Environmental Catastrophes and Mitigation Policies in a Multi-region World*

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1 Introduction

It is a truth almost universally acknowledged that greenhouse gas (GHG) accumulation is contributing to climate change and increasing the risk of catastrophes such as cyclones, floods, droughts, and wildfires. Several aspects of this are important: (1) Although the broad mechanism by which climate change occurs is well established, neither the occurrence nor the costs of a catastrophic event in any one year are precisely predictable. There is much uncertainty, and the policy issue is mitigation of large risks. (2) The probability of a catastrophe occurring in any one year increases as the levels of GHG in the atmosphere increase. (3) GHGs are a worldwide public bad; emissions from any one country or region increase the risks for all. (4) There is two-sided irreversibility of policies. If we do nothing and the problem proves serious, the climate, economic activity and human life will suffer permanent damage, but if we spend large sums on countermeasures and the problem turns out to be minor or even non-existent, we will have wasted resources unnecessarily. (5) Technological progress may yield partial or even complete solutions such as rapid and efficient carbon removal, injecting sulfur particles in the upper atmosphere, or some other form of geoengineering.

In this paper we present a simple model that includes all of these features, allows equally simple calculation of the expected economic costs of such environmental catastrophes, and yields upper bounds on the sums we should be willing to spend on countermeasures. The model is extremely tractable and applies to a multi-region world but with global externalities. This allows us to look at how willingness to pay in one region varies with measures taken towards mitigation in other regions. This is important since most examples of policies to combat climate change require international cooperation. Now that the U.S. has pulled out of the Paris climate accord, then this could affect the willingness of other parties to make sacrifices. The framework that we propose can give a sense of the quantitative significance of such effects.

This paper takes a different tack to much of the existing literature on mitigating environmental damage caused by climate change where the focus has largely been on trying to characterize the effect of optimal policies. While laudable, doing so requires knowledge of the whole function specifying the cost for every policy level. However, this function is very imprecisely known at best, and the optimum is liable to be sensitive to misspecification. Moreover, in the context of international negotiations, agreeing to and implementing optimal policies is also very unlikely compared to the broad accords that have been seen in recent years. We propose a simpler way of looking at things by calculating the cost it would be worth paying to achieve specified target levels of mitigation. Even being quite conservative, we find numbers upwards of 1% of GDP. This can justify substantial expenditures in pursuit of those targets; for example, for the U.S. this amounts to spending about $190 billion every year, far more than anyone has proposed for such policies.

The simplicity of our model makes the various effects and interactions directly interpretable, whereas in more complex models they remain opaque so the mechanisms and results must be taken on faith. This approach also greatly simplifies the calculations, to the point where they can be performed using a simple Excel file. We make our file publicly available, so other researchers and policymakers can use it to solve the model for their own preferred set of parameter values. Our approach is not meant to substitute for the kind of detailed modeling that is going on this area. Rather, it is a ready-reckoner to inform debate which can give a sense of the magnitudes involved without commitment to specific details of how the economy works.

The remainder of the paper is organized as follows. In the next section, we briefly discuss some of the existing literature. In section 3, we put for a simple and general approach where the compensating and equivalent variations from interventions are derived. We then extend the model to a multi-region setting with externalities across regions. Section 4 develops the numerical solution which we implement and results are in section 5. Some brief concluding reflections on the value of the approach are in section 6.

2 Related Literature

Existing research has identified some important features of the dynamic interaction between economic activity and climate change that were listed above. An excellent overview of the literature is in Hassler et al (2016). Here we give a few illustrative examples. (1) Fat-tailed uncertainty is highlighted by Stern (2007, p. xiv), who characterized the issue as "the economics of the management of very large risks"; see also Weitzman (2011, 2014), Barro (2015) and others. (2) Dynamics with a stock effect is studied in Brito and Intriligator (1987), Dietz (2011), and Kolstad (1996). (3) Hassler and Krusell (2012) develop a stochastic general equilibrium model with many regions, and discuss the spillovers of policies such as carbon taxes. There is also a large literature on pollution control in the context of optimal growth as reviewed in Xepapadeas (2005). (4) Two-sided uncertainty, which creates embedded options in both the decisions to act and to wait, features in Kolstad (1996).

