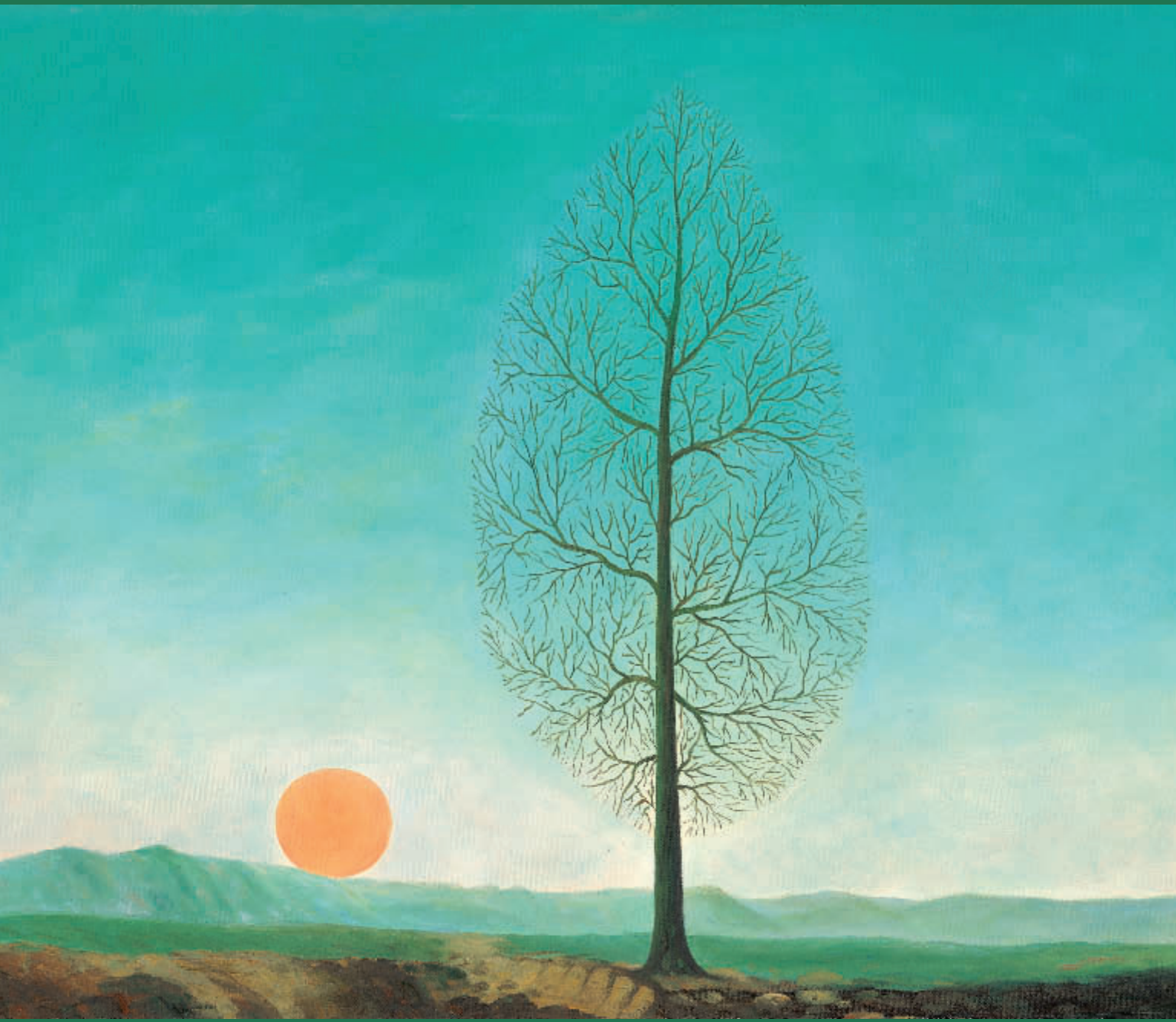
Intermediate Public Economics



Jean Hindriks and Gareth D. Myles

INTERMEDIATE PUBLIC ECONOMICS

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Jean Hindriks and Gareth D. Myles

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à Nathalie pour son amour et à mes adorables enfants Mattéo, Moïra, et Salomé (JH)

to Tracy, to Harriet, and to Georgina—it began before you could walk but was finished through your help with the typing (GDM)

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Preface

This book has been prepared as the basis for a final-year undergraduate or first-year graduate course in Public Economics. It is based on lectures given by the authors at several institutions over many years. It covers the traditional topics of efficiency and equity but also emphasizes more recent developments in information, games, and, especially, political economy.

The book should be accessible to anyone with a background of intermediate microeconomics and macroeconomics. We have deliberately kept the quantity of math as low as we could without sacrificing intellectual rigor. Even so, the book remains analytical rather than discursive.

To support the content, further reading is given for each chapter. This reading is intended to offer a range of material from the classic papers in each area through recent contributions to surveys and critiques. Exercises are included for each chapter. Most of the exercises should be possible for a good undergraduate but some may prove challenging.

There are many people who have contributed directly or indirectly to the preparation of this book. Nigar Hashimzade is entitled to special thanks for making incisive comments on the entire text and for assisting with the analyses in chapters 10 and 21. Thanks are also due to Jean Marie Baland, Paul Belleflamme, Tim Besley, Chuck Blackorby, Christopher Bliss, Craig Brett, John Conley, Richard Cornes, Philippe De Donder, Sanjit Dhami, Peter Diamond, Jean Gabszewicz, Peter Hammond, Arye Hillman, Norman Ireland, Michael Keen, François Maniquet, Jack Mintz, James Mirrlees, Frank Page Jr., Susana Peralta, Pierre Pestieau, Pierre M. Picard, Ian Preston, Maria Racionero, Antonio Rangel, Les Reinhorn, Elena del Rey, Todd Sandler, Kim Scharf, Hyun Shin, Michael Smart, Stephen Smith, Klaas Staal, Jacques Thisse, Harrie Verbon, John Weymark, David Wildasin, and Myrna Wooders. Jean also wishes to thank Fabienne Henry for her secretarial services.

Public Economics is about the government and the economic effects of its policies. This book offers an insight into what Public Economics says and what it can do. We hope that you enjoy it.

Jean Hindriks
Louvain La Neuve

Gareth Myles
Exeter
February 2005

I PUBLIC ECONOMICS AND ECONOMIC EFFICIENCY

1 An Introduction to Public Economics

1.1 Public Economics

The study of public economics has a long tradition. It developed out of the original political economy of John Stuart Mill and David Ricardo, through the public finance tradition of tax analysis into public economics, and has now returned to its roots with the development of the new political economy. From the inception of economics as a scientific discipline, public economics has always been one of its core branches. The explanation for why it has always been so central is the foundation that it provides for practical policy analysis. This has always been the motivation of public economists, even if the issues studied and the analytical methods employed have evolved over time. We intend the theory described in this book to provide an organized and coherent structure for addressing economic policy.

In the broadest interpretation, public economics is the study of economic efficiency, distribution, and government economic policy. The subject encompasses topics as diverse as responses to market failure due to the existence of externalities, the motives for tax evasion, and the explanation of bureaucratic decision-making. In order to reach into all of these areas, public economics has developed from its initial narrow focus upon the collection and spending of government revenues, to its present concern with every aspect of government interaction with the economy. Public economics attempts to understand both *how* the government makes decisions and *what* decisions it should make.

To understand how the government makes decisions, it is necessary to investigate the motives of the decision-makers within government, how the decision-makers are chosen, and how they are influenced by outside parties. Determining what decisions should be made involves studying the effects of the alternative policies that are available and evaluating the outcomes to which they lead. These aspects are interwoven throughout the text. By pulling them together, this book provides an accessible introduction to both these aspects of public economics.

1.2 Methods

The feature that most characterizes modern public economics is the use made of *economic models*. These models are employed as a tool to ensure that arguments

are conducted coherently with a rigorous logical basis. Models are used for analysis because the possibilities for experimentation are limited and past experience cannot always be relied on to provide a guide to the consequences of new policies. Each model is intended to be a simplified description of the part of the economy that is relevant for the analysis. What distinguishes economic models from those in the natural sciences is the incorporation of independent decision-making by the firms, consumers, and politicians that populate the economy. These actors in the economy do not respond mechanically but are motivated by personal objectives and are strategic in their behavior. Capturing the implications of this complex behavior in a convincing manner is one of the key skills of a successful economic modeler.

Once a model has been chosen, its implications have to be derived. These implications are obtained by applying logical arguments that proceed from the assumptions of the model to a set of formally correct conclusions. Those conclusions then need to be given an interpretation in terms that can be related to the original question of interest. Policy recommendations can then be derived but always with a recognition of the limitations of the model.

The institutional setting for the study of public economics is invariably the mixed economy where individual decisions are respected but the government attempts to affect these through the policies it implements. Within this environment many alternative objectives can be assigned to the government. For instance, the government can be assumed to care about the aggregate level of welfare in the economy and to act selflessly in attempting to increase this. Such a viewpoint is the foundation of optimal policy analysis that inquires how the government should behave. But there can be no presumption that actual governments act in this way. An alternative, and sometimes more compelling view, is that the government is composed of a set of individuals, each of whom is pursuing their own selfish agenda. Such a view provides a very different interpretation of the actions of the government and often provides a foundation for understanding how governments actually choose their policies. This perspective will also be considered in this book.

The focus on the mixed economy makes the analysis applicable to most developed and developing economies. It also permits the study of how the government behaves and how it should behave. To provide a benchmark from which to judge the outcome of the economy under alternative policies, the command economy with an omniscient planner is often employed. This, of course, is just an analytical abstraction.

1.3 Analyzing Policy

The method of policy analysis in public economics is to build a model of the economy and to find its equilibrium. Policy analysis is undertaken by determining the effect of a policy by tracing through the ways in which it changes the equilibrium of the economy relative to some *status quo*. Alternative policies are contrasted by comparing the equilibria to which they lead.

In conducting the assessment of policy, it is often helpful to emphasize the distinction between *positive* and *normative* analysis. The positive analysis of government investigates topics such as why there is a public sector, where government objectives emerge, and how government policies are chosen. It is also about understanding what effects policies have upon the economy. In contrast, normative analysis investigates what the best policies are, and aims to provide a guide to good government. These are not entirely disjoint activities. To proceed with a normative analysis, it is first necessary to conduct the positive analysis: it is not possible to say what is the best policy without knowing the effects of alternative policies upon the economy. It could also be argued that a positive analysis is of no value until used as a guide to policy.

Normative analysis is conducted under the assumption that the government has a specified set of objectives and its action are chosen in the way that best achieves these. Alternative policies (including the policy of *laissez faire* or, literally "leave to do") are compared by using the results of the positive analysis. The optimal policy is that which best meets the government's objective. Hence the equilibria for different policies are determined and the government's objective is evaluated for each equilibrium.

In every case restrictions are placed on the set of policies from which the government may choose. These restrictions are usually intended to capture limits on the information that the government has available. The information the government can obtain on the consumers and firms in the economy restricts the degree of sophistication that policy can have. For example, the extent to which taxes can be differentiated among different taxpayers depends on the information the government can acquire about each individual. Administrative and compliance costs are also relevant in generating restrictions on possible policies.

When the government's objective is taken to be some aggregate level of social welfare in the economy, important questions are raised as to how welfare can be measured. This issue is discussed in some detail in a later chapter, but it can be noted here that the answer involves invoking some degree of comparability

between the welfare levels of different individuals. It has been the willingness to proceed on the basis that such comparisons can be made that has allowed the development of public economics. While differences of opinion exist on the extent to which these comparisons are valid, it is still scientifically justifiable to investigate what they would imply if they could be made. Furthermore general principles can be established that apply to any degree of comparability.

1.4 Preview

Part I of the book, consisting of this chapter and chapter 2, introduces public economics and reviews the efficiency of the competitive equilibrium. The discussion of the methodology of public economics has shown that a necessary starting point for the development of the theory of policy analysis is an introduction to economic modeling. This represents the content of chapter 2 in which the basic model of a competitive economy is introduced. The chapter describes the agents involved in the economy and characterizes economic equilibrium. An emphasis is placed upon the assumptions on which the analysis is based since much of the subject matter of public economics follows from looking at how the government should respond if these are not satisfied. Having established the basic model, the chapter then investigates the efficiency of the competitive equilibrium. This leads into some fundamental results in welfare economics.

The analysis of government begins in part II. Chapter 3 provides an overview of the public sector. It first charts the historical growth of public sector expenditure over the previous century and then reviews statistics on the present size of the public sector in several of the major developed economies. The division of expenditure and the composition of income are then considered. Finally, issues involved in measuring the size of the public sector are addressed. The issues raised by the statistics of chapter 3 are addressed by the discussion of theories of the public sector in chapter 4. Reasons for the existence of the public sector are considered, as are theories that attempt to explain its growth. A positive analysis of how the government may have its objectives and actions determined is undertaken. An emphasis is given to arguments for why the observed size of government may be excessive.

The focus of part III is on the consequences of market failure. Chapter 5 introduces public goods into the economy and contrasts the allocation that is achieved when these are privately provided with the optimal allocation. Mechanisms for improving the allocation are considered and methods of preference revelation are also addressed. This is followed by an analysis of clubs and local public goods,

which are special cases of public goods in general, in chapter 6. The focus in this chapter returns to an assessment of the success of market provision. The treatment of externalities in chapter 7 relaxes another of the assumptions. It is shown why market failure occurs when externalities are present and reviews alternative policy schemes designed to improve efficiency. Imperfect competition and its consequences for taxation is the subject of chapter 8. The measurement of welfare loss is discussed and emphasis is given to the incidence of taxation. A distinction is also drawn between the effects of specific and ad valorem taxes. A symmetry of information between trading parties is required to sustain efficiency. When it is absent, inefficiency can arise. The implications of informational asymmetries and potential policy responses are considered in chapter 9.

Part IV provides an analysis of the public sector and its decision-making processes. This can be seen as a dose of healthy scepticism before proceeding into the body of normative analysis. An important practical method for making decisions and choosing governments is voting. Chapter 10 analyzes the success of voting as a decision mechanism and the tactical and strategic issues it involves. The main results that emerge are the Median Voter Theorem and the shortcomings of majority voting. The consequences of rent-seeking are then analyzed in chapter 11. The theory of rent-seeking provides an alternative perspective upon the policy-making process that is highly critical of the actions of government.

Part III focuses on economic efficiency. Part V complements this by considering issues of equity. Chapter 12 analyzes the policy implications of equity considerations and addresses the important restrictions placed on government actions by limited information. Several other fundamental results in welfare economics are also developed including the implications of alternative degrees of interpersonal comparability. Chapter 13 considers the measurement of economic inequality and poverty. The economics of these measures ultimately re-emphasizes the fundamental importance of utility theory.

Part VI is concerned with taxation. It analyzes the basic tax instruments and the economics of tax evasion. Chapters 14 and 15 consider commodity taxation and income taxation, which are the two main taxes levied on consumers. In both of these chapters the economic effects of the instruments are considered and rules for setting the taxes optimally are derived. The results illustrate the resolution of the equity/efficiency trade-off in the design of policy and the consequences of the limited information available to the government. In addition to the theoretical analysis, the results of application of the methods to data are considered. The numerical results are useful, since the theoretical analysis leads only to characterizations of

optimal taxes rather than explicit solutions. These chapters all assume that the taxes that are levied are paid honestly and in full. This empirically doubtful assumption is corrected in chapter 16, which looks at the extent of the hidden economy and analyzes the motives for tax evasion and its consequences.

Part VII studies public economics when there is more than one decision-making body. Chapter 17 on fiscal federalism addresses why there should be multiple levels of government and discusses the optimal division of responsibilities between different levels. The concept of tax competition is studied in chapter 18. It is shown how tax competition can limit the success of delegating tax-setting powers to independent jurisdictions.

