1 The economic environment

The model is characterized as a small, open and perfectly competitive economy in which agents produce and consume three final goods: an agricultural, manufacturing, and service good, indexed at each instant in time by $j = a, m, s$, and traded at price P_j. The economy is endowed with L units of labor, K units of capital and Z units of land. Capital and labor are used by each sector, while land used only in producing agricultural output. The agricultural and manufacturing goods are traded internationally, but the service good is non-traded (it is a "home good"). Manufacturing is also an investment good. Households earn income from providing labor services in exchange for wages w, earn interest income at rate r on capital assets K, and receive income from agricultural land rent.

1.1 Production

For $j = a, m, s$, let K_j and L_j represent the amount of labor demanded by sector j. Let $Y_m = \Psi_m (K_m)^{\alpha} (L_m)^{1-\alpha}$ represent the production technology for the manufacturing sector and let $Y_s = \Psi_s (K_s)^{\beta} (L_s)^{1-\beta}$ represent the production technology for the service sector. Ψ_m and Ψ_s are productivity (or scaling) parameters for manufacturing and services, respectively, while α and β are the capital cost shares (and also the production elasticities) for manufacturing and services. Then, the minimum unit cost of manufacturing production is given by

$$C^m (w, r) \equiv \min_{\{K_m, L_m\}} \left\{ wL_m + rK_m : 1 \leq \Psi_m (K_m)^{\alpha} (L_m)^{1-\alpha} \right\}$$

and the minimum unit cost of service sector production is given by

$$C^s (w, r) \equiv \min_{\{K_s, L_s\}} \left\{ wL_s + rK_s : 1 \leq \Psi_s (K_s)^{\beta} (L_s)^{1-\beta} \right\}$$

Finally, let $Y_a = \Psi_a (K_a)^{\alpha_1} (L_a)^{\alpha_2} (Z)^{\alpha_3}$ represent the poroduction function for agriculture, and assume $\alpha_1 + \alpha_2 + \alpha_3 = 1$. Then, maximum agricultural rent is given by

$$\Pi^a (p_a, w, r) Z \equiv \max_{\{K_a, L_a\}} \left\{ p_a \Psi_a (K_a)^{\alpha_1} (L_a)^{\alpha_2} (Z)^{\alpha_3} - wL_a - rK_a \right\}.$$

1.2 Households

The representative household receives utility from consuming the agricultural, manufacturing and service goods, denoted respectively by $Q_a, Q_m(t)$, and Q_s. Given normalized prices $p_a = P_a/P_m$ and $p_s = P_s/P_m$, income, I, and utility function $U = (Q_a)^{\gamma_a} (Q_m)^{\gamma m} (Q_s)^{\gamma s}$, the maximum utility the household can realize is given by the indirect utility function

$$V(p_a, p_s) I \equiv \max_{(Q_a, Q_m, Q_s)} \{(Q_a)^{\gamma_a} (Q_m)^{\gamma m} (Q_s)^{\gamma s} : p_m Q_m + p_a Q_a + p_s Q_s\}$$

2 The competitive equilibrium

Definition 1 *A competitive equilibrium is a set of factor and output prices,* $\{w, r, p_a, p_m, p_s\}$, *a set of factor allocations* $\{K_a, K_m, K_s, L_a, L_m, L_s\}$, *a set of production levels,* $\{Y_a, Y_m, Y_s\}$, *and a set of consumption levels,* $\{Q_a, Q_m, Q_s\}$ *such that: (i) firms maximize profit, (ii) households maximize utility, (iii) factor markets clear, and (iv) Walras law holds.*

A characterization of equilibrium:

(i) zero profits in manufacturing and services

$$C^j(w, r) = p_j, \;\; j = m, s \tag{1}$$

(ii) labor market clearing

$$\sum_{j=m,s} \frac{\partial}{\partial w} C^j(w, r) Y_j - \frac{\partial}{\partial w} \Pi^a(p_a, w, r) Z = L \tag{2}$$

(iii) capital market clearing

$$\sum_{j=m,s} \frac{\partial}{\partial r} C^j(w, r) Y_j - \frac{\partial}{\partial r} \Pi^a(p_a, w, r) Z = \hat{k} \tag{3}$$

(iv) non-traded good market clearing

$$Q_s = -\frac{V_{p_s}(p_a, p_s) I}{V(p_a, p_s)} = Y_s \tag{4}$$

where I will be maximum income, or gross domestic product (GDP).

This system has five endogenous variables $\{w, r, Y_m, Y_s, p_s\}$ and five equilibrium conditions, i.e., equations (1) – (4). So, in principle, you can set up Mathematica to solve for $\{w, r, Y_m, Y_s, p_s\}$, each as a function of the exogenous variables $\{p_a, p_m, K, L, Z\}$.

3 Problems

1. Create a Mathematica version of the three-sector model. As I warned in an earlier email, this is a challenging problem, and one on which you collaborate with at least one other person. Below are some hints at managing this project

   (a) Use the zero profit conditions for solve for w and r as functions of p_m and p_s.

   (b) Use the two factor market clearing conditions to solve for Y_m and Y_s.

      i. You might want to first represent the partials of the unit cost functions as a constant, e.g., $\frac{\partial}{\partial w} C^m (w, r) = cmw$. Then, after getting the expressions for Y^m and Y^s, substitute cmw with the general expression using the

      $$/. \; cmw \rightarrow \frac{\partial}{\partial w} C^m (w, r)$$

      command.

      ii. In general, if you have a problem getting Mathematica to solve a system of equations, look to see if you can represent some terms or expressions with constants, and then replace the term with the more complicated expressions.

      iii. Once you've done this, you should have w, r, Y_m and Y_s as a function of p_s and the exogenous variables.

   (c) Use the non-traded good market clearing condition to get the non-traded good price as a function of the exogenous variables.

   (d) Once you've done this, you can replace the p_s in w, r, Y_m and Y_s with the p_s^* you just derived.

   (e) Then you can do a variety of simulations to see what impact various policies can have on your model. There are other ways to perform simulations, and I'll leave it up to you to choose your approach.

2. Use the attached social accounting matrix (SAM) for Brazil, and calibrate the parameters of agricultural, manufacturing and service sectors, as well as the parameters for the Cobb-Douglas utility function. Confirm your Mathematica code replicates the SAM entries. If you've made any coding errors, this is where it will become obvious.

3. Conduct the following simulations:

   (a) Find out what happens to aggregate GDP when your capital endowment increases by 10% and when your labor endowment increases by 10%.

   (b) Find out what happens to each of the endogenous variables if the agricultural output price increases by 3%.

   (c) Find out what happens to each of the endogenous variables if the service sector output price increases by 3%.

   (d) Find out what happens to each of the endogenous variables if the manufacturing output price increases by 3%.

   (e) Explain the challenge in simulating a 5% subsidy on manufacturing wages (i.e., manufacturing wages are subsidized by 5%) in a general equilibrium world (as opposed to a partial equilibrium model).