Much of the discussion in the existing literature has focused on methodological issues of the appropriate choice of the discount rate, the shape of the distribution and the specification of risk aversion in a utility function; for example Weitzman (2014), Barro (2015), Dietz (2011, esp. pp. 524–7), Pindyck (2011) and Millner (2013). Brito and Intriligator (1987) do not consider uncertainty, while Kolstad (1996) has a two-period model that does not allow significant dynamics. Martin and Pindyck (2015) study the choice of which catastrophe(s) to avert when several threaten. In Hassler and Krusell (2012), total factor productivity is a decreasing function of GHG stocks multiplied by a stochastic shock. Therefore GHG accumulation actually reduces uncertainty in productivity equiproportionately with its level. Our model is about the increase in catastrophic risks that results from these accumulations. And most work does not allow for the possibility that technological progress may allow the problem to be avoided or solved much more cheaply in the future.

3 A general model

We begin with a single-region model and then extend it to multiple regions or countries.

Core Single Region Model Let x_t denote the logarithm of the cumulated GHG level in the atmosphere in year t. This will be our state variable; its dynamics are explained below. The expected GDP is denoted by y_t. This can be a decreasing function $y_t(x_t)$, interpreted as the certainty-equivalent of some normal (non-catastrophic) uncertainty caused by GHG accumulation, for example some loss of efficiency of production processes.

The loss caused by a catastrophic event (conditional on one occurring in year t) is denoted by K_t which can be an increasing function $K_t(x_t)$. It is interpreted as a comprehensive certainty-equivalent measure. For example, if the catastrophe lowers the path of GDP from its status quo for several years, K_t includes the discounted present value of the GDP gap. It is also intended to include the monetary equivalent of human costs such as loss of life and dislocation. Not surprisingly, this is a key parameter in our analysis.

The catastrophe is modeled as a Poisson process with arrival rate $\lambda(x_t)$, an increasing function. This is the simplest way to model fat-tailed risk that rises with GHG accumulation; in fact it is all tail! As the probability is bounded between 0 and 1, $\lambda(x_t)$ should be some form of a sigmoid. Below, we specify it parametrically for numerical calculations, but for the moment it is kept general.

The dynamics of carbon accumulation has been found to follow multiple paths; see Inman (2008). We adopt the formulation in Hassler and Krusell (2012). About 60% of the emissions dissipate very quickly, so we omit them from consideration. About 20%, i.e. half of the non-transient part, stay forever. The remaining stock dissipates with a depreciation rate of around 2.8% per year. We use these figures and base values for our model and calculations. Thus, if z_t denotes the emission flow in period t, and the fraction ϵ is permanent, the permanent stock P_t is

$$P_t = \sum_{\tau=0}^{t} \epsilon \, z_\tau$$

Writing δ for the dissipation rate of the remaining fraction $(1-\epsilon)$, the dissipating stock grows as[1]

$$D_0 = 0, \quad D_t = (1-\epsilon) \; z_t + (1-\delta) \; D_{t-1}$$

Then

$$x_t = \ln(P_t + D_t)$$

The emission flows z_t can in general have any specification; we expect this to be an increasing function of the GDP, i.e. y_t. In our numerical calculations we will make specific assumptions; these will be stated at that point.

A second and independent Poisson process represents a technological solution to the whole climate change problem. Its arrival rate is denoted by $\mu(x_t)$. This can be an increasing function of the GHG level – as the problem worsens, more resources are devoted to R&D – or decreasing – as the problem worsens, more resources are needed to solve it, but because GDP falls as GHG accumulation lowers productivity, fewer resources are available for R&D. If the technological solution arrives in year t, thereafter no catastrophes will occur. We assume that GDP will then go on growing at rate g:

$$y_{t+\tau} = y_t \; (1+g)^\tau$$

The expected net present value of the economy – NPV of the GDP minus the expected discounted costs of catastrophes – can be shown to be:[2]

$$V(x_0) = \sum_{t=0}^{\infty} D_t \; [\; b_t(x_t)\, y_t(x_t) \; - \lambda_t(x_t)\, K_t(x_t) \;] \tag{1}$$

[1]Taking the stock at $t=0$ to be zero is just a normalization, as this level gets incorporated into the parameters of the catastrophe hazard rate function $\lambda(x)$ defined below.

[2]See Appendix A in the Additional Supporting Material for the derivation

where $b_t(x_t) = [r - g + (1+g)\,\mu(x_t)]/(r-g)$, and $r > g$ is the discount rate.[3, 4] This has an intuitive interpretation with $\lambda_t\,(x_t)\,K_t\,(x_t)$ being deducted from the maximum payoff in each period.

We use a standard economic approach to measuring the willingness to pay for a change. This is the well-known compensating or equivalent variation. The former asks how much GDP a society would be willing to sacrifice to make it equally well off before and after a reduction in catastrophic risk. The latter asks the question in reverse; what increase in GDP would be needed without the reduction in catastropic risk to make the value the same. The first is the willingness to pay after the change and the other after the change has taken place.