Part VIII concentrates upon intertemporal issues in public economics. Chapter 19 describes the overlapping generations economy that is the main analytical tool of this part. The concept of the Golden Rule is introduced for economies with production and capital accumulation, and the potential for economic inefficiency is discussed. Chapter 20 analyses social security policy and relates this to the potential inefficiency of the competitive equilibrium. Both the motivation for the existence of social security programs and the determination of the level of benefits are addressed. Ricardian equivalence is linked to the existence of gifts and bequests. Finally, the book is completed by chapter 21, which considers the effects of taxation and public expenditure upon economic growth. Alternative models of economic growth are introduced and the evidence linking government policy to the level of growth is discussed.

1.5 Scope

This book is essentially an introduction to the theory of public economics. It presents a unified view of this theory and introduces the most significant results of the analysis. As such, it provides a broad review of what constitutes the present state of public economics.

What will not be found in the book are many details of actual institutions for the collection of taxes or discussion of existing tax codes and other economic policies, although relevant data are used to illuminate argument. There are several reasons for this. This book is much broader than a text focusing on taxation, and to extend the coverage in this way, something else has to be lost. Primarily, however, the book is about understanding the effects of public policy and how economists think about the analysis of policy. This should give an understanding of the

consequences of existing policies, but to benefit from the discussion does not require detailed institutional knowledge.

Furthermore tax codes and tax law are country-specific, and pages spent discussing in detail the rules of one particular country will have little value for those resident elsewhere. In contrast, the method of reasoning and the analytical results described here have value independent of country-specific detail. Finally there are many texts available that describe tax law and tax codes in detail. These are written for accountants and lawyers and have a focus rather distinct from that adopted by economists.

Further Reading

The history of political economy is described in the classic volume:

Blaug, M. 1996. *Economic Theory in Retrospect*. Cambridge: Cambridge University Press.

Two classic references on economic modeling are:

Friedman, M. 1953. *Essays on Positive Economics*. Chicago: University of Chicago Press.

Koopmans, T. C. 1957. *Three Essays on the State of Economic Science*. New York: McGraw-Hill.

The issues involved in comparing individual welfare levels are explored in:

Robbins, L. 1935. *An Essay on the Nature and Significance of Economic Science*. London: Macmillan.

Exercises

1.1. Should an economic model be judged on the basis of its assumptions or its conclusions?

1.2. Explain the economic implications of the imposition of quality standards for drinking water.

1.3. Can economics contribute to an understanding of how government decisions are made?

1.4. What should guide the choice of economic policy?

1.5. Are bureaucrats motivated by different factors than entrepreneurs?

1.6. What restricts the policies that a government can choose? Are there any arguments for imposing additional restrictions?

1.7. ''Physics is a simpler discipline than economics. This is because the objects of its study are bound by physical laws.'' Do you agree?

1.8. If individual welfare levels cannot be compared, how can it be possible to make social judgments?

1.9. "Poverty should be reduced to lessen the extent of malnutrition and raise economic growth." Distinguish the positive and normative components of this statement.

1.10. "It is economically efficient to maintain a pool of unemployed labor." Is this claim based on positive or normative reasoning?

1.11. "High income earners should pay a high rate of tax because their labor supply is inelastic and the revenue raised can be used to assist those on low incomes." Distinguish between the positive and normative components of this statement.

1.12. Consider two methods of dividing a cake between two people. Method 1 is to throw some of the cake away, and share what is left equally. Method 2 is to give one person 75 percent of the cake and the other 25 percent. Which method do you prefer, and why?

1.13. A cake has to be apportioned between two people. One is well-nourished, and the other is not. If the well-nourished person receives a share x, $0 \leq x \leq 1$, a share $y = [1 - x]^2$ is left for the other person (some is lost when the cake is divided). Plot the possible shares that the two people can have. What allocation of shares would you choose? How would your answer change if $y = [1 - x]^4$?

1.14. Can an economic model be acceptable if it assumes that consumers solve computationally complex maximization problems? Does your answer imply that Tiger Woods can derive the law of motion for a golf ball?

1.15. To analyze the effect of a subsidy to rice production, would you employ a partial equilibrium or a general equilibrium model?

1.16. If the European Union considered replacing the income tax with an increase in VAT, would you model this using partial equilibrium?

1.17. What proportion of the world's economies (by number, population, and wealth) can be described as "mixed"?

1.18. What problems may arise in setting economic policy if consumers know the economic model?

1.19. Should firms maximize profit?

1.20. To what extent is it possible to view the government as having a single objective?

1.21. Are you happier than your neighbor? How many times happier or less happy?

1.22. Assume that consumers are randomly allocated to either earn income M_ℓ or income M_h, where $M_h > M_\ell$. The probability of being allocated to M_ℓ is π. Prior to being allocated to an income level, consumers wish to maximize their expected income level. If it is possible to redistribute income costlessly, show that prior to allocation to income levels, no consumer would object to a transfer scheme. Now assume there is a cost Δ for each consumer of income M_h from whom income is taken. Find the maximum value of π for which there is still unanimous agreement that transfers should take place.

2 Equilibrium and Efficiency

2.1 Introduction

The link between competition and efficiency can be traced back, at least, to Adam Smith's eighteenth-century description of the working of the invisible hand. Smith's description of individually motivated decisions being coordinated to produce a socially efficient outcome is a powerful one that has found resonance in policy circles ever since. The expression of the efficiency argument in the language of formal economics, and the deeper understanding that comes with it, is a more recent innovation.

The focus of this chapter is to review what is meant by competition and to describe equilibrium in a competitive economy. The model of competition combines independent decision-making of consumers and firms into a complete model of the economy. Equilibrium is shown to be achieved in the economy by prices adjusting to equate demand and supply. Most important, the chapter employs the competitive model to demonstrate the efficiency theorems.

Surprisingly, equilibrium prices can always be found that simultaneously equate demand and supply for all goods. What is even more remarkable is that the equilibrium so obtained also has properties of efficiency. Why this is remarkable is that individual households and firms pursue their independent objectives with no concern other than their own welfare. Even so, the final state that emerges achieves efficiency solely through the coordinating role played by prices.

2.2 Economic Models

Prior to starting the analysis it is worth reflecting on why economists employ models to make predictions about the effects of economic policies. Models are used essentially because of problems of conducting experiments on economic systems and because the system is too large and complex to analyze in its entirety. Moreover formal modeling ensures that arguments are logically consistent with all the underlying assumptions exposed.

The models used, while inevitably being simplifications of the real economy, are designed to capture the essential aspects of the problem under study. Although many different models will be studied in this book, there are important common features that apply to all. Most models in public economics specify the objectives

of the individual agents in the economy (e.g., firms and consumers), and the constraints they face, and then aggregate individual decisions to arrive at market demand and supply. The equilibrium of the economy is then determined, and in a policy analysis the effects of government choice variables on this are calculated. This is done with various degrees of detail. Sometimes only a single market is studied—this is the case of *partial equilibrium* analysis. At other times *general equilibrium* analysis is used with many markets analyzed simultaneously. Similarly the number of firms and consumers varies from one or two to very many.

An essential consideration in the choice of the level of detail for a model is that its equilibrium must demonstrate a dependence on policy that gives insight into the functioning of the actual economy. If the model is too highly specified, it may not be capable of capturing important forms of response. On the other hand, if it is too general, it may not be able to provide any clear prediction. The theory described in this book will show how this trade-off can be successfully resolved. Achieving a successful compromise between these competing objectives is the "art" of economic modeling.

2.3 Competitive Economies

The essential feature of competition is that the consumers and firms in the economy do not consider their actions to have any effect on prices. Consequently, in making decisions, they treat the prices they observe in the market place as fixed (or *parametric*). This assumption can be justified when all consumers and firms are truly negligible in size relative to the market. In such a case the quantity traded by an individual consumer or firm is not sufficient to change the market price. But the assumption that the agents view prices as parametric can also be imposed as a modeling tool even in an economy with a single consumer and a single firm.

This defining characteristic of competition places a focus on the role of prices, as is maintained throughout the chapter. Prices measure values and are the signals that guide the decisions of firms and consumers. It was the exploration of what determined the relative values of different goods and services that led to the formulation of the competitive model. The adjustment of prices equates supply and demand to ensure that equilibrium is achieved. The role of prices in coordinating the decisions of independent economic agents is also crucial for the attainment of economic efficiency.

The secondary feature of the economies in this chapter is that all agents have access to the *same* information or—in formal terminology—that information is *symmetric*. This does not imply that there cannot be uncertainty but only that when there is uncertainty all agents are equally uninformed. Put differently, no agent is permitted to have an informational advantage. For example, by this assumption, the future profit levels of firms are allowed to be uncertain and shares in the firms to be traded on the basis of individual assessments of future profits. What the assumption does not allow is for the directors of the firms to be better informed than other shareholders about future prospects and to trade profitably on the basis of this information advantage.

Two forms of the competitive model are introduced in this chapter. The first form is an exchange economy in which there is no production. Initial stocks of goods are held by consumers and economic activity occurs through the trade of these stocks to mutual advantage. The second form of competitive economy introduces production. This is undertaken by firms with given production technologies who use inputs to produce outputs and distribute their profits as dividends to consumers.

2.3.1 The Exchange Economy

The exchange economy models the simplest form of economic activity: the trade of commodities between two parties in order to obtain mutual advantage. Despite the simplicity of this model it is a surprisingly instructive tool for obtaining fundamental insights about taxation and tax policy. This will become evident as we proceed. This section presents a description of a two-consumer, two-good exchange economy. The restriction on the number of goods and consumers does not alter any of the conclusions that will be derived—they will all extend to larger numbers. What restricting the numbers does is allow the economy to be displayed and analyzed in a simple diagram.

Each of the two consumers has an initial stock, or *endowment*, of the economy's two goods. The endowments can be interpreted literally as stocks of goods, or less literally as human capital, and are the quantities that are available for trade. Given the absence of production, these quantities remains constant. The consumers exchange quantities of the two commodities in order to achieve consumption plans that are preferred to their initial endowments. The rate at which one commodity can be exchanged for the other is given by the market prices. Both consumers believe that their behavior cannot affect these prices. This is the

fundamental assumption of competitive price-taking behavior. More will be said about the validity and interpretation of this in section 2.6.

A consumer is described by their endowments and their preferences. The endowment of consumer h is denoted by $\omega^h = (\omega_1^h, \omega_2^h)$, where $\omega_i^h \geq 0$ is h's initial stock of good i. When prices are p_1 and p_2, a consumption plan for consumer h, $x^h = (x_1^h, x_2^h)$, is affordable if it satisfies the budget constraint

$$p_1 x_1^h + p_2 x_2^h = p_1 \omega_1^h + p_2 \omega_2^h. \tag{2.1}$$

The preferences of each consumer are described by their utility function. This function should be seen as a representation of the consumer's indifference curves and does not imply any comparability of utility levels between consumers—the issue of comparability is taken up in chapter 12. The utility function for consumer h is denoted by

$$U^h = U^h(x_1^h, x_2^h). \tag{2.2}$$

It is assumed that the consumers enjoy the goods (so the marginal utility of consumption is positive for both goods) and that the indifference curves have the standard convex shape.

This economy can be pictured in a simple diagram that allows the role of prices in achieving equilibrium to be explored. The diagram is constructed by noting that the total consumption of the two consumers must equal the available stock of the goods, where the stock is determined by the endowments. Any pair of consumption plans that satisfies this requirement is called a *feasible plan* for the economy. A plan for the economy is feasible if the consumption levels can be met from the endowments, so

$$x_i^1 + x_i^2 = \omega_i^1 + \omega_i^2, \qquad i = 1, 2. \tag{2.3}$$

The consumption plans satisfying (2.3) can be represented as points in a rectangle with sides of length $\omega_1^1 + \omega_1^2$ and $\omega_2^1 + \omega_2^2$. In this rectangle the southwest corner can be treated as the zero consumption point for consumer 1 and the northeast corner as the zero consumption point for consumer 2. The consumption of good 1 for consumer 1 is then measured horizontally from the southwest corner and for consumer 2 horizontally from the northeast corner. Measurements for good 2 are made vertically.

The diagram constructed in this way is called an *Edgeworth box* and a typical box is shown in figure 2.1. It should be noted that the method of construction results in the endowment point, marked ω, being the initial endowment point for both consumers.

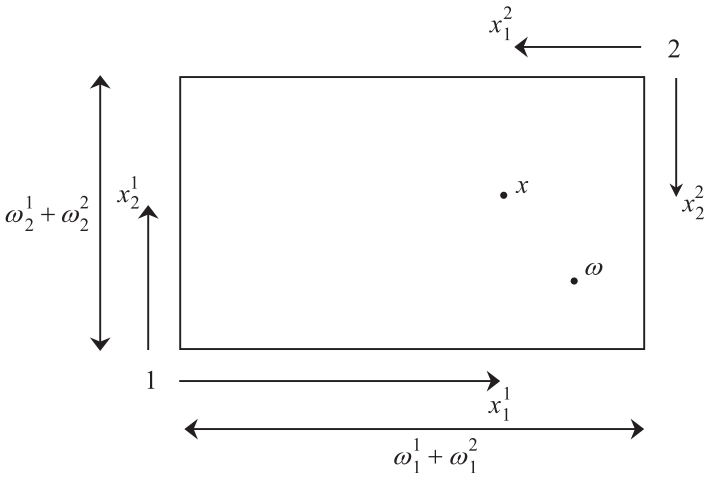


Figure 2.1
Edgeworth Box

The Edgeworth box is completed by adding the preferences and budget constraints of the consumers. The indifference curves of consumer 1 are drawn relative to the southwest corner and those of consumer 2 relative to the northeast corner. From (2.1) it can be seen that the budget constraint for both consumers must pass through the endowment point, since consumers can always afford their endowment. The endowment point is common to both consumers, so a single budget line through the endowment point with gradient $-\frac{p_1}{p_2}$ captures the market opportunities of the two consumers. Thus, viewed from the southwest, it is the budget line of consumer 1, and viewed from the northeast, the budget line of consumer 2. Given the budget line determined by the prices p_1 and p_2, the utility-maximizing choices for the two consumers are characterized by the standard tangency condition between the highest attainable indifference curve and the budget line. This is illustrated in figure 2.2, where x^1 denotes the choice of consumer 1 and x^2 that of 2.

In an *equilibrium* of the economy, supply is equal to demand. This is assumed to be achieved via the adjustment of prices. The prices at which supply is equal to demand are called *equilibrium prices*. How such prices are arrived at will be discussed later. For the present the focus will be placed on the nature of equilibrium and its properties. The consumer choices shown in figure 2.2 do not constitute an equilibrium for the economy. This can be seen by summing the demands and comparing these to the level of the endowments. Doing this shows that the demand for

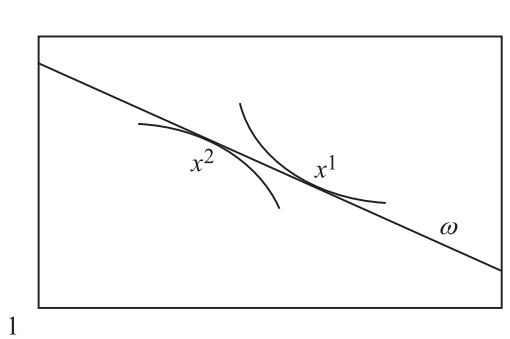


Figure 2.2
Preferences and demand

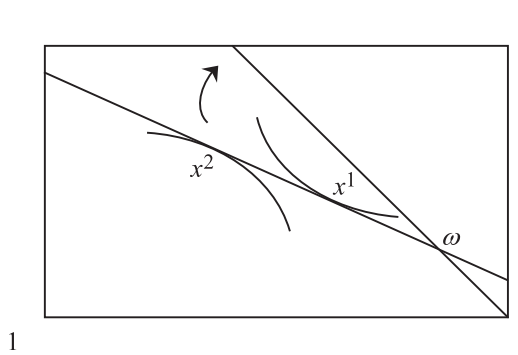


Figure 2.3
Relative price change

good 1 exceeds the endowment but the demand for good 2 falls short. To achieve an equilibrium position, the relative prices of the goods must change. An increase in the relative price of good 1 raises the absolute value of the gradient $-\frac{p_1}{p_2}$ of the budget line, making the budget line steeper. It becomes flatter if the relative price of good 1 falls. At all prices it continues to pass through the endowment point so a change in relative prices sees the budget line pivot about the endowment point.

