\ q2 \to \frac{wv\,\gamma2}{p2},\ q3 \to \frac{wv\,\gamma3}{p3},\ \lambda \to \frac{1}{wv}\}\}$

Substitute the above demand functions into the Cobb-Douglas utility function.

In[9]:=
```
V[p1_, p2_, p3_, wv_] = Exp[U] /. MarshSol[[1]] // ExpandAll // FullSimplify
```

Out[9]= $\left(\frac{wv\,\gamma1}{p1}\right)^{\gamma1}\left(\frac{wv\,\gamma2}{p2}\right)^{\gamma2}\left(\frac{wv\,\gamma3}{p3}\right)^{\gamma3}$

Further simplifying yields: $V[p1, p2, p3, wv] = \left(\frac{\gamma1}{p1}\right)^{\gamma1}\left(\frac{\gamma2}{p2}\right)^{\gamma2}\left(\frac{\gamma3}{p3}\right)^{\gamma3} wv$

Roy’s identity in action

In[57]:=

$-\frac{\partial_{p1} V[p1, p2, p3, wv]}{\partial_{wv} V[p1, p2, p3, wv]}$ // ExpandAll // FullSimplify

$-\frac{\partial_{p2} V[p1, p2, p3, wv]}{\partial_{wv} V[p1, p2, p3, wv]}$ // ExpandAll // FullSimplify

$-\frac{\partial_{p3} V[p1, p2, p3, wv]}{\partial_{wv} V[p1, p2, p3, wv]}$ // ExpandAll // FullSimplify

Out[57]= $\frac{wv\,\gamma1}{p1\,(\gamma1 + \gamma2 + \gamma3)}$

Out[58]= $\frac{wv\,\gamma2}{p2\,(\gamma1 + \gamma2 + \gamma3)}$

Out[59]= $\frac{wv\,\gamma3}{p3\,(\gamma1 + \gamma2 + \gamma3)}$

Expenditure Function

The optimization problem is $L = p1\,q1 + p2\,q2 + p3\,q3 + \lambda\,(u - q1^{\gamma1}\,q2^{\gamma2}\,q3^{\gamma3})$

In[15]:= $L = p1\,q1 + p2\,q2 + p3\,q3 + \lambda\,(u - q1^{\gamma1}\,q2^{\gamma2}\,q3^{\gamma3})$;

In[16]:=

$foc1 = \partial_{q1} L$

$foc2 = \partial_{q2} L$

$foc3 = \partial_{q3} L$

$foc\lambda = \partial_{\lambda} L$

Out[16]= $p1 - q1^{-1+\gamma1}\,q2^{\gamma2}\,q3^{\gamma3}\,\gamma1\,\lambda$

Out[17]= $p2 - q1^{\gamma1}\,q2^{-1+\gamma2}\,q3^{\gamma3}\,\gamma2\,\lambda$

Out[18]= $p3 - q1^{\gamma1}\,q2^{\gamma2}\,q3^{-1+\gamma3}\,\gamma3\,\lambda$

Out[19]= $-q1^{\gamma1}\,q2^{\gamma2}\,q3^{\gamma3} + u$

The solution gives the system of demands

In[29]:=
```
Solve[{foc1 == 0, foc2 == 0, foc3 == 0, focλ == 0}, {q1, q2, q3, λ}][[1]] // ExpandAll //
  FullSimplify
HickSol = % /. γ1 + γ2 + γ3 → 1
```