To capture these ideas formally and compute them numerically, suppose some parameters in the specification of various functions change, changing the value $V(x_0)$ to $\widetilde{V(x_0)}$. For the *compensating variation* we ask what fractional decrease θ in GDP at all times at the new parameters would yield the same value as before. That is, we want to find θ such that

$$V(x_0) = \sum_{t=0}^{\infty} D_t\ \left[\ b_t(x_t)\,\{\,(1-\theta)\,y_t(x_t)\}\ - \lambda_t(x_t)\,K_t(x_t)\ \right]$$

where the right hand side is evaluated at the new parameters. And for the *equivalent variation* we ask what fractional increase in GDP at the old parameters would yield the same value, i.e. we want to find θ such that

$$\widetilde{V(x_0)} = \sum_{t=0}^{\infty} D_t\ \left[\ b_t(x_t)\,\{\,(1+\theta)\,y_t(x_t)\}\ - \lambda_t(x_t)\,K_t(x_t)\ \right].$$

where the right hand side is evaluated at the old parameters. We show in (C) that in either case we can write

$$\theta = \frac{\widetilde{V(x_0)} - V(x_0)}{\sum_{t=0}^{\infty} D_t\ b_t(x_t)\ y_t(x_t)} \tag{2}$$

Although this makes the formula for the two variations look identical, the values may be different if the parameter changes affect the functions D_t and b_t so that the denominators in the two equations have different values, most importantly if they involve changes in the μ functions. However, in the numerical example that we solve, these variations turn out to be the same.

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[3] $r > g$ is the standard dynamic efficiency or convergence condition in growth models; see Dixit (1976, pp. 59, 109). For its empirical relevance, see Piketty (2014).

[4] Notice that only the expected loss $\lambda_t(x_t)\,K_t(x_t)$ from a catastrophe matters, not the probability and the loss separately. So we have some freedom in what follows in specifying the x-dependence of the two.

Multiple Regions Now consider a world with many regions indexed by superscript i where the externality from emissions and proness to catastrophe is global. We can think of regions as either countries or groups of countries.

Writing z_t^i for the emission flows in region i, the permanent and dissipating components P_t and D_t of the global stock now follow

$$P_t = \sum_{\tau=0}^{t} \sum_{i} \epsilon\, z_t^i$$

and

$$D_0 = 0, \quad D_t = (1-\epsilon) \sum_{i} z_t^i + (1-\delta)\ D_{t-1}$$

and then the state variable, namely log aggregate log-GHG accumulation X_t, is

$$X = \ln(P_t + D_t)$$

Thus emissions are a global public bad. To reflect this, region i's GDP is denoted by $y^i(X)$ and the cost of a catastrophe in region i is $K^i(X)$. The growth rate of these, g^i, can also be region-specific. The arrival rate of the catastrophe process is $\lambda^i(X)$ for region i; it can differ across the regions because although they are all affected by the worldwide X, their probabilities and costs can depend on whether they are in a hurricane-prone area or a flood zone etc. The technological solution, it materializes, likely to be global in which case the arrival rate function for that process would the same for all i, denoted by $\mu(X)$. However, the framework can cope with the possibility of more local solutions, such as levees or better rain capture to cope with droughts, in which case there would be separate functions $\mu^i(X)$.

It makes sense to have the discount rate r being common to all regions if capital markets are functioning well. But could also differ in the most general setting so as to capture any unmodeled region-specific capital market imperfections.

Putting this together, we can compute the value in any region i using the recursion relation specified above, yielding a solution very similar to (1) for the one-region or whole-world case:

$$V^i(X_0) = \sum_{t=0}^{\infty} D_t^i\ \left[\ b_t^i(X_t)\, y_t^i(X_t)\ - \lambda_t^i(X_t)\, K_t^i(X_t)\ \right] \tag{3}$$

where

$$D^i(0) = 1, \quad D_{T+1}^i = \prod_{t=0}^{T} \frac{1-\mu^i(X_t)}{1+r^i}$$

and

$$b_t^i(X_t) = \frac{r^i - g^i + (1+g^i)\,\mu^i(X_t)}{r^i - g^i}$$

The crucial difference is that X_t rather than the region specific x_t^i enters, reflecting the global interdependence. Hence the willingness to pay for reductions in emissions will be interdependent and depend on the time path of emissions in other countries.

4 Numerical solution

We will study the implications of the using a simple numerical solution and for that we make a few specific assumptions. We will then choose specific values of key parameters and provide a quantitative assessment of the willingness to pay to avoid the risks associated with climate change.