The effect of a relative price change on the budget constraint is shown in figure 2.3. In the figure the price of good 1 has increased relative to the price of good 2. This causes the budget constraint to pivot upward around the endowment point. As a consequence of this change the consumers will now select consumption plans on this new budget constraint.

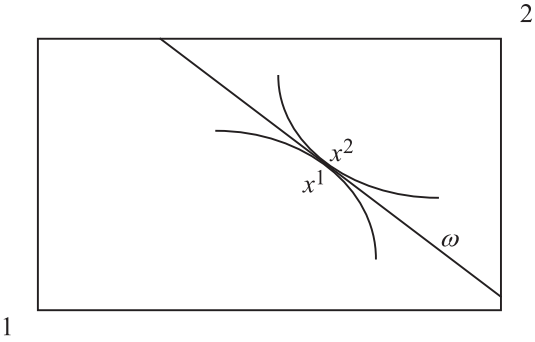


Figure 2.4
Equilibrium

The dependence of the consumption levels on prices is summarized in the consumers' demand functions. Taking the prices as given, the consumers choose their consumption plans to reach the highest attainable utility level subject to their budget constraints. The level of demand for good i from consumer h is $x_i^h = x_i^h(p_1, p_2)$. Using the demand functions, we see that demand is equal to supply for the economy when the prices are such that

$$x_i^1(p_1, p_2) + x_i^2(p_1, p_2) = \omega_i^1 + \omega_i^2, \qquad i = 1, 2. \tag{2.4}$$

Study of the Edgeworth box shows that such an equilibrium is achieved when the prices lead to a budget line on which the indifference curves of the consumers have a point of common tangency. Such an equilibrium is shown in figure 2.4. Having illustrated an equilibrium, we raise the question of whether an equilibrium is guaranteed to exist. As it happens, under reasonable assumptions, it will always do so. More important for public economics is the issue of whether the equilibrium has any desirable features from a welfare perspective. This is discussed in depth in section 2.4 where the Edgeworth box is put to substantial use.

Two further points now need to be made that are important for understanding the functioning of the model. These concern the number of prices that can be determined and the number of independent equilibrium equations. In the equilibrium conditions (2.4) there are two equations to be satisfied by the two equilibrium prices. It is now argued that the model can determine only the ratio of prices and not the actual prices. Accepting this, it would seem that there is one price ratio attempting to solve two equations. If this were the case, a solution would be unlikely, and we would be in the position of having a model that

generally did not have an equilibrium. This situation is resolved by noting that there is a relationship between the two equilibrium conditions that ensures that there is only one independent equation. The single price ratio then has to solve a single equation, making it possible for there to be always a solution.

The first point is developed by observing that the budget constraint always passes through the endowment point and its gradient is determined by the price ratio. The consequence of this is that only the value of p_1 relative to p_2 matters in determining demands and supplies rather than the absolute values. The economic explanation for this fact is that consumers are only concerned with the real purchasing power embodied in their endowment, and not with the level of prices. Since their nominal income is equal to the value of the endowment, any change in the level of prices raises nominal income just as much as it raises the cost of purchases. This leaves real incomes unchanged.

The fact that only relative prices matter is also reflected in the demand functions. If $x_i^h(p_1, p_2)$ is the level of demand at prices p_1 and p_2, then it must be the case that $x_i^h(p_1, p_2) = x_i^h(\lambda p_1, \lambda p_2)$ for $\lambda > 0$. A demand function having this property is said to be *homogeneous of degree* 0. In terms of what can be learned from the model, the homogeneity shows that only relative prices can be determined at equilibrium and not the level of prices. So, given a set of equilibrium prices, any scaling up or down of these will also be equilibrium prices because the change will not alter the level of demand. This is as it should be, since all that matters for the consumers is the rate at which they can exchange one commodity for another, and this is measured by the relative prices. This can be seen in the Edgeworth box. The budget constraint always goes through the endowment point so only its gradient can change, and this is determined by the relative prices.

In order to analyze the model, the indeterminacy of the level of prices needs to be removed. This is achieved by adopting a *price normalization*, which is simply a method of fixing a scale for prices. There are numerous ways to do this. The simplest way is to select a commodity as *numéraire*, which means that its price is fixed at one, and measure all other prices relative to this. The numéraire chosen in this way can be thought of as the *unit of account* for the economy. This is the role usually played by money, but formally, there is no money in this economy.

The second point is to demonstrate the dependence between the two equilibrium equations. It can be seen that at the disequilibrium position shown in figure 2.2 the demand for good 1 exceeds its supply, whereas the supply of good 2 exceeds demand. Considering other budget lines and indifference curves in the Edgeworth box will show that whenever there is an excess of demand for one

good there is a corresponding deficit of demand for the other. There is actually a very precise relationship between the excess and the deficit that can be captured in the following way: The level of *excess demand* for good i is the difference between demand and supply and is defined by $Z_i = x_i^1 + x_i^2 - \omega_i^1 - \omega_i^2$. Using this definition the value of excess demand can be calculated as

$$\begin{aligned} p_1 Z_1 + p_2 Z_2 &= \sum_{i=1}^{2} p_i [x_i^1 + x_i^2 - \omega_i^1 - \omega_i^2] \\ &= \sum_{h=1}^{2} [p_1 x_1^h + p_2 x_2^h - p_1 \omega_1^h - p_2 \omega_2^h] \\ &= 0, \end{aligned} \tag{2.5}$$

where the second equality is a consequence of the budget constraints in (2.1). The relationship in (2.5) is known as *Walras's law* and states that the value of excess demand is zero. This must hold for any set of prices, so it provides a connection between the extent of disequilibrium and prices. In essence, Walras's law is simply an aggregate budget constraint for the economy. Since all consumers are equating their expenditure to their income, so must the economy as a whole.

Walras's law implies that the equilibrium equations are interdependent. Since $p_1 Z_1 + p_2 Z_2 = 0$, if $Z_1 = 0$ then $Z_2 = 0$ (and vice versa). That is, if demand is equal to supply for good 1, then demand must also equal supply for good 2. Equilibrium in one market necessarily implies equilibrium in the other. This observation allows the construction of a simple diagram to illustrate equilibrium. Choose good 1 as the numéraire (so $p_1 = 1$) and plot the excess demand for good 2 as a function of p_2. The equilibrium for the economy is then found where the graph of excess demand crosses the horizontal axis. At this point excess demand for good 2 is zero, so by Walras's law, it must also be zero for good 1. An excess demand function is illustrated in figure 2.5 for an economy that has three equilibria. This excess demand function demonstrates why at least one equilibrium will exist. As p_2 falls toward zero then demand will exceed supply (good 2 becomes increasingly attractive to purchase), making excess demand positive. Conversely, as the price of good 2 rises, it will become increasingly attractive to sell, resulting in a negative value of excess demand for high values of p_2. Since excess demand is positive for small values of p_2 and negative for high values, there must be at least one point in between where it is zero.

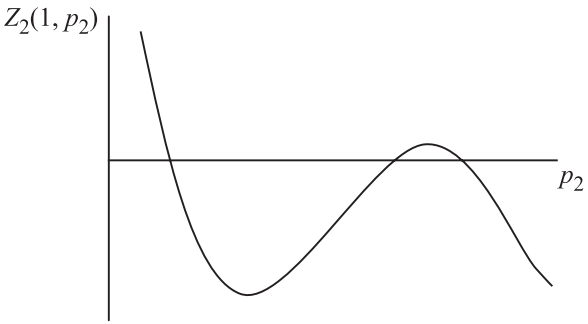


Figure 2.5
Equilibrium and excess demand

Finally, it should be noted that the arguments made above can be extended to include additional consumers and additional goods. Income, in terms of an endowment of many goods, and expenditure, defined in the same way, must remain equal for each consumer. The demand functions that result from the maximization of utility are homogeneous of degree zero in prices. Walras's law continues to hold so the value of excess demand remains zero. The number of price ratios and the number of independent equilibrium conditions are always one less than the number of goods.

2.3.2 Production and Exchange

The addition of production to the exchange economy provides a complete model of economic activity. Such an economy allows a wealth of detail to be included. Some goods can be present as initial endowments (e.g., labor), others can be consumption goods produced from the initial endowments, while some goods, intermediates, can be produced by one productive process and used as inputs into another. The fully developed model of competition is called the *Arrow-Debreu economy* in honor of its original constructors.

An economy with production consists of consumers (or households) and producers (or firms). The firms use inputs to produce outputs with the intention of maximizing their profits. Each firm has available a production technology that describes the ways in which it can use inputs to produce outputs. The consumers have preferences and initial endowments as they did in the exchange economy, but they now also hold shares in the firms. The firms' profits are distributed as dividends in proportion to the shareholdings. The consumers receive income from

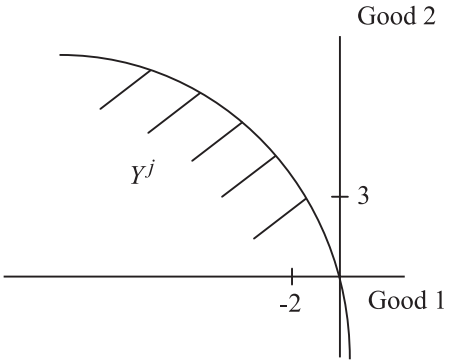


Figure 2.6
Typical production set

the sale of their initial endowments (e.g., their labor time) and from the dividend payments.

Each firm is characterized by its production set, which summarizes the production technology it has available. A production technology can be thought of as a complete list of ways that the firm can turn inputs into outputs. In other words, it catalogs all the production methods of which the firm has knowledge. For firm j operating in an economy with two goods a typical production set, denoted Y^j, is illustrated in figure 2.6. This figure employs the standard convention of measuring inputs as negative numbers and outputs as positive. The reason for adopting this convention is that the use of a unit of a good as an input represents a subtraction from the stock of that good available for consumption

Consider the firm shown in figure 2.6 choosing the production plan $y_1^j = -2$, $y_2^j = 3$. When faced with prices $p_1 = 2$, $p_2 = 2$, the firm's profit is

$$\pi^j = p_1 y_1^j + p_2 y_2^j = 2 \times [-2] + 2 \times 3 = 2. \tag{2.6}$$

The positive part of this sum can be given the interpretation of sales revenue, and the negative part that of production costs. This is equivalent to writing profit as the difference between revenue and cost. Written in this way, (2.6) gives a simple expression of the relation between prices and production choices.

The process of profit maximization is illustrated in figure 2.7. Under the competitive assumption the firm takes the prices p_1 and p_2 as given. These prices are used to construct *isoprofit curves*, which show all production plans that give a specific level of profit. For example, all the production plans on the isoprofit curve labeled $\pi = 0$ will lead to a profit level of 0. Production plans on higher isoprofit

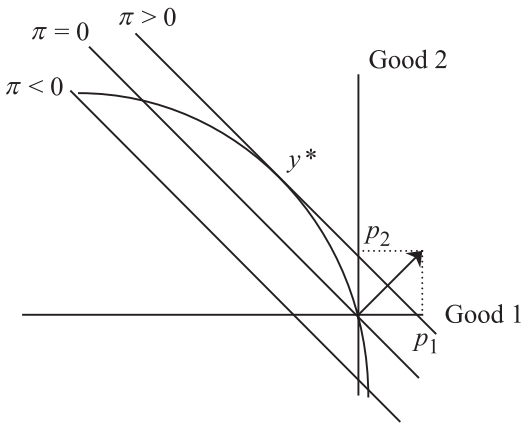


Figure 2.7
Profit maximization

curves lead to progressively larger profits, and those on lower curves to negative profits. Since doing nothing (which means choosing $y_1^j = y_2^j = 0$) earns zero profit, the $\pi = 0$ isoprofit curve always passes through the origin.

The profit-maximizing firm will choose a production plan that places it upon the highest attainable isoprofit curve. What restricts the choice is the technology that is available as described by the production set. In figure 2.7 the production plan that maximizes profit is shown by y^*, which is located at a point of tangency between the highest attainable isoprofit curve and the production set. There is no other technologically feasible plan that can attain higher profit.

It should be noted how the isoprofit curves are determined by the prices. The geometry in fact is that the isoprofit curves are at right angles to the price vector. The angle of the price vector is determined by the price ratio, $\frac{p_2}{p_1}$, so that a change in relative prices will alter the gradient of the isoprofit curves. The figure can be used to predict the effect of relative price changes. For instance, if p_1 increases relative to p_2, which can be interpreted as the price of the input (good 1) rising in comparison to the price of the output (good 2), the price vector become flatter. This makes the isoprofit curves steeper, so the optimal choice must move round the boundary of the production set toward the origin. The use of the input and the production of the output both fall.

Now consider an economy with n goods. The price of good i is denoted p_i. Production is carried out by m firms. Each firm uses inputs to produce outputs and maximizes profits given the market prices. Demand comes from the H consumers

in the economy. They aim to maximize their utility. The total supply of each good is the sum of the production of it by firms and the initial endowment of it held by the consumers.

Each firm chooses a production plan $y^j = (y_1^j, \ldots, y_n^j)$. This production plan is chosen to maximize profits subject to the constraint that the chosen plan must be in the production set. From this maximization can be determined firm j's supply function for good i as $y_i^j = y_i^j(p)$, where $p = (p_1, \ldots, p_n)$. The level of profit is $\pi^j = \sum_{i=1}^n p_i y_i^j(p) = \pi^j(p)$, which also depends on prices.

Aggregate supply from the production sector of the economy is obtained from the supply decisions of the individual firms by summing across the firms. This gives the aggregate supply of good i as

$$Y_i(p) = \sum_{j=1}^{m} y_i^j(p). \tag{2.7}$$

Since some goods must be inputs, and others outputs, aggregate supply can be positive (the total activity of the firms adds to the stock of the good) or negative (the total activity of the firms subtracts from the stock).

Each consumer has an initial endowment of commodities and also a set of shareholdings in firms. The latter assumption makes this a *private ownership economy* in which the means of production are ultimately owned by individuals. In the present version of the model, these shareholdings are exogenously given and remain fixed. A more developed version would introduce a stock market and allow them to be traded. For consumer h the initial endowment is denoted ω^h and the shareholding in firm j is θ_j^h. The firms must be fully owned by the consumers, so $\sum_{h=1}^H \theta_j^h = 1$. That is, the shares in the firms must sum to one. Consumer h chooses a consumption plan x^h to maximize utility subject to the budget constraint

$$\sum_{i=1}^{n} p_i x_i^h = \sum_{i=1}^{n} p_i \omega_i^h + \sum_{j=1}^{m} \theta_j^h \pi^j. \tag{2.8}$$

This budget constraint requires that the value of expenditure be not more than the value of the endowment plus income received as dividends from firms. Since firms always have the option of going out of business (and hence earning zero profit), dividend income must be nonnegative. The profit level of each firm is dependent on prices. A change in prices therefore affects a consumer's budget constraint through a change in the value of their endowment and through a change in

dividend income. The maximization of utility by the consumer results in demand for good i from consumer h of the form $x_i^h = x_i^h(p)$. The level of aggregate demand is found by summing the individual demands of the consumers to give

$$X_i(p) = \sum_{h=1}^{H} x_i^h(p). \tag{2.9}$$

The same notion of equilibrium that was used for the exchange economy can be applied in this economy with production. That is, equilibrium occurs when supply is equal to demand. The distinction between the two is that supply, which was fixed in the exchange economy, is now variable and dependent on the production decisions of firms. Although this adds a further dimension to the question of the existence of equilibrium, the basic argument why such an equilibrium always exists is essentially the same as that for the exchange economy.

As already noted, the equilibrium of the economy occurs when demand is equal to supply or, equivalently, when excess demand is zero. Excess demand for good i, $Z_i(p)$, can be defined by

$$Z_i(p) = X_i(p) - Y_i(p) - \sum_{h=1}^{H} \omega_i^h. \tag{2.10}$$

Here excess demand is the difference between demand and the sum of initial endowment and firms' supply. The equilibrium occurs when $Z_i(p) = 0$ for all of the goods $i = 1, \ldots, n$. There are standard theorems that prove such an equilibrium must exist under fairly weak conditions.