Out[29]=

$$\left\{q1 \to p1^{-1+\frac{\gamma 1}{\gamma 1+\gamma 2+\gamma 3}}\, p2^{\frac{\gamma 2}{\gamma 1+\gamma 2+\gamma 3}}\, p3^{\frac{\gamma 3}{\gamma 1+\gamma 2+\gamma 3}}\, u^{\frac{1}{\gamma 1+\gamma 2+\gamma 3}}\, \gamma 1^{\frac{\gamma 2+\gamma 3}{\gamma 1+\gamma 2+\gamma 3}}\, \gamma 2^{-\frac{\gamma 2}{\gamma 1+\gamma 2+\gamma 3}}\, \gamma 3^{-\frac{\gamma 3}{\gamma 1+\gamma 2+\gamma 3}},\right.$$
$$q2 \to p1^{\frac{\gamma 1}{\gamma 1+\gamma 2+\gamma 3}}\, p2^{-1+\frac{\gamma 2}{\gamma 1+\gamma 2+\gamma 3}}\, p3^{\frac{\gamma 3}{\gamma 1+\gamma 2+\gamma 3}}\, u^{\frac{1}{\gamma 1+\gamma 2+\gamma 3}}\, \gamma 1^{-\frac{\gamma 1}{\gamma 1+\gamma 2+\gamma 3}}\, \gamma 2^{\frac{\gamma 1+\gamma 3}{\gamma 1+\gamma 2+\gamma 3}}\, \gamma 3^{-\frac{\gamma 3}{\gamma 1+\gamma 2+\gamma 3}},$$
$$q3 \to p1^{\frac{\gamma 1}{\gamma 1+\gamma 2+\gamma 3}}\, p2^{\frac{\gamma 2}{\gamma 1+\gamma 2+\gamma 3}}\, p3^{-\frac{\gamma 1+\gamma 2}{\gamma 1+\gamma 2+\gamma 3}}\, u^{\frac{1}{\gamma 1+\gamma 2+\gamma 3}}\, \gamma 1^{-\frac{\gamma 1}{\gamma 1+\gamma 2+\gamma 3}}\, \gamma 2^{-\frac{\gamma 2}{\gamma 1+\gamma 2+\gamma 3}}\, \gamma 3^{\frac{\gamma 1+\gamma 2}{\gamma 1+\gamma 2+\gamma 3}},$$
$$\left.\lambda \to p1^{\frac{\gamma 1}{\gamma 1+\gamma 2+\gamma 3}}\, p2^{\frac{\gamma 2}{\gamma 1+\gamma 2+\gamma 3}}\, p3^{\frac{\gamma 3}{\gamma 1+\gamma 2+\gamma 3}}\, u^{-1+\frac{1}{\gamma 1+\gamma 2+\gamma 3}}\, \gamma 1^{-\frac{\gamma 1}{\gamma 1+\gamma 2+\gamma 3}}\, \gamma 2^{-\frac{\gamma 2}{\gamma 1+\gamma 2+\gamma 3}}\, \gamma 3^{-\frac{\gamma 3}{\gamma 1+\gamma 2+\gamma 3}}\right\}$$

Out[30]=

$$\left\{q1 \to p1^{-1+\gamma 1}\, p2^{\gamma 2}\, p3^{\gamma 3}\, u\, \gamma 1^{\gamma 2+\gamma 3}\, \gamma 2^{-\gamma 2}\, \gamma 3^{-\gamma 3},\ q2 \to p1^{\gamma 1}\, p2^{-1+\gamma 2}\, p3^{\gamma 3}\, u\, \gamma 1^{-\gamma 1}\, \gamma 2^{\gamma 1+\gamma 3}\, \gamma 3^{-\gamma 3},\right.$$
$$\left. q3 \to p1^{\gamma 1}\, p2^{\gamma 2}\, p3^{-\gamma 1-\gamma 2}\, u\, \gamma 1^{-\gamma 1}\, \gamma 2^{-\gamma 2}\, \gamma 3^{\gamma 1+\gamma 2},\ \lambda \to p1^{\gamma 1}\, p2^{\gamma 2}\, p3^{\gamma 3}\, \gamma 1^{-\gamma 1}\, \gamma 2^{-\gamma 2}\, \gamma 3^{-\gamma 3}\right\}$$

One of the demand functions

$$q1demand = \left(\frac{p1}{\gamma 1}\right)^{\gamma 2+\gamma 3} \left(\frac{p2}{\gamma 2}\right)^{\gamma 2} \left(\frac{p3}{\gamma 3}\right)^{\gamma 3} u$$

The cost function

In[35]:=
```
p1 q1 + p2 q2 + p3 q3 /. HickSol // ExpandAll // FullSimplify
```

Out[35]=

$$p1^{\gamma 1}\, p2^{\gamma 2}\, p3^{-\gamma 1-\gamma 2}\, u\, \gamma 1^{-\gamma 1}\, \gamma 2^{-\gamma 2}\, \gamma 3^{-\gamma 3}\left(p3^{\gamma 1+\gamma 2+\gamma 3}\left(\gamma 1^{\gamma 1+\gamma 2+\gamma 3} + \gamma 2^{\gamma 1+\gamma 2+\gamma 3}\right) + p3\, \gamma 3^{\gamma 1+\gamma 2+\gamma 3}\right)$$