Parametrization For the moment, we revert of the case of single region and hence drop the i superscript. Assume that under status quo policies the GDP and the cost of a catastrophe keep on growing at fixed rate rate g so that

$$y_t = y_0\ (1+g)^t, \quad K_t = K_0\ (1+g)^t$$

We also assume that arrival rate for the saviour technology is constant at μ which captures a rough balance of the two forces mentioned above. Then (1) simplifies to[5]

$$V(x_0) = y_0\,\frac{1+r}{r-g} - K_0\ \Lambda \tag{4}$$

where

$$\Lambda = \sum_{t=0}^{\infty} \left[\frac{(1+g)(1-\mu)}{1+r} \right]^t\ \lambda(x_t)$$

which is a kind of expected present value operator that captures the influence of the key parameters embedded in $\lambda(x_t)$ (through the growth parameters in x_t, and of μ. The solution (4) has a nice interpretation: it is the full discounted present value of GDP absent any catastrophes, minus the expected discounted cost of catastrophes.

In our base case numerical calculations we assume that emission flows z_t grow at a rate α equal to the GDP rate of growth g; then we examine the effects of various policies, for example a Kyoto-style reduction of α by 30%.

[5]The derivation is in Appendix B.

We specify the arrival rate of the catastrophe Poisson process as the usual logistic function with two parameters:

$$\lambda(x) = e^{\gamma x}/[\, J + e^{\gamma x}\,]$$

A convenient feature of this special case is that the compensating and equivalent variations are the same as each other. So in what follows, we do not need to differentiate between them. We derive their formula in Appendix C of the Additional Supporting Material:

$$\theta = \frac{r-g}{1+r}\,\frac{K_0}{y_0}\,(\Lambda - \Lambda') \tag{5}$$

for some parameter shift change Λ to Λ'. Note that the "loss ratio", K_0/y_0, simply multiplies the expression for the compensating variation up or down.

The generalization of this to regional differences is straightforward. We suppose that ϵ and δ pertain to global carbon dynamics so they are the same for all regions; then so is the state variable X. But the GDP levels y_0^i, the costs of catastrophes K_0^i, the functional form of the catastrophe hazard functions $\lambda^i(X)$, and the parameters g^i, r^i, and μ^i can be region-specific. Then (5) in a multi-region world becomes:

$$\theta^i = \frac{r^i - g^i}{1+r^i}\,\left(\frac{K_0^i}{y_0^i}\right)^i (\Lambda^i - \Lambda^{i\prime})$$

where

$$\Lambda^i = \sum_{t=0}^{\infty}\left[\frac{(1+g^i)(1-\mu^i)}{1+r^i}\right]^t \lambda^i(X_t).$$

This has the neat feature that all the interdependence is captured entirely through $\lambda^i\,(X_t)$.

Choice of parameter values We specify central values for the parameters, and consider a range around them in numerical solutions. We err towards assuming somewhat optimistic parameter values implying that our conclusions are conservative in the sense that advocating significant expenditures based on the chosen parameter values would hold *a fortiori* for the specifications and parameter values favored those with more alarmist views of climate change problems.

As our baseline, we set

$$r = 0.05, \quad g = 0.03, \quad \alpha = 0.03, \quad \delta = 0.03, \quad \epsilon = 0.5, \quad \mu = 0.01, \quad y_0 = 1, \quad K_0 = 2 \tag{6}$$

The discount rate r is much higher than the near-zero rate advocated by many advocates of strong policies to counter climate change, for example Stern (2007), and close to the 5% or more that was advocated by critics of the Stern Review, for example Weitzman (2007). This is in the spirit of making conservative assumptions as indicated above: the lower the discount rate, the more future damage weighs in the calculation and the greater the justification for countermeasures. The 3% status quo growth rate is again optimistic. We have set $\alpha = g$, so under the status quo GHG emissions would keep step with economic growth. The values of the permanent component of emissions ϵ and the dissipation rate δ of the rest are in broad agreement with Inman (2008) and Hassler and Krusell (2012). Very little is known about the likelihood of a total technological solution, but the choice $\mu = 0.01$ implies that the probability of such a solution having arrived rises to 50% in 70 years, which seems if anything optimistic. Setting $y_0 = 1$ is a normalization and we discuss the justification of K_0 below.

In the logistic specification of the function $\lambda(x)$ our base values are $\gamma = 1.5$ and $J = 20000$. With these, the probability of at least one catastrophe occurring by time T rises to 50% in $T = 56$ years and to 90% in 81 years; again these are fairly optimistic figures. It should be clear to the reader that all of these magnitudes could be varied and an Excel spreadsheet allows the interested reader to do so.