The properties established for the exchange economy also apply to this economy with production. Demand is determined only by relative prices (so it is homogeneous of degree zero). Supply is also determined by relative prices. Together, these imply that excess demand is homogeneous of degree zero. To determine the equilibrium prices that equate supply to demand, a normalization must again be used. Typically one of the goods will be chosen as numéraire, and its price set to one. Equilibrium prices are then those that equate excess demand to zero.

2.4 Efficiency of Competition

Economics is often defined as the study of scarcity. This viewpoint is reflected in the concern with the efficient use of resources that runs throughout the core of

the subject. Efficiency would seem to be a simple concept to characterize: if more cannot be achieved, then the outcome is efficient. This is certainly the case when an individual decision-maker is considered. The individual will employ their resources to maximize utility subject to the constraints they face. When utility is maximized, the efficient outcome has been achieved.

Problems arise when there is more than one decision-maker. To be unambiguous about efficiency, it is necessary to resolve the potentially competing needs of different decision-makers. This requires efficiency to be defined with respect to a set of aggregate preferences. Methods of progressing from individual to aggregate preferences will be discussed in chapters 10 and 12. The conclusions obtained there are that the determination of aggregate preferences is not a simple task. There are two routes we can use to navigate around this difficulty. The first is to look at a single-consumer economy so that there is no conflict between competing preferences. But with more than one consumer some creativity has to be used to describe efficiency. The second route is met in section 2.4.2 where the concept of Pareto-efficiency is introduced. The trouble with such creativity is that it leaves the definition of efficiency open to debate. We will postpone further discussion of this until chapter 12.

Before proceeding some definitions are needed. A *first-best* outcome is achieved when only the production technology and the limited endowments restrict the choice of the decision-maker. The first-best is essentially what would be chosen by an omniscient planner with complete command over resources. A *second-best* outcome arises whenever constraints other than technology and resources are placed on what the planner can do. Such constraints could be limits on income redistribution, an inability to remove monopoly power, or a lack of information.

2.4.1 Single Consumer

With a single consumer there is no doubt as to what is good and bad from a social perspective: the single individual's preferences can be taken as the social preferences. To do otherwise would be to deny the validity of the consumer's judgments. Hence, if the individual prefers one outcome to another, then so must society. The unambiguous nature of preferences provides significant simplification of the discussion of efficiency in the single-consumer economy. In this case the "best" outcome must be first-best because no constraints on policy choices have been invoked nor is there an issue of income distribution to consider.

If there is a single firm and a single consumer, the economy with production can be illustrated in a helpful diagram. This is constructed by superimposing the

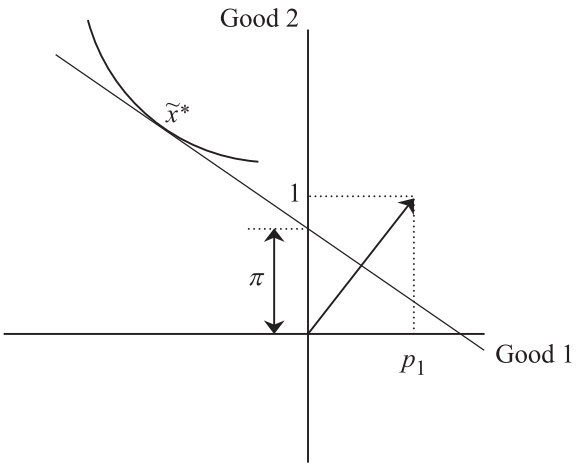


Figure 2.8
Utility maximization

profit-maximization diagram for the firm over the choice diagram for the consumer. Such a model is often called the *Robinson Crusoe economy*. The interpretation is that Robinson acts as a firm carrying out production and as a consumer of the product of the firm. It is then possible to think of Robinson as a social planner who can coordinate the activities of the firm and producer. It is also possible (though in this case less compelling!) to think of Robinson as having a split personality and acting as a profit-maximizing firm on one side of the market and as a utility-maximizing consumer on the other. In the latter interpretation the two sides of Robinson's personality are reconciled through the prices on the competitive markets. The important fact is that these two interpretations lead to exactly the same levels of production and consumption.

The budget constraint of the consumer needs to include the dividend received from the firm. With two goods, the budget constraint is

$$p_1[x_1 - \omega_1] + p_2[x_2 - \omega_2] = \pi, \tag{2.11}$$

or

$$p_1\tilde{x}_1 + p_2\tilde{x}_2 = \pi, \tag{2.12}$$

where $\tilde{x}_i$, the change from the endowment point, is the *net consumption* of good i. This is illustrated in figure 2.8 with good 2 chosen as numéraire. The budget constraint (2.12) is always at a right angle to the price vector and is displaced

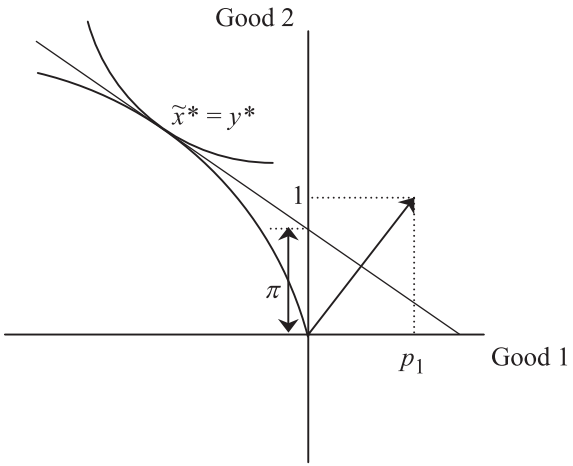


Figure 2.9
Efficient equilibrium

above the origin by the value of profit. Utility maximization occurs where the highest indifference curve is reached given the budget constraint. This results in net consumption plan $\tilde{x}^*$.

The equilibrium for the economy is shown in figure 2.9, which superimposes figure 2.7 onto 2.8. At the equilibrium the net consumption plan from the consumer must match the supply from the firm. The feature that makes this diagram work is the fact that the consumer receives the entire profit of the firm so the budget constraint and the isoprofit curve are one and the same. The height above the origin of both is the level of profit earned by the firm and received by the consumer. Equilibrium can only arise when the point on the economy's production set that equates to profit maximization is the same as that of utility maximization. This is point $\tilde{x}^* = y^*$ in figure 2.9.

It should be noted that the equilibrium is on the boundary of the production set so that it is efficient: it is not possible for a better outcome to be found in which more is produced with the same level of input. This captures the efficiency of production at the competitive equilibrium, about which much more is said soon. The equilibrium is also the first-best outcome for the single-consumer economy, since it achieves the highest indifference curve possible subject to the restriction that it is feasible under the technology. This is illustrated in figure 2.9 where $\tilde{x}^*$ is the net level of consumption relative to the endowment point in the first-best and at the competitive equilibrium.

A simple characterization of this first-best allocation can be given by using the fact that it is at a tangency point between two curves. The gradient of the indifference curve is equal to the ratio of the marginal utilities of the two goods and is called the *marginal rate of substitution*. This measures the rate at which good 1 can be traded for good 2 while maintaining constant utility. The marginal rate of substitution is given by $MRS_{1,2} = \frac{U_1}{U_2}$, with subscripts used to denote the marginal utilities of the two goods. Similarly the gradient of the production possibility set is termed the marginal rate of transformation and denoted $MRT_{1,2}$. The $MRT_{1,2}$ measures the rate at which good 1 has to be given up to allow an increase in production of good 2. At the tangency point the two gradients are equal, so

$$MRS_{1,2} = MRT_{1,2}. \tag{2.13}$$

The reason why this equality characterizes the first-best equilibrium can be explained as follows: The MRS captures the marginal value of good 1 to the consumer relative to the marginal value of good 2, while the MRT measures the marginal cost of good 1 relative to the marginal cost of good 2. The first-best is achieved when the marginal value is equal to the marginal cost.

The market achieves efficiency through the coordinating role of prices. The consumer maximizes utility subject to their budget constraint. The optimal choice occurs when the budget constraint is tangential to highest attainable indifference curve. The condition describing this is that ratio of marginal utilities is equal to the ratio of prices. Expressed in terms of the MRS, this is

$$MRS_{1,2} = \frac{p_1}{p_2}. \tag{2.14}$$

Similarly profit maximization by the firm occurs when the production possibility set is tangential to the highest isoprofit curve. Using the MRT, we write the profit-maximization condition as

$$MRT_{1,2} = \frac{p_1}{p_2}. \tag{2.15}$$

Combining these conditions, we find that the competitive equilibrium satisfies

$$MRS_{1,2} = \frac{p_1}{p_2} = MRT_{1,2}. \tag{2.16}$$

The condition in (2.16) demonstrates that the competitive equilibrium satisfies the same condition as the first-best and reveals the essential role of prices. Under the competitive assumption, both the consumer and the producer are guided in their

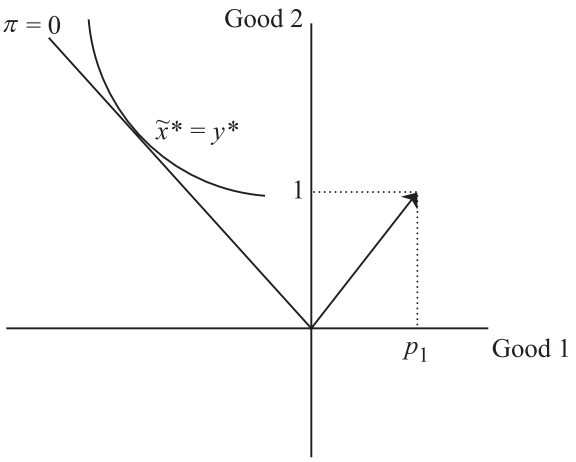


Figure 2.10
Constant returns to scale

decisions by the same price ratio. Each optimizes relative to the price ratio; hence their decisions are mutually efficient.

There is one special case that is worth noting before moving on. When the firm has constant returns to scale the efficient production frontier is a straight line through the origin. The only equilibrium can be when the firm makes zero profits. If profit was positive at some output level, then the constant returns to scale allows the firm to double profit by doubling output. Since this argument can be repeated, there is no limit to the profit that the firm can make. Hence we have the claim that equilibrium profit must be zero. Now the isoprofit curve at zero profit is also a straight line through the origin. The zero-profit equilibrium can only arise when this is coincident with the efficient production frontier. At this equilibrium the price vector is at right angles to both the isoprofit curve and the production frontier. This is illustrated in figure 2.10.

There are two further implications of constant returns. First, the equilibrium price ratio is determined by the zero-profit condition alone and is independent of demand. Second, the profit income of the consumer is zero, so the consumer's budget constraint also passes through the origin. As this is determined by the same prices as the isoprofit curve, the budget constraint must be coincident with the production frontier.

In this single-consumer context the equilibrium reached by the market simply cannot be bettered. Such a strong statement cannot be made when further

consumers are introduced because issues of distribution between consumers then arise. However, what will remain is the finding that the competitive market ensures that firms produce at an efficient point on the frontier of the production set and that the chosen production plan is what is demanded at the equilibrium prices by the consumer. The key to this coordination are the prices that provide the signals guiding choices.

2.4.2 Pareto-Efficiency

When there is more than one consumer, the simple analysis of the Robinson Crusoe economy does not apply. Since consumers can have differing views about the success of an allocation, there is no single, simple measure of efficiency. The essence of the problem is that of judging among allocations with different distributional properties. What is needed is some process that can take account of the potentially diverse views of the consumers and separate efficiency from distribution.

To achieve this, economists employ the concept of *Pareto-efficiency*. The philosophy behind this concept is to interpret efficiency as meaning that there must be no unexploited economic gains. Testing the efficiency of an allocation then involves checking whether there are any such gains available. More specifically, Pareto-efficiency judges an allocation by considering whether it is possible to undertake a reallocation of resources that can benefit at least one consumer without harming any other. If it were possible to do so, then there would exist unexploited gains. When no improving reallocation can be found, then the initial position is deemed to be Pareto-efficient. An allocation that satisfies this test can be viewed as having achieved an efficient distribution of resources. For the present chapter this concept will be used uncritically. The interpretations and limitations of this form of efficiency will be discussed in chapter 12.

To provide a precise statement of Pareto-efficiency that applies in a competitive economy, it is first necessary to extend the idea of feasible allocations of resources that was used in (2.3) to define the Edgeworth box. When production is included, an allocation of consumption is feasible if it can be produced given the economy's initial endowments and production technology. Given the initial endowment, ω, the consumption allocation x is feasible if there is production plan y such that

$$x = y + \omega. \tag{2.17}$$

Pareto-efficiency is then tested using the feasible allocations. A precise definition follows.

Definition 1 A feasible consumption allocation $\hat{x}$ is Pareto-efficient if there does not exist an alternative feasible allocation $\bar{x}$ such that:

i. Allocation $\bar{x}$ gives all consumers at least as much utility as $\hat{x}$;
ii. Allocation $\bar{x}$ gives at least one consumer more utility than $\hat{x}$.

These two conditions can be summarized as saying that allocation $\hat{x}$ is Pareto-efficient if there is no alternative allocation (a move from $\hat{x}$ to $\bar{x}$) that can make someone better off without making anyone worse off. It is this idea of being able to make someone better off without making someone else worse off that represents the unexploited economic gains in an inefficient position.

It should be noted even at this stage how Pareto-efficiency is defined by the negative property of being unable to find anything better than the allocation. This is somewhat different from a definition of efficiency that looks for some positive property of the allocation. Pareto-efficiency also sidesteps issues of distribution rather than confronting them. This is why it works with many consumers. More will be said about this in chapter 12 when the construction of social welfare indicators is discussed.

2.4.3 Efficiency in an Exchange Economy

The welfare properties of the economy, which are commonly known as the *Two Theorems of Welfare Economics*, are the basis for claims concerning the desirability of the competitive outcome. In brief, the First Theorem states that a competitive equilibrium is Pareto-efficient and the Second Theorem that any Pareto-efficient allocation can be decentralized as a competitive equilibrium. Taken together, they have significant implications for policy and, at face value, seem to make a compelling case for the encouragement of competition.

The Two Theorems are easily demonstrated for a two-consumer exchange economy by using the Edgeworth box diagram. The first step is to isolate the Pareto-efficient allocations. Consider figure 2.11 and the allocation at point a. To show that a is not a Pareto-efficient allocation, it is necessary to find an alternative allocation that gives at least one of the consumers a higher utility level and neither consumer a lower level. In this case, moving to the allocation at point b raises the utility of both consumers when compared to a—we say in such a case that b is *Pareto-preferred* to a. This establishes that a is not Pareto-efficient. Although b improves on a, it is not Pareto-efficient either: the allocation at c provides higher utility for both consumers than b.

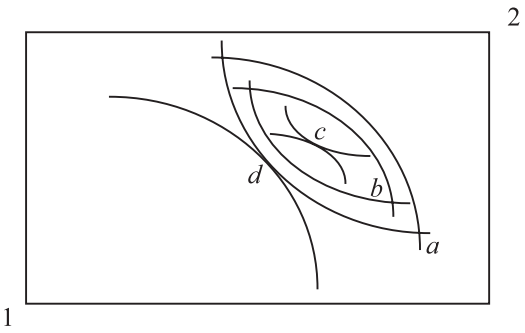


Figure 2.11
Pareto-efficiency

The allocation at c is Pareto-efficient. Beginning at c, any change in the allocation must lower the utility of at least one of the consumers. The special property of point c is that it lies at a point of tangency between the indifference curves of the two consumers. As it is a point of tangency, moving away from it must lead to a lower indifference curve for one of the consumers if not both. Since the indifference curves are tangential, their gradients are equal, so

$$MRS^1_{1,2} = MRS^2_{1,2}. \tag{2.18}$$

This equality ensures that the rate at which consumer 1 will want to exchange good 1 for good 2 is equal to the rate at which consumer 2 will want to exchange the two goods. It is this equality of the marginal valuations of the two consumers at the tangency point that results in there being no further unexploited gains and so makes c Pareto-efficient.

