Kolstad Chapter 15, Problem #2 (page 311)

Given :

1. damage due to pollution = $D(p) = p^2/2$
2. $p = 2e_A + e_M$ [e_A = automobile emissions; e_M = steel mill emissions]

q_A, q_M = abatement (reduction) in auto/steel mill emissions from uncontrolled level

3. $TC(q_A) = (q_A)^2$, therefore $MAC(q_A) = 2q_A$
4. $TC(q_M) = (q_M)^2$, therefore $MAC(q_M) = 2q_M$
5. $e_A + q_A = 10$ or $q_A = 10 - e_A$
6. $e_M + q_M = 10$ or $q_M = 10 - e_M$

a) $a_A = 2$; $a_M = 1$

b) $TC(e) = (10 - e_A)^2 + (4 – e_M)^2$

c) $p = p_A + p_M = 2e_A + e_M$

Convert ‘e’ to ‘p’ using ‘a’.

$TC(p) = (10 – p_A/2)^2 + (4 – p_M)^2$

d) $MD(p) = \partial D(p)/\partial p = \mathbf{p}$ (\$/ppm)

e) Each source has a different impact on the damage (based on a_i)

So, $MD(e_i) = a_i\, MD(p)$

$MD\,(e_A) = 2p = 2(2e_A + e_M)$; $MD\,(e_M) = p = 2e_A + e_M$

f) $–MAC(e_A)/a_A = –MAC(e_M)/a_M = MD(p)$

g) $MAC(e_A) = -\partial(C(e_A))/\partial(e_A) = 20 – 2e_A$

$MAC(e_M) = -\partial(C(e_M))/\partial(e_M) = 8 – 2e_M$

Using (f): $(20 - e_A)/2 = (8 – 2e_M)/1 = 2e_A + e_M$

Solve (2 equations, 2 unknowns)

$e_A^* = 22/7$; $e_M^* = 4/7$

Kolstad Chapter 15, Problem #4 (page 312)

4(a) $a_1 = ½$; $a_2 = 1$

4(b) $10 – 2e_1 = 10 – 2e_2$ or $e_1 = e_2$ (see Fig. 11.2 on page 221 in Chapter. 11)

Using this in following equation:

$e_1 + e_2 = 6$

gives the solution:

$e_1* = e_2* = 3$

Permit price (π) = MS or MAC

$\pi = 10 - 2e = 10 – 2(3) = 10 – 6 = \$4$

4(c) $(10 – 2e_1)/(1/2) = (10 – 2e_2)/1$

Firm one needs $(1/2)e_1$ ambient pollution permits

Firm two needs e_2 ambient pollution permits

So, $½ e_1 + e_2 = 4$

$e_1* = 18/5$; $e_2* = 11/5$

So, firm 1 needs (1/2)(18/5) = 9/5 permits (it emits more but it's '*a*'; is smaller)

and firm 2 needs 11/5 permits.

price = $10\text{-}2e_2 = 10 – 2(11/5) = \$ 28/5$ per ambient pollution permit

Kolstad Chapter 15, Problem #7 (page 313)

(a) D(S) = 0.01S

(b) $a_E = 1$; $a_D = 3$

q_E = abatement in England; q_D = abatement in Denmark

(c) Not enough information to answer this

(d) $q_E + q_D = 12$ and $q_D = 2q_E$

So, $q_D* = 8$;$q_E* = 4$

(e) $q_E + q_D = 12$ and $2q_E/1 = q_D/3$

So, $q_D*=72/7$;$q_E* = 12/7$

Two practice questions

1. Two firms can control emissions of particulate matter at the following marginal costs:

   $MC_1 = 200q_1$ $MC_2 = 100q_2$,

   where q_1 and q_2 are, respectively, the amount of emissions <u>controlled</u> or <u>reduced</u> (i.e., the amount of <u>abatement</u>) by the first and second firm, respectively. Assume that with no control at all, each firm would be emitting 20 units of emissions for a total of 40 units by both firms.