In our Excel file, we carry out the sum defining Λ in (4) to 1000 years, when the terms generally become of the order of 10^{-12}. Again readers can easily alter the file as they wish.

Specifying the expected cost of a catastrophe As emphasized in reviews of Integrated Assessment Models such as Metcalf (2015), there is a considerable uncertainty about the right assumptions to make about the likely damages from higher carbon emissions. We anchor our estimates around the U.S. experience of Hurricane Katrina, which hit New Orleans and other parts of southeastern United States in late August 2005, had many of the features that are expected to figure in future environmental catastrophes – flooding, wind and water damage to structures, loss of life, dislocation of populations and disruption of economic activity, and so on. That storm cannot be attributed directly to climate change, but a rough quantification of its effects gives us a useful starting point for thinking about costs of catastrophes. Although this is a specific case to help fix ideas, it would be straightforward to assess the sensitivity of the results to alternative scenarios.

We begin with the loss of GDP. Prior to Katrina, New Orleans' GDP was growing fast, from \$52.38 billion in 2001 to \$72.91 billion in 2005, which is an annual growth rate of 8.6%.[6]

[6]These figures come from an article on the Atlanta Federal Reserve web site, "New Orleans, 10 Years af-

To assess the shortfall of GDP below what it would have been without Katrina, let us take a very conservative approach by assuming that the GDP would have grown for the next three years at the slower rate experienced by the U.S. as a whole. Table 1 shows the calculations. The cumulative shortfall for the three years 2006-08 is $24.8 billion, which is 34% of the 2005 GDP level. Effects of Katrina continued for much longer than 3 years, but after 2009 the calculation gets trickier because of the need to separate the effects of Katrina from those of the Great Recession. We again take a conservative approach by omitting any GDP losses beyond three years.

Table 1: GDP loss due to Katrina in New Orleans

Year	Actual GDP	US growth rate	Hypothetical GDP	Shortfall
2005	72.91			
2006	71.18	5.8	77.14	5.96
2007	70.93	4.5	80.61	9.68
2008	72.82	1.7	81.98	9.16

There was also considerable loss of, and damage to, capital. For the whole region affected by Katrina, which comprises the states of Louisiana, Florida and Mississippi, private insured and uninsured losses are estimated at $108 billion, of which about half occurred in New Orleans alone. In addition, restoration of the damaged levies and coastal restoration and urban water management projects in New Orleans required about $29 billion. These costs – $54 billion + $29 billion = $83 billion – amount to over 113% of that city's 2005 GDP.[7]

Thus the capital costs (113%) and GDP losses (34%) taken together come to 147% of one year's GDP in New Orleans. And these calculations do not include human costs – 1,836 lives lost, over 100,000 people displaced and their lives disrupted, trauma suffered by pretty much the whole population (over 400,000) of that city, and much more. Our conclusion from this exercise is that a reasonable baseline case is $K_0 = 2\, y_0$. In assessing the sensitivity of our conclusions, we consider variation in K_0/y_0 between 1 and 3.

ter Katrina," https://www.frbatlanta.org/economy-matters/2015/08/20/new-orleans-10-years-after-katrina, accessed May 10, 2017.

[7]From https://www.thebalance.com/hurricane-katrina-facts-damage-and-economic-effects-3306023, accessed May 10, 2017.

5 Results

Single Region We begin with the single region (or unified world) case. This will help to get a feel for the quantitative magnitudes which come out of the model. In the first instance, we use the base parameters above. But we will also see how our conclusions vary with some of these, usually one at a time.

First consider the Kyoto reduction in gross GHG emissions, lowering α by 30%, i.e. from 0.03 to 0.021. This has a variation (compensating or equivalent) of 0.033. That is, we should be willing to pay a cost of 3.3% of GDP each year to bring about the Kyoto reduction. This is a large number; for the US it amounts to about $500 billion a year (and growing at 3% in step with GDP growth). However, it put it in perspective, it is only equivalent to permanently sacrificing one good year of economic growth.