The Pareto-efficient allocation at c is not unique. There are in fact many points of tangency between the two consumers' indifference curves. A second Pareto-efficient allocation is at point d in figure 2.11. Taken together, all the Pareto-efficient allocations form a locus in the Edgeworth box that is called the *contract curve*. This is illustrated in figure 2.12. With this construction it is now possible to demonstrate the First Theorem.

A competitive equilibrium is given by a price line through the initial endowment point, ω, that is tangential to both indifference curves at the same point. The common point of tangency results in consumer choices that lead to the equilibrium levels of demand. Such an equilibrium is indicated by point e in figure 2.12. As the equilibrium is a point of tangency of indifference curves, it must also be Pareto-efficient. For the Edgeworth box, this completes the demonstration that a competitive equilibrium is Pareto-efficient.

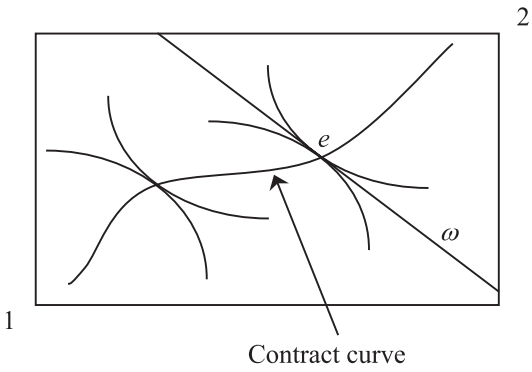


Figure 2.12
First Theorem

The alternative way of seeing this result is to recall that each consumer maximizes utility at the point where their budget constraint is tangential to the highest indifference curve. Using the *MRS*, we can write this condition for consumer h as $MRS^h_{1,2} = \frac{p_1}{p_2}$. The competitive assumption is that both consumers react to the same set of prices, so it follows that

$$MRS^1_{1,2} = \frac{p_1}{p_2} = MRS^2_{1,2}. \tag{2.19}$$

Comparing this condition with (2.18) provides an alternative demonstration that the competitive equilibrium is Pareto-efficient. It also shows again the role of prices in coordinating the independent decisions of different economic agents to ensure efficiency.

This discussion can be summarized in the precise statement of the theorem.

Theorem 1 (First Theorem of Welfare Economics) The allocation of commodities at a competitive equilibrium is Pareto-efficient.

This theorem can be formally proved by assuming that the competitive equilibrium is not Pareto-efficient and deriving a contradiction. Assuming the competitive equilibrium is not Pareto-efficient implies there is a feasible alternative that is at least as good for all consumers and strictly better for at least one. Now take the consumer who is made strictly better off. Why did they not choose the alternative consumption plan at the competitive equilibrium? The answer has to be because it was more expensive than their choice at the competitive equilibrium and not

affordable with their budget. Similarly for all other consumers the new allocation has to be at least as expensive as their choice at the competitive equilibrium. (If it were cheaper, they could afford an even better consumption plan that made them strictly better off than at the competitive equilibrium.) Summing across the consumers, the alternative allocation has to be strictly more expensive than the competitive allocation. But the value of consumption at the competitive equilibrium must equal the value of the endowment. Therefore the new allocation must have greater value than the endowment, which implies it cannot be feasible. This contradiction establishes that the competitive equilibrium must be Pareto-efficient.

The theorem demonstrates that the competitive equilibrium is Pareto-efficient, but it is not the only Pareto-efficient allocation. Referring back to figure 2.12, we have that any point on the contract curve is also Pareto-efficient because all are defined by a tangency between indifference curves. The only special feature of e is that it is the allocation reached through competitive trading from the initial endowment point ω. If ω were different, then another Pareto-efficient allocation would be achieved. There is in fact an infinity of Pareto-efficient allocations. Observing these points motivates the Second Theorem of Welfare Economics.

The Second Theorem is concerned with whether any chosen Pareto-efficient allocation can be made into a competitive equilibrium by choosing a suitable location for the initial endowment. Expressed differently, can a competitive economy be constructed that has a selected Pareto-efficient allocation as its competitive equilibrium? In the Edgeworth box this involves being able to choose any point on the contract curve and turning it into a competitive equilibrium.

From the Edgeworth box diagram it can be seen that this is possible in the exchange economy if the households' indifference curves are convex. The common tangent to the indifference curves at the Pareto-efficient allocation provides the budget constraint that each consumer must face if they are to afford the chosen point. The convexity ensures that given this budget line, the Pareto-efficient point will also be the optimal choice of the consumers. The construction is completed by choosing a point on this budget line as the initial endowment point. This process of constructing a competitive economy to obtain a selected Pareto-efficient allocation is termed *decentralization*.

This process is illustrated in figure 2.13 where the Pareto-efficient allocation e' is made a competitive equilibrium by selecting ω' as the endowment point. Starting from ω', trading by consumers will take the economy to its equilibrium allocation e'. This is the Pareto-efficient allocation that was intended to be reached. Note that if the endowments of the households are initially given by ω and the equilib-

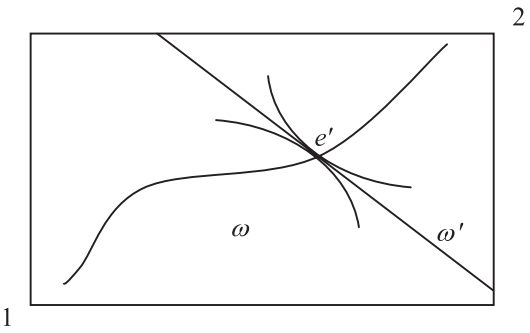


Figure 2.13
Second Theorem

rium at e' is to be decentralized, it is necessary to redistribute the initial endowments of the consumers in order to begin from ω'.

The construction described above can be given a formal statement as the Second Theorem of Welfare Economics.

Theorem 2 (Second Theorem of Welfare Economics) With convex preferences, any Pareto-efficient allocation can be made a competitive equilibrium.

The statement of the Second Theorem provides a conclusion but does not describe the mechanism involved in the decentralization. The important step in decentralizing a chosen Pareto-efficient allocation is placing the economy at the correct starting point. For now it is sufficient to observe that behind the Second Theorem lies a process of redistribution of initial wealth. How this can be achieved is discussed later. Furthermore the Second Theorem determines a set of prices that make the chosen allocation an equilibrium. These prices may well be very different from those that would have been obtained in the absence of the wealth redistribution.

2.4.4 Extension to Production

The extension of the Two Theorems to an economy with production is straightforward. The major effect of production is to make supply variable: it is now the sum of the initial endowment plus the net outputs of the firms. In addition a consumer's income includes the profit derived from their shareholdings in firms.

Section 2.4.1 has already demonstrated efficiency for the Robinson Crusoe economy that included production. It was shown that the competitive equilibrium achieved the highest attainable indifference curve given the production possibilities of the economy. Since the single consumer cannot be made better off by any change, the equilibrium is Pareto-efficient and the First Theorem applies. The Second Theorem is of limited interest with a single consumer because there is only one Pareto-efficient allocation, and this is attained by the competitive economy.

When there is more than one consumer, the proof of the First Theorem follows the same lines as for the exchange economy. Given the equilibrium prices, each consumer is maximizing utility, so their marginal rate of substitution is equated to the price ratio. This is true for all consumers and all goods, yielding

$$MRS_{i,j}^{h} = \frac{p_i}{p_j} = MRS_{i,j}^{h'} \tag{2.20}$$

for any pair of consumers h and h' and any pair of goods i and j. This is termed *efficiency in consumption*. In an economy with production this condition alone is not sufficient to guarantee efficiency; it is also necessary to consider production. The profit-maximization decision of each firm ensures that it equates its marginal rate of transformation between any two goods to the ratio of prices. For any two firms m and m' this gives

$$MRT_{i,j}^{m} = \frac{p_i}{p_j} = MRT_{i,j}^{m'} \tag{2.21}$$

a condition that characterizes *efficiency in production*. The price ratio also coordinates consumers and firms, giving

$$MRS_{i,j}^{h} = MRT_{i,j}^{m} \tag{2.22}$$

for any consumer and any firm for all pairs of goods. As for the Robinson Crusoe economy, the interpretation of this condition is that it equates the relative marginal values to the relative marginal costs. Since (2.20) through (2.22) are the conditions required for efficiency, this shows that the First Theorem extends to the economy with production.

The formal proof of this claim mirrors that for the exchange economy, except for the fact that the value of production must also be taken into account. Given this fact, the basis of the argument remains that since the consumers chose the competitive equilibrium quantities, anything that is preferred must be more expensive and hence can be shown not to be feasible.

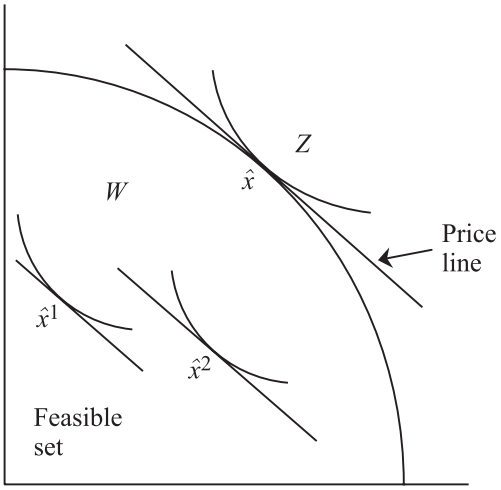


Figure 2.14
Proof of the Second Theorem

The extension of the Second Theorem to include production is illustrated in figure 2.14. The set W describes the feasible output plans for the economy, with each point in W being the sum of a production plan and the initial endowment; hence $w = y + \omega$. Set Z describes the quantities of the two goods that would allow a Pareto-improvement (a re-allocation that makes neither consumer worse off and makes one strictly better off) over the allocation $\hat{x}^1$ to consumer 1 and $\hat{x}^2$ to consumer 2. If W and Z are convex, which occurs when firms' production sets and preferences are convex, then a common tangent to W and Z can be found. This makes $\hat{x}$ an equilibrium. Individual income allocations, the sum of the value of endowment plus profit income, can be placed anywhere on the budget lines tangent to the indifference curves at the individual allocations $\hat{x}_1$ and $\hat{x}_2$ provided that they sum to the total income of the economy. This decentralizes the consumption allocation $\hat{x}_1, \hat{x}_2$.

Before proceeding further, it is worth emphasizing that the proof of the Second Theorem requires more assumptions than the proof of the First, so there may be situations in which the First Theorem is applicable but the Second is not. The Second Theorem requires that a common tangent can be found that relies on preferences and production sets being convex. A competitive equilibrium can exist with some nonconvexity in the production sets of the individual firms or the preferences of the consumers, so the First Theorem will apply, but the Second Theorem will not apply.

2.5 Lump-Sum Taxation

The discussion of the Second Theorem noted that it does not describe the mechanism through which the decentralization is achieved. It is instead implicit in the statement of the theorem that the consumers are given sufficient income to purchase the consumption plans forming the Pareto-efficient allocation. Any practical value of the Second Theorem depends on the government being able to allocate the required income levels. The way in which the theorem sees this as being done is by making what are called *lump-sum transfers* between consumers.

A transfer is defined as lump sum if no change in a consumer's behavior can affect the size of the transfer. For example, a consumer choosing to work less hard or reducing the consumption of a commodity must not be able to affect the size of the transfer. This differentiates a lump-sum transfer from other taxes, such as income or commodity taxes, for which changes in behavior do affect the value of the tax payment. Lump-sum transfers have a very special role in the theoretical analysis of public economics because, as we will show, they are the idealized redistributive instrument.

The lump-sum transfers envisaged by the Second Theorem involve quantities of endowments and shares being transferred between consumers to ensure the necessary income levels. Some consumers would gain from the transfers; others would lose. Although the value of the transfer cannot be changed, lump-sum transfers do affect consumers' behavior because their incomes are either reduced or increased by the transfers—the transfers have an income effect but do not lead to a substitution effect between commodities. Without recourse to such transfers, the decentralization of the selected allocation would not be possible.

The illustration of the Second Theorem in an exchange economy in figure 2.15 makes clear the role and nature of lump-sum transfers. The initial endowment point is denoted ω, and this is the starting point for the economy. Assuming that the Pareto-efficient allocation at point e is to be decentralized, the income levels have to be modified to achieve the new budget constraint. At the initial point the income level of h is $\hat{p}\omega^h$ when evaluated at the equilibrium prices $\hat{p}$. The value of the transfer to consumer h that is necessary to achieve the new budget constraint is $M^h - \hat{p}\omega^h = \hat{p}\hat{x}^h - \hat{p}\omega^h$. One way of ensuring this is to transfer a quantity $\tilde{x}_1^1$ of good 1 from consumer 1 to consumer 2. But any transfer of commodities with the same value would work equally well.

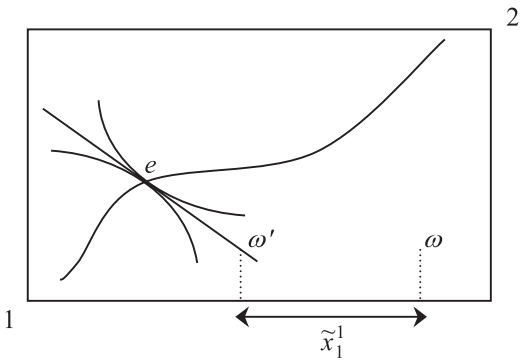


Figure 2.15
Lump-sum transfer

There is a problem, though, if we attempt to interpret the model this literally. For most people, income is earned almost entirely from the sale of labor so that their endowment is simply lifetime labor supply. This makes it impossible to transfer the endowment since one person's labor cannot be given to another. Responding to such difficulties leads to the reformulation of lump-sum transfers in terms of *lump-sum taxes*. Suppose that the two consumers both sell their entire endowments at prices $\hat{p}$. This generates incomes $\hat{p}\omega^1$ and $\hat{p}\omega^2$ for the two consumers. Now make consumer 1 pay a tax of amount $T^1 = \hat{p}\tilde{x}_1^1$ and give this tax revenue to consumer 2. Consumer 2 therefore pays a negative tax (or, in simpler terms, receives a subsidy) of $T^2 = -\hat{p}\tilde{x}_1^1 = -T^1$. The pair of taxes (T^1, T^2) moves the budget constraint in exactly the same way as the lump-sum transfer of endowment. The pair of taxes and the transfer of endowment are therefore economically equivalent and have the same effect on the economy. The taxes are also lump sum because they are determined without reference to either consumers' behavior and their values cannot be affected by any change in behavior.

Lump-sum taxes have a central role in public economics due to their efficiency in achieving distributional objectives. It should be clear from the discussion above that the economy's total endowment is not reduced by the application of the lump-sum taxes. This point applies to lump-sum taxes in general. As households cannot affect the level of the tax by changing their behavior, lump-sum taxes do not lead to any distortions in choice. There are also no resources lost due to the imposition of lump-sum taxes, so redistribution is achieved with no efficiency cost. In short, if they can be employed in the manner described they are the perfect taxes.

2.6 Discussion of Assumptions

The description of the competitive economy introduced a number of assumptions concerning the economic environment and how trade was conducted. These are important since they bear directly on the efficiency properties of competition. The interpretation and limitation of these assumptions is now discussed. This should help to provide a better context for evaluating the practical relevance of the efficiency theorems.

The most fundamental assumption was that of competitive behavior. This is the assumption that both consumers and firms view prices as fixed when they make their decisions. The natural interpretation of this assumption is that the individual economic agents are small relative to the total economy. When they are small, they naturally have no consequence. This assumption rules out any kind of market power such as monopolistic firms or trade unions in labor markets.