a) Compute the cost-effective allocation of control responsibility if a total reduction of 21 units of emissions is necessary.

b) Compute the cost-effective allocation of control responsibility if the ambient standard for particulate matter is 27 parts per million (ppm), and the dispersion coefficients which translate a unit of emissions into a ppm concentration at the receptor are, respectively, $a_1 = 2$ and $a_2 = 1$. [Note the total amount of control should satisfy the equation $\Sigma_i a_i (20 - q_i) = s^*$, where s^* is the ambient standard and i represents the firm.]

a) Apply the equimarginal principle and equate marginal abatement costs:

$MC_1 = MC_2$

So, $q_1 = q_2/2$ (i)

Also given: $q_1 + q_2 = 21$

So, $q_1^* = 7$; $q_2^* = 14$

b) $a_1(20-q_1) + a_2(20-q_2) = s^*$

or $2(20-q_1) + 1(20-q_2) = 27$

or $2q_1 + q_2 = 33$ (ii)

We also know that cost effective now implies using the modified equimarginal principle:

$MAC_1/a_1 = MAC_2/a_2$

$200q_1/2 = 100q_2/1$ (iii)

So, $q_1 = q_2$

Combining (ii) and (iii) $q_1 = q_2 = 11$

Note, earlier q_1 was less than q_2 (half of q_2) because MAC_1 was twice as high as MAC_2. Now because the impact of firm 1's emissions is higher, abatement by firm 1 increases (from 7 to 11) and abatement by firm 2 decreases (from 14 to 11).

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2. Marginal abatement cost (MAC) for two sources of pollution (1 and 2) affecting a single receptor are:

$MAC_1 = 0.3q_1$ $\quad\quad$ $MAC_2 = 0.5q_2$,

where q_1 and q_2 are, respectively, the amount of <u>abatement</u> by the first and second firm. Their respective transfer coefficients are $a_1 = 1.5$ and $a_2 = 1$. With no control they would emit 20 units of emissions each. The ambient standard is 12 ppm (parts per million).

a) If an ambient permit system (APS) were established, how many permits would be issued and what price would prevail?
b) How much would each source spend on permits if they were auctioned off? How much would each source ultimately spend on permits if each source was initially given, free-of-charge, half of the permits?

a) Since ambient standard is 12 ppm, 12 permits will be issued

From equation 15.16 in Kolstad (page 301)

$MAC_1/a_1 = MAC_2/a_2 = -\pi$ (ambient permit price)

Also, $a_1 e_1 + a_2 e_2 = 12$

$e_i = 20 - q_i$ $(i = 1,2)$

Rewriting,

$a_1(20 - q_1) + a_2(20 - q_2) = 12$ (i)

$0.3q_1/1.5 = p$ (ii)

$0.5q_2/1 = p$ (iii)

Three equations and three unknowns (p, q_1, q_2)

Rewrite (i) as $1.5(20 - q_1) + 1(20 - q_2) = 12$

$1.5q_1 + q_2 = 38$

Rewrite (ii) and (iii) as

$0.3q_1 = 1.5p$

$0.5q_2 = p$

or $0.3q_1 = 1.5(0.5q_2)$

$q_1 = 5q_2/2$ or $q_2 = 2q_1/5$

So, $1.5q_1 + 0.4q_1 = 38$

$q_1^* = 20$; $q_2^* = 8$; $p = \$4$ (per unit pollution or per ppm)

Pollution generated by firm 1 = $1.5e_1 = 1.5(0) = 0$

Pollution generated by firm 2 = $1.e_2 = 1(12) = 12$

b) Firm 1's demand for ambient permits is zero and firm 2 will spend 12($4) = $48 on permits. If each firm gets 6 permits free, then firm 2 will buy permits from firm 1 and pay 6($4) = $24