This number is quite sensitive to the outlook for growth and if the growth projection were 1% instead of 3% which is more in line with recent growth pessimism, then this number also falls to around a third, namely 1.1%, of current GDP. Perhaps not surprisingly, the willingness to pay for a the Kyoto reduction is increased significantly by having a smaller discount rate with an increase in the willingness to pay to 4.8% if the discount rate is $r = 0.04$ (while keeping g at its base level of 0.03. One could also argue that the Hurricane Katrina output loss is too conservative to capture the kind of catastrophic change that could be envisaged. Suppose that instead that K_0/y_0 is equal to 3 instead of 2, then the willingness to pay would increase to 5% of GDP. The bottom line in all cases, is that the plausible willingness to pay for Kyoto-style reductions is in the range of $1\% - 5\%$. While one would seek to design policies which do this both fairly and efficiently, the sizes of the sacrifices in consumption indicated here are small in comparison to historic increases in material living standards.

Next consider increasing μ from 0.01 to 0.015. This would raise the probability that a solution has been found in 25 years time from 22% to 31%. This might be a feasible with the kind of investment in science that have been scene in the past in pursuit of military ends or space travel. But how much does our model suggest would be a reasonable commit of aggregate resources to achieve this end? The willingness to pay for this in our baseline cases 0.033, i.e. we should be willing to invest 3.3% of GDP, or $500 billion a year for the US (and growing at 3%), to raise the probability of a complete technological solution by 50%.[8] To

[8] This CV is almost equal to that of the Kyoto reduction in emissions stated above. But that is an accident; the two CVs do differ in significant figures beyond the second.

put this perspective, note that this is less than the roughly 4% of GDP that the government spends on national defense although considerably in excess of the (around) $20 billion a year spent on NASA and the total NSF budget of around $6 billion.

Multiple Countries We will consider four regions which we call China, USA, Europe and the Rest of the World (RoW).[9] Their GDP shares and share of CO_2 emissions are:

	GDP share	CO_2 share
China	15%	30%
USA	15%	15%
Europe	20%	15%
RoW	50%	50%

The world GDP and initial emission level are both normalized to 1. And in the baseline we assume that the parameter values in (6) are maintained.

What makes the multi-country case interesting is how the willingness to pay in one country is affected by actions taken elsewhere. Due to the global nature of the externality, the willingness of the US or China to take a Kyoto style cut would depend on the path of emissions taken elsewhere. Associated with any proposal, therefore, would be an associated vector $\{\theta^1, \theta^2, \theta^3, \theta^4\}$ denoting the willinness to pay in each region. The critical issue in negotiations over emissions reduction is how the benefits and costs are shared. Aldy et al (2009) discuss the complex issues that are involved in aligning this. Our ready-reckoner approach will be useful in giving an insight into how the heterogeneity in willingness to pay depends upon underlying differences in economic prospects for the regions of the world. If the willingness to pay is similar, then it should be easier to achieve consensus.

To provide a benchmark, we consider a world in which all countries follow the Kyoto benchmark with a cut in emissions such that α^i falls 0.03 to 0.021. In this case, the willingness to pay is more or less identical and equal to about 3.4% in all regions of the world. But if one region decided to opt out of the deal and free-ride, then how big would the gain be to the participating countries? In the case where either China or the US opts out then the willingness to pay for participators falls to around 2.1%. Another way of looking at this is that around two thirds of the benefit from emissions reductions is available to a free-rider. Hence, the *marginal* willingness to pay for China or the US, assuming that other

[9]Hassler and Krusell (2012) specifies Africa as the fourth region in their approach.

countries go along with a Kyoto style cut, is around 1.3% of GDP which is quite a bit below the average gain of 3.4%.

Another interesting question is to ask what should be China's or the US's willingness to pay for unilateral action? For this we suppose that the rest of the world maintain's $\alpha^i = 0.03$ and that either China or the US cuts α^i to 0.21. This yields a willingness to pay for the country that is cutting of only 0.0076 or less than 1% of GDP. This is still a substantial number but less than a third a of the benefit under multilateral action as the benefits dissipate around the world.

Taken together, these results suggest that, in our baseline case, there is still a safe presumption of a willingness to pay of around 1% for a Kyoto-sized emissions reduction even in a world where no multilateral action is assumed.

Given the baseline parameter values, the proportionate gains and losses are similar in all regions. But different assumptions about growth prospects would have an effect on this. Suppose, for example, that one is pessimistic about growth in Europe and the U.S. while there will broadly be catch up among the remaining economies around the world of the kind that we have seen in recent years. If we assume that growth will only be around 1% future then the gains from participating in a Kyoto style cut fall to only 1% of GDP. And the gain from unilateral action by the US falls to only 0.3% of GDP if growth is only projected at 1%. This illustrates the potential power of growth pessimism in shaping willingness to participate in global action.