We can further simplify

In[37]:=
```
% /. γ1 + γ2 + γ3 → 1 // ExpandAll // FullSimplify
```

Out[37]=

$$p1^{\gamma 1}\, p2^{\gamma 2}\, p3^{1-\gamma 1-\gamma 2}\, u\, \gamma 1^{-\gamma 1}\, \gamma 2^{-\gamma 2}\, \gamma 3^{-\gamma 3}\left(\gamma 1 + \gamma 2 + \gamma 3\right)$$

So the cost function is:

In[60]:=

$$CC = \left(\frac{p1}{\gamma 1}\right)^{\gamma 1} \left(\frac{p2}{\gamma 2}\right)^{\gamma 2} \left(\frac{p3}{\gamma 3}\right)^{\gamma 3} u$$

Out[60]=

$$u \left(\frac{p1}{\gamma 1}\right)^{\gamma 1} \left(\frac{p2}{\gamma 2}\right)^{\gamma 2} \left(\frac{p3}{\gamma 3}\right)^{\gamma 3}$$

Applying Shepard’s lemma to p1 gives

In[61]:=

$$\partial_{p1}\, CC$$

Out[61]=

$$u \left(\frac{p1}{\gamma 1}\right)^{-1+\gamma 1} \left(\frac{p2}{\gamma 2}\right)^{\gamma 2} \left(\frac{p3}{\gamma 3}\right)^{\gamma 3}$$

Since $-1 + \gamma 1 = \gamma 2 + \gamma 3$, this is the same as q1demand

APEC 5152

Mathematica Code that Derives Cost and Value-Added Functions

1. Cobb-Doublas production function, cost minimization problem, setup and first-order conditions

Introduce a three-input (k, l, z) Cobb-Douglas production function, and use it to derive a cost function. We do this by solving a cost minimization problem, with the objective function defined by “cst” and setting up the typical lagrangian, and then taking the first-order conditions.

```
yy = ψ k^α1 l^α2 z^α3 (*/.{α1→0.3,α2→0.3,α3→0.4}*);
cst = r k + w l + τ z;
Lagrangian = cst + λ (y - yy);
fc1 = ∂k Lagrangian;
fc2 = ∂l Lagrangian;
fc3 = ∂z Lagrangian;
fc4 = ∂λ Lagrangian;
```

Now solve the system of equations for optimal levels of all inputs (k, l, z) and for the Lagrangian multiplier, λ. This yields the factor demand curves, fdk, fdl, fdz. Using // ExpandAll eliminates the exponential type answers.

```
cf = Solve[{fc1 == 0, fc2 == 0, fc3 == 0, fc4 == 0}, {k, l, z, λ}] // ExpandAll // FullSimplify;
```

```
fdk = cf[[1, 1, 2]]
fdl = cf[[1, 2]]
fdz = cf[[1, 3]]
```

$$r^{-1+\frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}} w^{\frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3}} y^{\frac{1}{\alpha_1+\alpha_2+\alpha_3}} \tau^{\frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3}} \psi^{-\frac{1}{\alpha_1+\alpha_2+\alpha_3}} \alpha_1^{\frac{\alpha_2+\alpha_3}{\alpha_1+\alpha_2+\alpha_3}} \alpha_2^{-\frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3}} \alpha_3^{-\frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3}}$$

$$l \to r^{\frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}} w^{-1+\frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3}} y^{\frac{1}{\alpha_1+\alpha_2+\alpha_3}} \tau^{\frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3}} \psi^{-\frac{1}{\alpha_1+\alpha_2+\alpha_3}} \alpha_1^{-\frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}} \alpha_2^{\frac{\alpha_1+\alpha_3}{\alpha_1+\alpha_2+\alpha_3}} \alpha_3^{-\frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3}}$$

$$z \to r^{\frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}} w^{\frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3}} y^{\frac{1}{\alpha_1+\alpha_2+\alpha_3}} \tau^{-\frac{\alpha_1+\alpha_2}{\alpha_1+\alpha_2+\alpha_3}} \psi^{-\frac{1}{\alpha_1+\alpha_2+\alpha_3}} \alpha_1^{-\frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}} \alpha_2^{-\frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3}} \alpha_3^{\frac{\alpha_1+\alpha_2}{\alpha_1+\alpha_2+\alpha_3}}$$