Competitive behavior leads to the problem of who actually sets prices in the economy. In the exchange model it is possible for equilibrium prices to be achieved via a process of barter and negotiation between the trading parties. However barter cannot be a credible explanation of price determination in an advanced economic environment. One theoretical route out of this difficulty is to assume the existence of a fictitious ''Walrasian auctioneer'' who literally calls out prices until equilibrium is achieved. Only at this point trade is allowed to take place. Obviously this does not provide a credible explanation of reality. Although there are other theoretical explanations of price setting, none is entirely consistent with the competitive assumption. How to integrate the two remains an unsolved puzzle.

The second assumption was symmetry of information. In a complex world there are many situations in which this does not apply. For instance, some qualities of a product, such as reliability (I do not know when my computer will next crash, but I expect it will be soon), are not immediately observable but are discovered only through experience. When it comes to re-sale, this causes an asymmetry of information between the existing owner and potential purchasers. The same can be true in labor markets where workers may know more about their attitudes to work and potential productivity than a prospective employer. An asymmetry of information provides a poor basis for trade because the caution of those transacting prevents the full gains from trade being realized.

When any of the assumptions underlying the competitive economy fail to be met, and as a consequence efficiency is not achieved, we say that there is *market*

failure. Situations of market failure are of interest to public economics because they provide a potential role for government policy to enhance efficiency. A large section of this book is in fact devoted to a detailed analysis of the sources of market failure and the scope for policy response.

As a final observation, notice that the focus has been on positions of equilibrium. Several explanations can be given for this emphasis. Historically economists viewed the economy as self-correcting so that, if it were ever away from equilibrium, forces existed that move it back toward equilibrium. In the long run, equilibrium would then always be attained. Although such adjustment can be justified in simple single-market contexts, both the practical experience of sustained high levels of unemployment and the theoretical study of the stability of the price adjustment process have shown that the self-adjusting equilibrium view is not generally justified. The present justifications for focusing on equilibrium are more pragmatic. The analysis of a model must begin somewhere, and the equilibrium has much merit as a starting point. In addition, even if the final focus is on disequilibrium, there is much to be gained from comparing the properties of points of disequilibrium to those of the equilibrium. Finally, no positions other than those of equilibrium have any obvious claim to prominence.

2.7 Summary

This chapter described competitive economies and demonstrated the Two Theorems of Welfare Economics. To do this, it was necessary to introduce the concept of Pareto-efficiency. While Pareto-efficiency was simply accepted in this chapter, it will be considered very critically in chapter 12. The Two Theorems characterize the efficiency properties of the competitive economy and show how a selected Pareto-efficient allocation can be decentralized. It was also shown how prices are central to the achievement of efficiency through their role in coordinating the choices of individual agents. The role of lump-sum transfers or taxes in supporting the Second Theorem was highlighted. These transfers constitute the ideal tax system because they cause no distortions in choice and have no resource costs.

The subject matter of this chapter has very strong implications that are investigated fully in later chapters. An understanding of the welfare theorems, and of their limitations, is fundamental to appreciating many of the developments of public economics. Since claims about the efficiency of competition feature routinely in economic debate, it is important to subject it to the most careful scrutiny.

Further Reading

The two fundamental texts on the competitive economy are:

Arrow, K. J., and Hahn, F. H. 1971. *General Competitive Analysis*. Amsterdam: North-Holland.

Debreu, G. 1959. *The Theory of Value*. New Haven: Yale University Press.

A textbook treatment can be found in:

Ellickson, B. 1993. *Competitive Equilibrium: Theory and Applications*. Cambridge: Cambridge University Press.

The competitive economy has frequently been used as a practical tool for policy analysis. A survey of applications is in:

Shoven, J. B., and Whalley, J. 1992. *Applying General Equilibrium Theory*. Cambridge: Cambridge University Press.

A historical survey of the development of the model is given in:

Duffie, D., and Sonnenschein, H. 1989. Arrow and general equilibrium theory. *Journal of Economic Literature* 27: 565–98.

Some questions concerning the foundations of the model are addressed in:

Koopmans, T. C. 1957. *Three Essays on the State of Economic Science*. New York: McGraw-Hill.

The classic proof of the Two Theorems is in:

Arrow, K. J. 1951. An extension of the basic theorems of welfare economics. In J. Neyman, ed., *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*. Berkeley: University of California Press.

A formal analysis of lump-sum taxation can be found in:

Mirrlees, J. A. 1986. The theory of optimal taxation. In K. J. Arrow and M. D. Intrilligator, eds., *Handbook of Mathematical Economics*. Amsterdam: North-Holland.

An extensive textbook treatment of Pareto-efficiency is:

Ng, Y.-K. 2003. *Welfare Economics*. Basingstoke: Macmillan.

Exercises

2.1. Distinguish between partial equilibrium analysis and general equilibrium analysis. Briefly describe a model of each kind.

2.2. Keynesian models in macroeconomics are identified by the assumption of a fixed price for output. Are such models partial or general equilibrium?

2.3. You are requested to construct a model to predict the effect on the economy of the discovery of new oil reserves. How would you model the discovery? Discuss the number of goods that should be included in the model.

2.4. Let a consumer have preferences described by the utility function

$$U = \log(x_1) + \log(x_2),$$

and an endowment of 2 units of good 1 and 2 units of good 2.

a. Construct and sketch the consumer's budget constraint. Show what happens when the price of good 1 increases.

b. By maximizing utility, determine the consumer's demands.

c. What is the effect of increasing the endowment of good 1 upon the demand for good 2? Explain your finding.

2.5. How would you model an endowment of labor?

2.6. Let two consumers have preferences described by the utility function

$$U^h = \log(x_1^h) + \log(x_2^h), \qquad h = 1, 2,$$

and the endowments described below:

	Good 1	Good 2
Consumer 1	3	2
Consumer 2	2	3

a. Calculate the consumers' demand functions.

b. Selecting good 2 as the numéraire, find the equilibrium price of good 1. Hence find the equilibrium levels of consumption.

c. Show that the consumers' indifference curves are tangential at the equilibrium.

2.7. Consider an economy with two goods and two consumers with preferences

$$U^h = \min\{x_1^h, x_2^h\}, \qquad h = 1, 2.$$

Assume that the endowments are as follows:

	Good 1	Good 2
Consumer 1	1	2
Consumer 2	2	1

a. Draw the Edgeworth box for the economy.

b. Display the equilibrium in the Edgeworth box.

c. What is the effect on the equilibrium price of good 2 relative to good 1 of an increase in each consumer's endowment of good 1 by 1 unit?

2.8. Consumer 1 obtains no pleasure from good 1, and consumer 2 obtains no pleasure from good 2. At the initial endowment point both consumers have endowments of both goods.

a. Draw the preferences of the consumers in an Edgeworth box.

b. By determining the trades that improve both consumers' utilities, find the equilibrium of the economy.

c. Display the equilibrium budget constraint.

2.9. Demonstrate that the demands obtained in exercise 2.4 are homogeneous of degree zero in prices. Show that doubling prices does not affect the graph of the budget constraint.

2.10. It has been argued that equilibrium generally exists on the basis that there must be a point where excess demand for good 2 is zero if excess demand is positive as the price of good 2 tends to zero and negative as it tends to infinity.

a. Select good 1 as numéraire and show that these properties hold when preferences are given by the utility function

$$U^h = \log(x_1^h) + \log(x_2^h),$$

and the consumer's endowment of both goods is positive.

b. Show that they do not hold if the consumer has no endowment of good 2.

c. Consider the implications of the answer to part b for proving the existence of equilibrium.

2.11. Consider an economy with 2 consumers, A and B, and 2 goods, 1 and 2. The utility function of A is $U^A = \gamma \log(x_1^A) + [1-\gamma]\log(x_2^A)$, where x_i^A is consumption of good i by A. A has endowments $\omega^A = (\omega_1^A, \omega_2^A) = (2,1)$. For B, $U^B = \gamma \log(x_1^B) + [1-\gamma]\log(x_2^B)$ and $\omega^B = (3,2)$.

a. Use the budget constraint of A to substitute for x_2^A in U^A, and by maximizing over x_1^A, calculate the demands of A. Repeat for B.

b. Choosing good 2 as the numéraire, graph the excess demand for good 1 as a function of p_1.

c. Calculate the competitive equilibrium allocation by equating the demand for good 1 to the supply and then substituting for M^A and M^B. Verify that this is the point where excess demand is zero.

d. Show how the equilibrium price of good 2 is affected by a change in γ and in ω_1^A. Explain the results.

2.12. A firm has a production technology that permits it to turn 1 unit of good 1 into 2 units of good 2. If the price of good 1 is 1, at what price for good 2 will the firm just break even? Graph the firm's profit as a function of the price of good 2.

2.13. How can the existence of fixed costs be incorporated into the production set diagram? After paying its fixed costs a firm has constant returns to scale. Can it earn zero profits in a competitive economy?

2.14. Consider an economy with 2 goods, H consumers and m firms. Each consumer, h, has an endowment of 2 units of good 1 and none of good 2, preferences described by

$U^h = x_1^h x_2^h$, and a share $\theta_j^h = \frac{1}{H}$ in firm $j = 1, \ldots, m$. Each firm has a technology characterized by the production function $y_2^j = [-y_1^j]^{1/2}$.

a. Calculate a firm's profit-maximizing choices, a consumer's demands and the competitive equilibrium of the economy.

b. What happens to $\frac{p_2}{p_1}$ as (i) m increases; (ii) H increases? Why?

c. Suppose that each consumer's endowment of good 1 increases to $2 + 2\delta$. Explain the change in relative prices.

d. What is the effect of changing:

i. The distribution of endowments among consumers;

ii. The consumers' preferences to $U^h = \alpha \log(x_1^h) + \beta \log(x_2^h)$?

2.15. Reproduce the diagram for the Robinson Crusoe economy for a firm that has constant returns to scale. Under what conditions will it be efficient for the firm not to produce? What is the consumption level of the consumer in such a case? Provide an interpretation of this possibility.

2.16. After the payment of costs, fishing boat captains distribute the surplus to the owner and crew. Typically the owner receives 50 percent, the captain 30 percent, and the remaining 20 percent is distributed to crew according to status. Is this distribution Pareto-efficient? Is it equitable?

2.17. A box of chocolates is to be shared by two children. The box contains ten milk chocolates and ten plain chocolates. Neither child likes plain chocolates. Describe the Pareto-efficient allocations.

2.18. As economists are experts in resource allocation, you are invited by two friends to resolve a dispute about the shared use of a car. By applying Pareto-efficiency, what are you able to advise them?

2.19. Two consumers have utility functions

$U^h = \log(x_1^h) + \log(x_2^h)$.

a. Calculate the marginal rate of substitution between good 1 and good 2 in terms of consumption levels.

b. By equating the marginal rates of substitution for the two consumers, characterize a Pareto-efficient allocation.

c. Using the solution to part b, construct the contract curve for an economy with 2 units of good 1 and 3 units of good 2.

2.20. A consumer views two goods as perfect substitutes.

a. Sketch the indifference curves of the consumer.

b. If an economy is composed of two consumers with these preferences, demonstrate that any allocation is Pareto-efficient.

c. If an economy has one consumer who views its two goods as perfect substitutes and a second that considers each unit of good 1 to be worth 2 units of good 2, find the Pareto-efficient allocations.

2.21. Consider an economy in which preferences are given by

$U^1 = x_1^1 + x_2^1$ and $U^2 = \min\{x_1^2, x_2^2\}$.

Given the endowments $\omega^1 = (1, 2)$ and $\omega^2 = (3, 1)$, construct the set of Pareto-efficient allocations and the contract curve. Which allocations are also competitive equilibria?

2.22. Take the economy in the exercise above, but change the preferences of consumer 2 to

$U^2 = \max\{x_1^2, x_2^2\}$.

Is there a Pareto-efficient allocation?

2.23. Consider an economy with two consumers, A and B, and two goods, 1 and 2. Using x_i^h to denote the consumption of good i by consumer h, assume that both consumers have the utility function $U^h = \min\{x_1^h, x_2^h\}$.

a. By drawing an Edgeworth box, display the Pareto-efficient allocations if the economy has an endowment of 1 unit of each good.

b. Display the Pareto-efficient allocations if the endowment is 1 unit of good 1 and 2 units of good 2.

c. What would be the competitive equilibrium prices for parts a and b?

2.24. Consider the economy in exercise 2.11.

a. Calculate the endowments required to make the equal-utility allocation a competitive equilibrium.

b. Discuss the transfer of endowment necessary to support this equilibrium.

2.25. Provide an example of a Pareto-efficient allocation that cannot be decentralized.

2.26. Let an economy have a total endowment of two units of the two available goods. If the two consumers have preferences

$U^h = \alpha \log(x_1^h) + [1 - \alpha] \log(x_2^h)$,

find the ratio of equilibrium prices at the allocation where $U^1 = U^2$. Hence find the value of the lump-sum transfer that is needed to decentralize the allocation if the initial endowments are $\left(\frac{1}{2}, \frac{3}{4}\right)$ and $\left(\frac{3}{2}, \frac{5}{4}\right)$.

2.27. Are the following statements true or false? Explain in each case.

a. If one consumer gains from a trade, the other consumer involved in the trade must lose.

b. The gains from trade are based on comparative advantage, not absolute advantage.

c. The person who can produces the good with less input has an absolute advantage in producing this good.

d. The person who has the smaller opportunity cost of producing the good has a comparative advantage in producing this good.

e. The competitive equilibrium is the only allocation where the gains from trade are exhausted.

7 Externalities

7.1 Introduction

An externality is a link between economic agents that lies outside the price system of the economy. Everyday examples include the pollution from a factory that harms a local fishery and the envy that is felt when a neighbor proudly displays a new car. Such externalities are not controlled directly by the choices of those affected—the fishery cannot choose to buy less pollution nor can you choose to buy your neighbor a worse car. This prevents the efficiency theorems described in chapter 2 from applying. Indeed, the demonstration of market efficiency was based on the following two presumptions:

- The welfare of each consumer depended solely on her own consumption decision.
- The production of each firm depended only on its own input and output choices.

In reality, these presumptions may not be met. A consumer or a firm may be directly affected by the actions of other agents in the economy; that is, there may be external effects from the actions of other consumers or firms. In the presence of such externalities the outcome of a competitive market is unlikely to be Pareto-efficient because agents will not take account of the external effects of their (consumption/production) decisions. Typically the economy will generate too great a quantity of "bad" externalities and too small a quantity of "good" externalities.

The control of externalities is an issue of increasing practical importance. Global warming and the destruction of the ozone layer are two of the most significant examples, but there are numerous others, from local to global environmental issues. Some of these externalities may not appear immediately to be economic problems, but economic analysis can expose why they occur and investigate the effectiveness of alternative policies. Economic analysis can generate surprising conclusions and challenge standard policy prescriptions. In particular, it shows how government intervention that induces agents to internalize the external effects of their decisions can achieve a Pareto improvement.

The starting point for the chapter is to provide a working definition of an externality. Using this, it is shown why market failure arises and the nature of the resulting inefficiency. The design of the optimal set of corrective, or *Pigouvian*,

taxes is then addressed and related to missing markets for externalities. The use of taxes is contrasted with direct control through tradable licenses. Internalization as a solution to externalities is considered. Finally these methods of solving the externality problem are set against the claim of the Coase theorem that efficiency will be attained by trade even when there are externalities.

7.2 Externalities Defined

An externality has already been described as an effect on one agent caused by another. This section provides a formal statement of this description, which is then used to classify the various forms of externality. The way of representing these forms of externalities in economic models is introduced.

There have been several attempts at defining externalities and of providing classifications of various types of externalities. From among these the following definition is the most commonly adopted. Its advantages are that it places the emphasis on recognizing externalities through their effects and it leads to a natural system of classification.

Definition 4 (Externality) An externality is present whenever some economic agent's welfare (utility or profit) is "directly" affected by the action of another agent (consumer or producer) in the economy.

By "directly" we exclude any effects that are mediated by prices. That is, an externality is present if a fishery's productivity is affected by the river pollution of an upstream oil refinery but not if the fishery's profitability is affected by the price of oil (which may depend on the oil refinery's output of oil). The latter type of effect (often called a *pecuniary externality*) is present in any competitive market but creates no inefficiency (since price mediation through competitive markets leads to a Pareto-efficient outcome). We will present later an illustration of a pecuniary externality.