We next consider what happens if the losses from catastrophes are unevenly distributed. Suppose that the U.S. and Europe have reasons to be sanguine about the cost of catastrophes and their willingness to pay is based on $K_0/y_0 = 1$ while in China and the Rest of the World, the losses are larger with $K_0/y_0 = 3$. Then how do we think that this will affect the geopolitics of reaching an agreement? First, consider a multilateral agreement to $\alpha = 0.021$. The willingness to pay in China and the Rest of the World now increases to around 5% of GDP while that in the U.S. and Europe would fall to around 1.6%. Unless based on altruism, this makes multi-lateral action less likely. Another way to look at this is that the marginal benefit to the US and Europe to join such a cut conditional on China and the Rest of the World making is is just 0.5% of GDP.

Of course, all of these numbers are only illustrative but they show that the perceptions around the distribution of damages due to climate change affect the potential for self-interest to motivate action and our framework allows us to think about the magnitudes involved and sensitivity to parameter values.

6 Concluding Comments

This paper has put forward a model to evaluate the risks of climate catastrophes in a multi-region world. We have developed a simple formulation of the costs of catastrophic risk and the willingeness to pay for mitigation. The model is simple, transparent and can be solved on a spread sheet, thereby giving a simple way of thinking about the kinds of sacrifice that a society might make to mitigate these risks. In our baseline, the numbers turn out to be quite large (typically in excess of 1% of GDP). Of course, ensuring that reductions in consumption brought about by taxation are actually spent wisely and effectively to bring about emissions reduction and/or investments in technology is by no means easy. And there are complicated issues in policy design that we have not tackled here.

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Additional Supporting Material

A Derivation of Equation (1)

The present discounted value of GDP as of time t, using a discount rate $r > g$.

$$\sum_{\tau=0}^{\infty} y_t \, (1+g)^{\tau} \, (1+r)^{-\tau} = \frac{y_t}{1 - \frac{1+g}{1+r}} = y_t \, \frac{1+r}{r-g} \tag{7}$$

Let $V(x_t)$ denote the expected net present value of all relevant economic and money-equivalent non-economic benefits and costs starting with GHG log-level x_t. This satisfies the recurrence relation

$$\begin{aligned}
V(x_t) = {} & \lambda(x_t)\,\mu(x_t)\, \left[-K_t(x_t) + y_t(x_t)\, \frac{1+r}{r-g} \right] \\
& + \lambda(x_t)\,[1-\mu(x_t)]\, \left[-K_t(x_t) + y_t(x_t) + \frac{1}{1+r}\, V(x_{t+1}) \right] \\
& + [1-\lambda(x_t)]\,\mu(x_t)\, \left[y_t(x_t)\, \frac{1+r}{r-g} \right] \\
& + [1-\lambda(x_t)]\,[1-\mu(x_t)]\, \left[y_t(x_t) + \frac{1}{1+r}\, V(x_{t+1}) \right] \\
& = y_t(x_t)\, \left\{ (1-\mu(x_t) + \mu(x_t)\, \frac{1+r}{r-g} \right\} - \lambda_t(x_t)\, K_t(x_t) + \frac{1-\mu(x_t)}{1+r}\, V(x_{t+1}) \\
& = y_t(x_t)\, \frac{r-g+(1+g)\mu(x_t)}{r-g} - \lambda_t(x_t)\, K_t(x_t) + \frac{1-\mu(x_t)}{1+r}\, V(x_{t+1})
\end{aligned} \tag{8}$$

A brief explanation of this expression is as follows. On the right hand side, each line corresponds to one outcome of the two Poisson processes. In the first line, both the catastrophe and the technological solution occur; this incurs the cost $K_t(x_t)$ and generates the present value of the GDP stream calculated in (7). In the second line, the catastrophe occurs but the technological solution does not arrive. This incurs cost $K_t(x_t)$, has the immediate GPD flow $y_t(x_t)$, and yields the continuation value $V(x_{t+1})$ starting next year so discounted back by the factor $1/(1+r)$. In the third line there is no catastrophe in period t and the technological solution arrives, so we simply get the present value of the GDP stream. In the fourth line neither the catastrophe nor the technological solution occur, so we have the immediate GDP flow and the continuation value. The following two lines collect terms and rearrange them suitable for iteration.