1.1 Substitute factor demand curves into cost function to derive producer’s cost function

Here, we substitute the factor demand curves derived above into the original objective function - the result is the parameterized version of the cost function associated with the Cobb-Douglas production function.

```
pcost = cst /. cf[[1]] // Simplify
```

$$r^{\frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}} w^{\frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3}} y^{\frac{1}{\alpha_1+\alpha_2+\alpha_3}} \tau^{\frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3}} \psi^{-\frac{1}{\alpha_1+\alpha_2+\alpha_3}} \alpha_1^{-\frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}} \alpha_2^{-\frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3}} \alpha_3^{-\frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3}} (\alpha_1+\alpha_2+\alpha_3)$$

- **1.2 Apply Shepard's lemma to the producer's cost function to derive factor demand curves**

```
kshep = ∂r pcost // Simplify
lshep = ∂w pcost // FullSimplify
zshep = ∂τ pcost // FullSimplify
```

$$r^{-1+\frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}} w^{\frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3}} y^{\frac{1}{\alpha_1+\alpha_2+\alpha_3}} \tau^{\frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3}} \psi^{-\frac{1}{\alpha_1+\alpha_2+\alpha_3}} \alpha_1^{\frac{\alpha_2+\alpha_3}{\alpha_1+\alpha_2+\alpha_3}} \alpha_2^{-\frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3}} \alpha_3^{-\frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3}}$$

$$r^{\frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}} w^{-1+\frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3}} y^{\frac{1}{\alpha_1+\alpha_2+\alpha_3}} \tau^{\frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3}} \psi^{-\frac{1}{\alpha_1+\alpha_2+\alpha_3}} \alpha_1^{-\frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}} \alpha_2^{\frac{\alpha_1+\alpha_3}{\alpha_1+\alpha_2+\alpha_3}} \alpha_3^{-\frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3}}$$

$$r^{\frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}} w^{\frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3}} y^{\frac{1}{\alpha_1+\alpha_2+\alpha_3}} \tau^{-1+\frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3}} \psi^{-\frac{1}{\alpha_1+\alpha_2+\alpha_3}} \alpha_1^{-\frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}} \alpha_2^{-\frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3}} \alpha_3^{\frac{\alpha_1+\alpha_2}{\alpha_1+\alpha_2+\alpha_3}}$$

Note that these are identical to the factor demand curves derived above in 1.0. To confirm, subtract one from the other. Notice how I'm storing the first order conditions.

```
kshep - fdk // Simplify
lshep - l /. fdl // Simplify
zshep - z /. fdz // Simplify
```

0

0

0

2. Value-added function, profit maximization problem, setup and first-order conditions

Again, we start with a three-input (k, l, z) Cobb-Douglas production functin. Define the revenue from selling output, here "rev", and get the first-order conditions of rev.

```
y = ψ k^α1 l^α2 z^α3 (*/.{α1→0.3,α2→0.3,α3→0.4} *);
rev = p y;
Rent = rev - r k - w l;
(* First order conditions *)
FOCk = ∂k Rent
FOCl = ∂l Rent;
```

$-r + k^{-1+\alpha_1} l^{\alpha_2} p z^{\alpha_3} \psi \alpha_1$

Now solve for the factor demand curves.

```
factdem = Solve[{FOCk == 0, FOCl == 0}, {k, l}] // ExpandAll // FullSimplify;
FDk = factdem[[1, 1]]
FDl = factdem[[1, 2]]
```

$k \to p^{-\frac{1}{-1+\alpha_1+\alpha_2}} r^{-1+\frac{\alpha_1}{-1+\alpha_1+\alpha_2}} w^{\frac{\alpha_2}{-1+\alpha_1+\alpha_2}} z^{-\frac{\alpha_3}{-1+\alpha_1+\alpha_2}} \psi^{-\frac{1}{-1+\alpha_1+\alpha_2}} \alpha_1^{\frac{-1+\alpha_2}{-1+\alpha_1+\alpha_2}} \alpha_2^{-\frac{\alpha_2}{-1+\alpha_1+\alpha_2}}$

$l \to p^{-\frac{1}{-1+\alpha_1+\alpha_2}} r^{\frac{\alpha_1}{-1+\alpha_1+\alpha_2}} w^{\frac{1-\alpha_1}{-1+\alpha_1+\alpha_2}} z^{-\frac{\alpha_3}{-1+\alpha_1+\alpha_2}} \psi^{-\frac{1}{-1+\alpha_1+\alpha_2}} \alpha_1^{-\frac{\alpha_1}{-1+\alpha_1+\alpha_2}} \alpha_2^{\frac{-1+\alpha_1}{-1+\alpha_1+\alpha_2}}$