This definition of an externality implicitly distinguishes between two broad categories. A *production externality* occurs when the effect of the externality is on a profit relationship and a *consumption externality* whenever a utility level is affected. Clearly, an externality can be both a consumption and a production externality simultaneously. For example, pollution from a factory may affect the profit of a commercial fishery and the utility of leisure anglers.

Using this definition of an externality, it is possible to move on to how they can be incorporated into the analysis of behavior. Denote, as in chapter 2, the consumption levels of the households by $x = \{x^1, \ldots, x^H\}$ and the production plans of the firms by $y = \{y^1, \ldots, y^m\}$. It is assumed that consumption externalities enter the utility functions of the households and that production externalities enter the production sets of the firms. At the most general level, this assumption implies that the utility functions take the form

$$U^h = U^h(x, y), \qquad h = 1, \ldots, H, \tag{7.1}$$

and the production sets are described by

$$Y^j = Y^j(x, y), \qquad j = 1, \ldots, m. \tag{7.2}$$

In this formulation the utility functions and the production sets are possibly dependent upon the entire arrays of consumption and production levels. The expressions in (7.1) and (7.2) represent the general form of the externality problem, and in some of the discussion below a number of further restrictions will be employed.

It is immediately apparent from (7.1) and (7.2) that the actions of the agents in the economy will no longer be independent or determined solely by prices. The linkages via the externality result in the optimal choice of each agent being dependent on the actions of others. Viewed in this light, it becomes apparent why competition will generally not achieve efficiency in an economy with externalities.

7.3 Market Inefficiency

It has been accepted throughout the discussion above that the presence of externalities will result in the competitive equilibrium failing to be Pareto-efficient. The immediate implication of this fact is that incorrect quantities of goods, and hence externalities, will be produced. It is also clear that a non–Pareto-efficient outcome will never maximize welfare. This provides scope for economic policy to improve the outcome. The purpose of this section is to demonstrate how inefficiency can arise in a competitive economy. The results are developed in the context of a simple two-consumer model, since this is sufficient for the purpose and also makes the relevant points as clear as possible.

Consider a two-consumer, two-good economy where the consumers have utility functions

$$U^1 = x^1 + u_1(z^1) + v_1(z^2) \tag{7.3}$$

and

$$U^2 = x^2 + u_2(z^2) + v_2(z^1). \tag{7.4}$$

The externality effect in (7.3) and (7.4) is generated by consumption of good z by the consumers. The externality will be *positive* if $v_h(\cdot)$ is increasing in the consumption level of the other consumer and *negative* if it is decreasing.

To complete the description of the economy, it is assumed that the supply of good x comes from an endowment ω_h to consumer h, whereas good z is produced from good x by a competitive industry that uses one unit of good x to produce one unit of good z. Normalizing the price of good x at 1, the structure of production ensures that the equilibrium price of good z must also be 1. Given this, all that needs to be determined for this economy is the division of the initial endowment into quantities of the two goods.

Incorporating this assumption into the maximization decision of the consumers, the competitive equilibrium of the economy is described by the equations

$$u_h'(z^h) = 1, \qquad h = 1, 2, \tag{7.5}$$

$$x^h + z^h = \omega^h, \qquad h = 1, 2, \tag{7.6}$$

and

$$x^1 + z^1 + x^2 + z^2 = \omega^1 + \omega^2. \tag{7.7}$$

It is equations (7.5) that are of primary importance at this point. For consumer h these state that the private marginal benefit from each good, determined by the marginal utility, is equated to the private marginal cost. The external effect does not appear directly in the determination of the equilibrium. The question we now address is whether this competitive market equilibrium is efficient.

The Pareto-efficient allocations are found by maximizing the total utility of consumers 1 and 2, subject to the production possibilities. The equations that result from this will then be contrasted to (7.5). In detail, a Pareto-efficient allocation solves

$$\max_{\{x^h, z^h\}} U^1 + U^2 = [x^1 + u_1(z^1) + v_1(z^2)] + [x^2 + u_2(z^2) + v_2(z^1)] \tag{7.8}$$

subject to

$$\omega^1 + \omega^2 - x^1 - z^1 - x^2 - z^2 \geq 0. \tag{7.9}$$

The solution is characterized by the conditions

$$u_1'(z^1) + v_2'(z^1) = 1 \tag{7.10}$$

and

$$u_2'(z^2) + v_1'(z^2) = 1. \tag{7.11}$$

In (7.10) and (7.11) the externality effect can be seen to affect the optimal allocation between the two goods via the derivatives of utility with respect to the externality. If the externality is positive then $v_h' > 0$ and the externality effect will raise the value of the left-hand terms. It will decrease them if there is a negative externality, so $v_h' < 0$. It can then be concluded that at the optimum with a positive externality the marginal utilities of both consumers are below their value in the market outcome. The converse is true with a negative externality. The externality leads to a divergence between the private valuations of consumption given by (7.5) and the corresponding social valuations in (7.10) and (7.11). This observation has the implication that the market outcome is not Pareto-efficient.

In general, it can also be concluded that if the externality is positive then more of good z will be consumed at the optimum than under the market outcome. The converse holds for a negative externality. This situation is illustrated in figure 7.1. The market outcome is represented by equality between the private marginal benefit of the good (PMB) and its marginal cost (MC). The social marginal benefit (SMB) of the good is the sum of the private marginal benefit, $u_h'(z^h)$, and the marginal external effect, $v_{\tilde{h}}'(z^h)$. When $v_{\tilde{h}}'(z^h)$ is positive, SMB is above PMB. The converse holds when $v_{\tilde{h}}'(z^h)$ is negative. The Pareto-efficient outcome equates the social marginal benefit to marginal cost. The market failure is characterized by

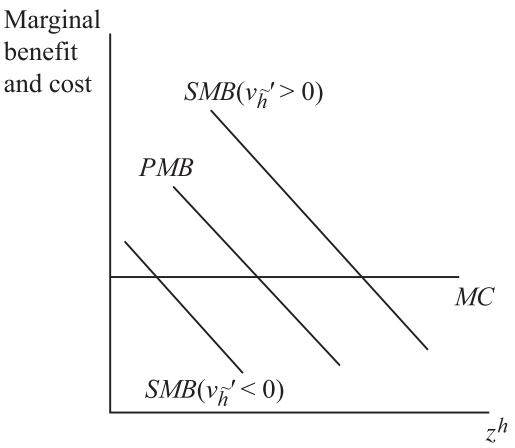


Figure 7.1
Deviation of private from social benefits

too much consumption of a good causing a negative externality and too little consumption of a good generating a positive externality.

7.4 Externality Examples

The previous section has discussed externalities at a somewhat abstract level. We now consider some more-concrete examples of externalities. Some of the examples are very simple because of the binary nature of the choice and the assumption of identical individuals. This modeling choice was widely used by Schelling to achieve an extremely simple exposition that brings out the line of the argument very clearly. In addition it will illustrate the range of situations that fall under the general heading of externalities.

7.4.1 River Pollution

This example, from Louis Gevers, is one of the simplest examples that can be described using only two agents. Assume that two firms are located along the same river. The upstream firm u pollutes the river, which reduces the production (e.g., the output of fish) of the downstream firm d. Both firms produce the same output, which they sell at a constant unit price of 1 so that total revenue coincides with production.

Labor and water are used as inputs. Water is free, but the equilibrium wage w on the competitive labor market is paid for each unit of labor. The production technologies of the firms are given by $F^u(L^u)$ and $F^d(L^d, L^u)$, with $\frac{\partial F^d}{\partial L^u} < 0$ to reflect that the pollution reduces downstream output. Decreasing returns to scale are assumed with respect to own labor input. Each firm acts independently and seeks to maximize its own profit $\pi^i = F^i(\cdot) - wL^i$, taking prices as given.

The equilibrium is illustrated in figure 7.2. The total stock of labor is allocated between the two firms. The labor input of the upstream firm is measured from the left, that of the downstream from the right. Each point on the horizontal axis represents a different allocation between the firms. The upstream firm's profit maximization process is represented in the upper part of the diagram and the downstream firm's in the lower part. As the input of the upstream firm increases the production function of the downstream firm moves progressively in toward the horizontal axis. Given the profit-maximizing input level of the upstream firm, denoted L^{u*}, the downstream firm can do no better than choose L^{d*}. At these choices the firms earn profits π^u and π^d respectively. This is the competitive equi-

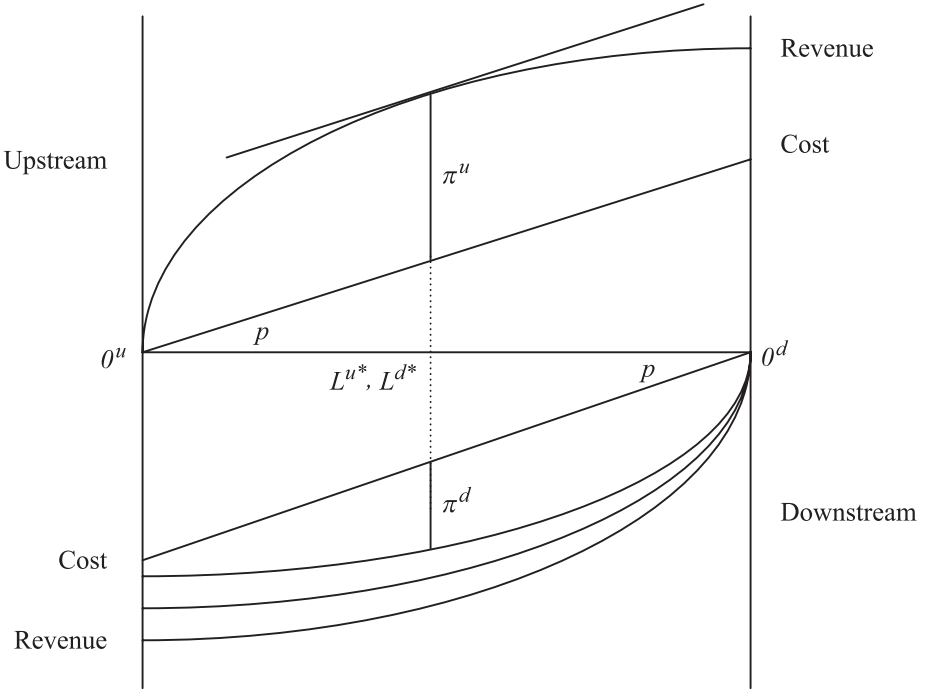


Figure 7.2
Equilibrium with river pollution

librium. We now show that this is inefficient and that reallocating labor between the firms can increase total profit and reduce pollution.

Consider starting at the competitive equilibrium and make a small reduction in the labor input to the upstream firm. Since the choice was optimal for the upstream firm, the change has no effect on profit for the upstream firm (recall that $\frac{\partial \pi^u}{\partial L^u} = 0$). However, it leads to an outward shift of the downstream firm's production function. This raises its profits. Hence the change raises aggregate profit. This demonstrates that the competitive equilibrium is not efficient and that the externality results in the upstream firm using too much labor and the downstream too little. Shifting labor to the downstream firm raises total production and reduces pollution.

7.4.2 Traffic Jams

The next example considers the externalities imposed by drivers on each other. Let there be N commuters who have the choice of commuting by train or by

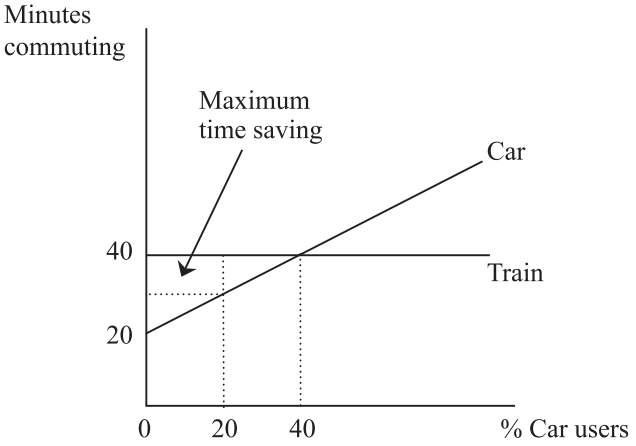


Figure 7.3
Choice of commuting mode

car. Commuting by train always takes 40 minutes regardless of the number of travelers. The commuting time by car increases as the number of car users increases. This congestion effect, which raises the commuting time, is the externality for travelers. Individuals must each make decisions to minimize their own transportation time.

The equilibrium in the choice of commuting mode is depicted in figure 7.3. The number of car users will adjust until the travel time by car is exactly equal to the travel time by train. For the travel time depicted in the figure, the equilibrium occurs when 40 percent of commuters travel by car. The optimum occurs when the aggregate time saving is maximized. This occurs when only 20 percent of commuters use a car.

The externality in this situation is that the car drivers take into account only their own travel time but not the fact that they will increase the travel time for all other drivers. As a consequence too many commuters choose to drive.

7.4.3 Pecuniary Externality

Consider a set of students each of whom must decide whether to be an economist or a lawyer. Being an economist is great when there are few economists, and not so great when the labor market becomes crowded with economists (due to price competition). If the number of economists grows high enough, they will eventually earn less than their lawyer counterparts. Suppose that each person chooses

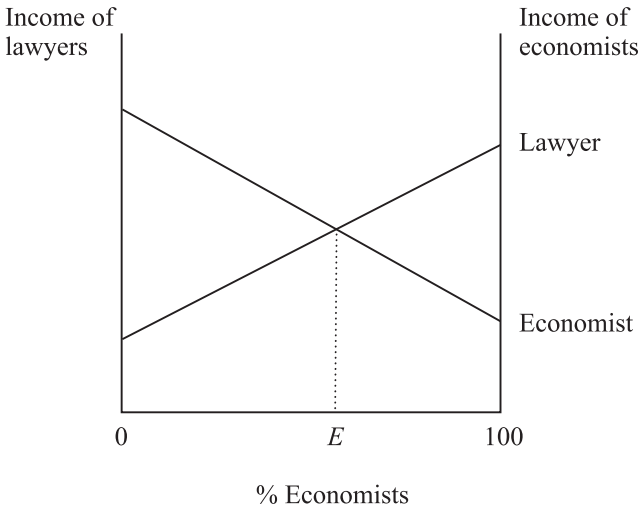


Figure 7.4
Job choice

the profession with the best earnings prospects. The externality (a pecuniary one!) comes from the fact that when one more person decides to become an economist, he lowers all other economists' incomes (through competition), imposing a cost on the existing economists. When making his decision, he ignores this external effect imposed on others. The question is whether the invisible hand will lead to the correct allocation of students across different jobs.

The equilibrium depicted in figure 7.4 determines the allocation of students between jobs. The number of economists will adjust until the earnings of an economist are exactly equal to the earnings of a lawyer. The equilibrium is given by the percentage of economists at point E. To the right of point E, lawyers would earn more and the number of economists would decrease. Alternatively, to the left of point E economists are relatively few in number and will earn more than lawyers, attracting more economists into the profession.

The laissez-faire equilibrium is efficient because the external effect is a change in price. The cost to an economists of a lower income is a benefit to employers. Since employers' benefits equals employees' costs, there is zero net effect. The policy implication is that there is no need for government intervention to regulate the access to professions. It follows that any public policy that aims to limit the access to some profession (like the *numerus clausus*) is not justified. Market forces will correctly allocate the right number of people to each of the different professions.

		Player 2: Low	Player 2: High
Player 1	Low	1/2 1/2	$1-c$ 0
	High	0 $1-c$	$1/2-c$ $1/2-c$

Figure 7.5
Rat race

7.4.4 The Rat Race Problem

The rat race problem is a contest for relative position as pointed out by George Akerlof. It can help explain why students work too hard when final marking takes the form of a ranking. It can also explain the intense competition for a promotion in the workplace when candidates compete with each other and only the best is promoted. We take the classroom example here. Assume that performance is judged not in *absolute* terms but in *relative* terms so that what matters is not how much is known but how much is known compared to what other students know.