Now define the risk-corrected discount factors D_t by $D_0 = 1$ and

$$D_{t+1} = D_t \, \frac{1 - \mu(x_t)}{1 + r} \text{ for } t = 0,\, 1,\, 2,\, \ldots$$

that is,

$$D_{T+1} = \prod_{t=0}^{T} \frac{1 - \mu(x_t)}{1 + r}$$

Then iterating (8) starting at $t = 0$ and going up to T yields

$$V(x_0) = \sum_{t=0}^{T} D_t \, \left[\, y_t(x_t) \, \frac{r - g + (1 + g)\mu(x_t)}{r - g} - \lambda_t(x_t)\, K_t(x_t) \, \right] + D_{T+1} \, V(x_{T+1})$$

Assume that there exist $\overline{y}$ and $\overline{K}$ such that

$$y_t \leq \overline{y} \, (1 + g)^t, \qquad K_t \leq \overline{K} \, (1 + g)^t$$

Then in the best possible case is where $\lambda(x) \equiv 0$ and $\mu(x) \equiv 1$,

$$V(x_t) \leq \overline{y} \, \frac{1 + r}{r - g} \, (1 + g)^t$$

and in the worst possible case $\lambda(x) \equiv 1$ and $\mu(x) \equiv 0$,

$$V(x_t) \geq -\overline{K} \, (1 + g)^t$$

Also $D_t \leq (1 + r)^{-t}$ for all t. Therefore, using $r > g$, we have

$$\lim_{T \to \infty} \, D_{T+1} \, V(x_{T+1}) = 0.$$

Putting his together gives (1).

B Derivation of Equation (4)

Let

$$D_t = \left(\frac{1 - \mu}{1 + r} \right)^t$$

and

$$\sum_{t=0}^{\infty} D_t\, b_t(x_t)\, y_t(x_t) = y_0\ \frac{r-g+(1+g)\mu}{r-g} \sum_{t=0}^{\infty} (1+g)^t\, \left(\frac{1-\mu}{1+r}\right)^t$$

$$= y_0\ \frac{r-g+(1+g)\mu}{r-g} \frac{1}{1-\frac{(1+g)(1-\mu)}{1+r}}$$

$$= y_0\ \frac{r-g+(1+g)\mu}{r-g}\ \frac{1+r}{r-g+\mu(1+g)}$$

$$= y_0\ \frac{1+r}{r-g} \tag{9}$$

Therefore

$$V(x_0) = y_0\ \frac{1+r}{r-g} - K_0 \sum_{t=0}^{\infty} \left[\frac{(1+g)(1-\mu)}{1+r}\right]^t\ \lambda(x_t)$$

which is (4).

C Compensating and Equivalent Variations

For the compensating variation, we want to solve

$$V(x_0) = \widetilde{V(x_0)} - \theta\ \sum_{t=0}^{\infty} D_t\ b_t(x_t)\ y_t(x_t)$$

and hence

$$\theta = \frac{\widetilde{V(x_0)} - V(x_0)}{\sum_{t=0}^{\infty} D_t\ b_t(x_t)\ y_t(x_t)} \tag{10}$$

And we for the equivalent variation, we want to solve

$$\widetilde{V(x_0)} = V(x_0) + \theta\ \sum_{t=0}^{\infty} D_t\ b_t(x_t)\ y_t(x_t)$$

Therefore

$$\theta = \frac{\widetilde{V(x_0)} - V(x_0)}{\sum_{t=0}^{\infty} D_t\ b_t(x_t)\ y_t(x_t)}. \tag{11}$$

In the numerical solution which changes Λ to Λ', then compensatingly, change y_0 to $(1-\theta)\, y_0$ to keep V_0 unchanged:

$$V_0 = (1-\theta)\, y_0\ \frac{1+r}{r-g} - K_0\ \Lambda'$$

Subtracting,

$$0 = \theta\ y_0\ \frac{1+r}{r-g} - K_0\ (\Lambda - \Lambda')$$

Therefore the compensating variation is

$$\theta = \frac{r-g}{1+r}\ \frac{K_0}{y_0}\ (\Lambda - \Lambda')$$

To calculate the equivalent variation, let the parameter change shift V_0 to $\widetilde{V}_0$, and let an equivalent change in y_0 to $(1+\theta)\, y_0$ achieve the same thing at the old parameter values:

$$\begin{aligned}\widetilde{V}_0 &= y_0\ \frac{1+r}{r-g} - K_0\ \Lambda' \\ &= (1+\theta)\, y_0\ \frac{1+r}{r-g} - K_0\ \Lambda\end{aligned}$$

Subtracting,

$$0 = \theta\, y_0\ \frac{1+r}{r-g} - K_0\ (\Lambda - \Lambda')$$

So the equivalent variation is

$$\theta = \frac{r-g}{1+r}\ \frac{K_0}{y_0}\ (\Lambda - \Lambda') = CV$$

In this special case, in the calculation that led to (9) the μ canceled out. Therefore there is no difference between the denominators in (10) and (11) and the two variations are equal.