2.1 Substitue factor demand curves into net revenue function to derive producer’s value-added function

```
vaf = Rent /. factdem[[1]] // PowerExpand // FullSimplify
```

$-p^{-\frac{1}{-1+\alpha_1+\alpha_2}} r^{\frac{\alpha_1}{-1+\alpha_1+\alpha_2}} w^{\frac{\alpha_2}{-1+\alpha_1+\alpha_2}} z^{-\frac{\alpha_3}{-1+\alpha_1+\alpha_2}} \psi^{-\frac{1}{-1+\alpha_1+\alpha_2}} \alpha_1^{-\frac{\alpha_1}{-1+\alpha_1+\alpha_2}} \alpha_2^{-\frac{\alpha_2}{-1+\alpha_1+\alpha_2}} (-1+\alpha_1+\alpha_2)$

2.2 Apply Hotelling’s lemma to the value-added function to derive factor demand curves

```
khotel = -∂r vaf
lhotel = -∂w vaf
```

$p^{-\frac{1}{-1+\alpha_1+\alpha_2}} r^{-1+\frac{\alpha_1}{-1+\alpha_1+\alpha_2}} w^{\frac{\alpha_2}{-1+\alpha_1+\alpha_2}} z^{-\frac{\alpha_3}{-1+\alpha_1+\alpha_2}} \psi^{-\frac{1}{-1+\alpha_1+\alpha_2}} \alpha_1^{1-\frac{\alpha_1}{-1+\alpha_1+\alpha_2}} \alpha_2^{-\frac{\alpha_2}{-1+\alpha_1+\alpha_2}}$

$p^{-\frac{1}{-1+\alpha_1+\alpha_2}} r^{\frac{\alpha_1}{-1+\alpha_1+\alpha_2}} w^{-1+\frac{\alpha_2}{-1+\alpha_1+\alpha_2}} z^{-\frac{\alpha_3}{-1+\alpha_1+\alpha_2}} \psi^{-\frac{1}{-1+\alpha_1+\alpha_2}} \alpha_1^{-\frac{\alpha_1}{-1+\alpha_1+\alpha_2}} \alpha_2^{1-\frac{\alpha_2}{-1+\alpha_1+\alpha_2}}$

Now test to see if Hotelling’s lemma holds for factor demands

```
khotel - FDk // Simplify
lhotel - FDl // Simplify
```

0

0

2.3 Apply Hotelling’s lemma to derive the supply function for this single-sector model

```
∂p vaf
```

$p^{-1-\frac{1}{-1+\alpha_1+\alpha_2}} r^{\frac{\alpha_1}{-1+\alpha_1+\alpha_2}} w^{\frac{\alpha_2}{-1+\alpha_1+\alpha_2}} z^{-\frac{\alpha_3}{-1+\alpha_1+\alpha_2}} \psi^{-\frac{1}{-1+\alpha_1+\alpha_2}} \alpha_1^{-\frac{\alpha_1}{-1+\alpha_1+\alpha_2}} \alpha_2^{-\frac{\alpha_2}{-1+\alpha_1+\alpha_2}}$

```
D[vaf, w] /. {α1 → 0.2, α2 → 0.3, α3 → 0.5}
D[%, w] /. {α1 → 0.2, α2 → 0.3, α3 → 0.5}
D[vaf, p] /. {α1 → 0.2, α2 → 0.3, α3 → 0.5}
D[%, p] /. {α1 → 0.2, α2 → 0.3, α3 → 0.5}
```

$-\frac{0.0765255\, p^{2.} z^{1.} \psi^{2.}}{r^{0.4} w^{1.6}}$

$\frac{0.122441\, p^{2.} z^{1.} \psi^{2.}}{r^{0.4} w^{2.6}}$

$\frac{0.255085\, p^{1.} z^{1.} \psi^{2.}}{r^{0.4} w^{0.6}}$

$\frac{0.255085\, z^{1.} \psi^{2.}}{r^{0.4} w^{0.6}}$

```
rK = 26460.4 + 51410.2 + 9930.2 + 151297.3
wL = 6902.7 + 44125.4 + 7798.6 + 242620.1
Solve[rK / K == 0.1, K]
Solve[wL / L == 3257, L]
```

239098.

301447.

{{K → 2.39098 × 10^6}}

{{L → 92.5535}}

```
rK / 2390981 // N
```

0.1