In this situation an advantage over other students can only be gained by working harder than they do. Since this applies to all students, all must work harder. But since performance is judged in relative terms, all the extra effort cancels out. The result of this is an inefficient rat race in which each student works too hard to no ultimate advantage. If all could agree to work less hard, the same grades would be obtained with less work. Such an agreement to work less hard cannot be self-supporting, since each student would then have an incentive to cheat on the agreement and work harder.

A simple variant of the rat race with two possible effort levels is shown in figure 7.5. In this figure, c, $0 < c < \frac{1}{2}$, denotes the cost of effort. For both students high effort is a dominant strategy. In contrast, the Pareto-efficient outcome is low effort. This game is an example of the Prisoners' Dilemma in which a Pareto-improvement could be made if the players could make a commitment to the low-effort strategy.

Another example of rat race is the use of performance-enhancing drugs by athletes. In the absence of effective drug regulations, many athletes will feel com-

pelled to enhance their performance by using anabolic steroids, and the failure to use steroids might seriously reduce their success in competition. Since the rewards in athletics are determined by performance relative to others, anyone that uses such drugs to increase their chance of winning must necessarily reduce the chances of others (an externality effect). The result is that when the stakes are high in the competition, unregulated contests almost always lead to a race for using more and more performance-enhancing drugs. However, when everyone does so, the use of such drugs yields no real benefits for the contestants as a whole: the performance-enhancing actions cancel each other. At the same time the race imposes substantial risks. Anabolic steroids have been shown to cause cancer of the liver and other serious health problems. Given what is at stake, voluntary restraint is unlikely to be an effective solution, and public intervention now requires strict drug testing of all competing athletes.

The rat race problem is present in almost every contest where something important is at stake and rewards are determined by relative position. In an electoral competition race, contestants spend millions on advertising, and governing bodies have now put strict limits on the amount of campaign advertising. Similarly a ban on cigarette advertising has been introduced in many countries. Surprisingly enough, this ban turned out to be beneficial to cigarette companies. The reason is that the ban helped them out of the costly rat race in defensive advertising where a company had to advertise because the others did.

7.4.5 The Tragedy of the Commons

The *Tragedy of the Commons* arises from the common right of access to a resource. The inefficiency to which it leads results again from the divergence between the individual and social incentives that characterizes all externality problems.

Consider a lake that can be used by fishermen from a village located on its banks. The fishermen do not own boats but instead can rent them for daily use at a cost c. If B boats are hired on a particular day, the number of fish caught by each boat will be $F(B)$, which is decreasing in B. A fisherman will hire a boat to fish if they can make a positive profit. Let w be the wage if they choose to undertake paid employment rather than fish, and let $p = 1$ be the price of fish so that total revenue coincide with fish catch $F(B)$. Then the number of boats that fish will be such as to ensure that profit from fishing activity is equal to the opportunity cost of fishing, which is the forgone wage w from the alternative job (if profit

were greater, more boats would be hired and the converse if it were smaller). The equilibrium number of boats, B^*, then satisfies

$$\pi = F(B^*) - c = w. \tag{7.12}$$

The optimal number of boats for the community, B°, must be that which maximizes the total profit for the village, net of the opportunity cost from fishing. Hence B° satisfies

$$\max_{\{B\}} B[F(B) - c - w]. \tag{7.13}$$

This gives the necessary condition

$$F(B^\circ) - c - w + BF'(B^\circ) = 0. \tag{7.14}$$

Since an increase in the number of boats reduces the quantity of fish caught by each, $F'(B^\circ) < 0$. Therefore contrasting (7.12) and (7.14) shows that $B^\circ < B^*$ so the equilibrium number of boats is higher than the optimal number. This situation is illustrated in figure 7.6.

The externality at work in this example is that each fisherman is concerned only with their own profit. When deciding whether to hire a boat they do not take account of the fact that they will reduce the quantity of fish caught by every other fisherman. This negative externality ensures that in equilibrium too many boats

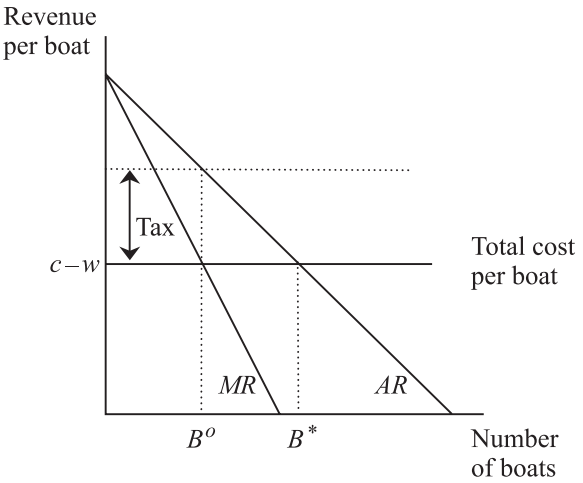


Figure 7.6
Tradegy of the Commons

are operating on the lake. Public intervention can take two forms. There is the price-based solution consisting of a tax per boat so as to internalize the external effect of sending a boat on the lake. As indicated in the figure a correctly chosen tax will reduce the number of boats so as to restore the optimal outcome. Alternatively, the quantity-based solution consists of setting a quota of fishing equal to the optimal outcome.

7.4.6 Bandwagon Effect

The bandwagon effect studies the question of how standards are adopted and, in particular, how it is possible for the wrong standard to be adopted. The standard application of this is the choice of arrangement for the keys on a keyboard.

The current standard, Qwerty, was designed in 1873 by Christopher Scholes in order to deliberately slow down the typist by maximizing the distance between the most used letters. The motivation for this was the reduction of key-jamming problems (remember this would be for mechanical typewriters in which metal keys would have to strike the ink ribbon). By 1904 the Qwerty keyboard was mass produced and became the accepted standard. The key-jamming problem is now irrelevant, and a simplified alternative keyboard (Dvorak's keyboard) has been devised that reduces typing time by 5 to 10 percent.

Why has this alternative keyboard not been adopted? The answer is that there is a switching cost. All users are reluctant to switch and bear the cost of retraining, and manufacturers see no advantage in introducing the alternative. It has therefore proved impossible to switch to the better technology.

This problem is called a *bandwagon effect* and is due to a *network externality*. The decision of a typist to use the Qwerty keyboard makes it more attractive for manufacturers to produce Qwerty keyboards, and hence for others to learn Qwerty. No individual has any incentive to switch to Dvorak. The nature of the equilibrium is displayed in figure 7.7. This shows the intertemporal link between the percentage using Qwerty at time t and the percentage at time $t+1$. The natural advantage of Dvorak is captured in the diagram by the fact that the number of Qwerty users will decline over time starting from a position where 50 percent use Qwerty at time t. There are three equilibria. Either all will use Qwerty or Dvorak or else a proportion p^*, $p^* > 50$ percent, will use Qwerty and $1 - p^*$ Dvorak However, this equilibrium is unstable and any deviation from it will lead to one of the corner equilibria. The inefficient technology, Qwerty, can dominate in equilibrium if the initial starting point is to the right of p^*.

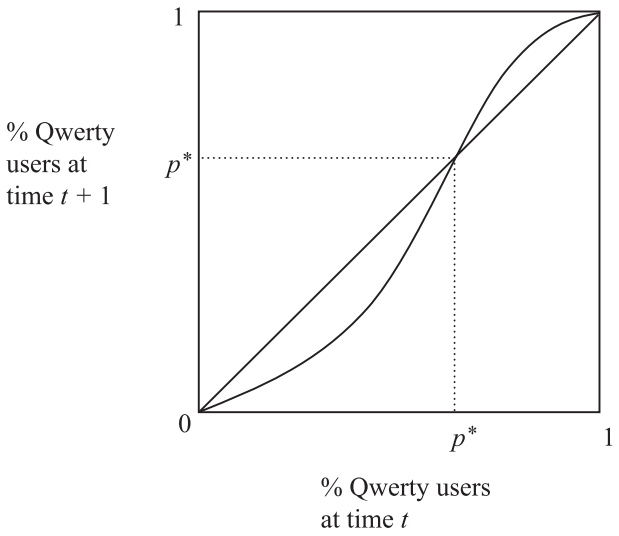


Figure 7.7
Equilibrium keyboard choice

7.5 Pigouvian Taxation

The description of market inefficiency has shown that its basic source is the divergence between social and private benefits (or between social and private costs). This fact has been reinforced by the examples. A natural means of eliminating such divergence is to employ appropriate taxes or subsidies. By modifying the decision problems of the firms and consumers these can move the economy closer to an efficient position.

To see how a tax can enhance efficiency, consider the case of a negative consumption externality. With a negative externality the private marginal benefit of consumption is always in excess of the social marginal benefit. These benefits are depicted by the PMB and SMB curves respectively in figure 7.8. In the absence of intervention, the equilibrium occurs where the PMB intersects the private marginal cost (PMC). This gives a level of consumption x^m. The efficient consumption level equates the PMC with the SMB; this is at point x^o. As already noted, with a negative externality the market outcome involves more consumption of the good than is efficient. The market outcome can be improved by placing a tax on consumption. What it is necessary to do is to raise the PMC so that it intersects

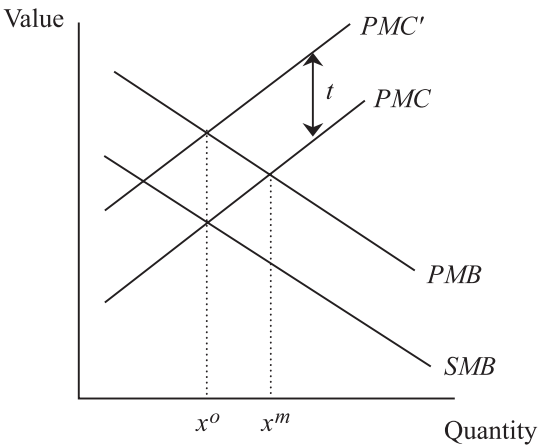


Figure 7.8
Pigouvian taxation

the SMB vertically above x^o. This is what happens for the curve PMC', which has been raised above PMC by a tax of value t. This process, often termed *Pigouvian taxation*, allows the market to attain efficiency for the situation shown in figure 7.8.

Based on arguments like that exhibited above, Pigouvian taxation has been proposed as a simple solution to the externality problem. The logic is that the consumer or firm causing the externality should pay a tax equal to the marginal damage the externality causes (or a subsidy if there is a marginal benefit). Doing so makes them take account of the damage (or benefit) when deciding how much to produce or consume. In many ways this is a compellingly simple conclusion.

The previous discussion is informative but leaves a number of issues to be resolved. Foremost among these is the fact that the figure implicitly assumes there is a single agent generating the externality whose marginal benefit and marginal cost are exhibited and that there is a single externality. The single tax works in this case, but will it still do so with additional externalities and agents? This is an important question to be answered if Pigouvian taxation is to be proposed as a serious practical policy.

To address these issues, we use our example from the market failure section again. This example involved two consumers and two goods with the consumption of one of the goods, z, causing an externality. The optimal structure of Pigouvian taxes is determined by characterizing the social optimum and inferring from

that what the taxes must be. Recall from (7.10) and (7.11) that the social optimum is characterized by the conditions

$$u_1'(z^1) + v_2'(z^1) = 1 \tag{7.15}$$

and

$$u_2'(z^2) + v_1'(z^2) = 1. \tag{7.16}$$

It is from contrasting these conditions to those for individual choice that the optimal taxes can be derived.

Utility maximization by consumer 1 will equate their private marginal benefit, $u_1'(z^1)$, to the consumer price q_1. Given that the producer price is equal to 1 in this example, (7.15) shows that efficiency will be achieved if the price, q_1, facing consumer 1 satisfies

$$q_1 = 1 - v_2'(z^1). \tag{7.17}$$

Similarly from (7.16) efficiency will be achieved if the price facing consumer 2 satisfies

$$q_2 = 1 - v_1'(z^2). \tag{7.18}$$

These identities reveal that the taxes that ensure the correct difference between consumer and producer prices are given by

$$t_1 = -v_2'(z^1) \tag{7.19}$$

and

$$t_2 = -v_1'(z^2). \tag{7.20}$$

Therefore the tax on consumer 1 is the negative of the externality effect their consumption of good z inflicts on consumer 2. Hence, if the good causes a negative externality ($v_2'(z^1) < 0$), the tax is positive. The converse holds if it causes a positive externality. The same construction and reasoning can be applied to the tax facing consumer 2, t_2, to show that this is the negative of the externality effect caused by the consumption of good z by consumer 2. The argument is now completed by noting that these externality effects will generally be different, and so the two taxes will generally not be equal. Another way of saying this is that efficiency can only be achieved if the consumers face personalized prices that fully capture the externalities that they generate.

So what does this say for Pigouvian taxation? Put simply, the earlier conclusion that a single tax rate could achieve efficiency was misleading. In fact the general

outcome is that there must be a different tax rate for each externality-generating good for each consumer. Achieving efficiency needs taxes to be differentiated across consumers and goods. Naturally this finding immediately shows the practical difficulties involved in implementing Pigouvian taxation. The same arguments concerning information that were placed against the Lindahl equilibrium for public good provision with personalized pricing are all relevant again here. In conclusion, Pigouvian taxation can achieve efficiency but needs an unachievable degree of differentiation.

If the required degree of differentiation is not available, for instance, information limitations require that all consumers must pay the same tax rate, then efficiency will not be achieved. In such cases the chosen taxes will have to achieve a compromise. They cannot entirely correct for the externality but can go some way toward doing so. Since the taxes do not completely offset the externality, there is also a role for intervening in the market for goods related to that causing the externality. For instance, pollution from car use may be lessened by subsidizing alternative mode of transports. These observations are meant to indicate that once the move is made from full efficiency, many new factors become relevant, and there is no clean and general answer as to how taxes should be set.

A final comment is that the effect of the tax or subsidy is to put a price (respectively positive or negative) on the externality. This leads to the conclusion, which will be discussed in detail below, that if there are competitive markets for the externalities, efficiency will be achieved. In other words, efficiency does not require intervention but only the creation of the necessary markets.

7.6 Licenses

The reason why Pigouvian taxation can raise welfare is that the unregulated market will produce incorrect quantities of externalities. The taxes alter the cost of generating an externality and, if correctly set, will ensure that the optimal quantity of externality is produced. An apparently simpler alternative is to control externalities directly by the use of licenses. This can be done by legislating that externalities can only be generated up to the quantity permitted by licenses held. The optimal quantity of externality can then be calculated and licenses totaling this quantity distributed. Permitting these licenses to be traded will ensure that they are eventually used by those who obtain the greatest benefit.

Administratively, the use of licenses has much to recommend it. As was argued in the previous section, the calculation of optimal Pigouvian taxes requires

considerable information. The tax rates will also need to be continually changed as the economic environment evolves. The use of licenses only requires information on the aggregate quantity of externality that is optimal. Licenses to this value are released and trade is permitted. Despite these apparently compelling arguments in favor of licenses, when the properties of licenses and taxes are considered in detail, the advantage of the former is not quite so clear.

The fundamental issue involved in choosing between taxes and licenses revolves around information. There are two sides to this. The first is what must be known to calculate the taxes or determine the number of licenses. The second is what is known when decisions have to be taken. For example, does the government know costs and benefits for sure when it sets taxes or issues licenses?

Taking the first of these, although licenses may appear to have an informational advantage this is not really the case. Consider what must be known to calculate the Pigouvian taxes. The construction of section 7.5 showed that taxation required the knowledge of the preferences of consumers and, if the model had included production, the production technologies of firms. Such extensive information is necessary to achieve the personalization of the taxes. But what of licenses? The essential feature of licenses is that they must total to the optimal level of externality. To determine the optimal level requires precisely the same information as is necessary for the tax rates. Consequently taxes and licenses are equivalent in their informational demands.

Now consider the issue of the information that is known when decisions must be made. When all costs and benefits are known with certainty by both the government and individual agents, licenses and taxation are equivalent in their effects. This result is easily seen by reconsidering figure 7.8. The optimal level of externality is x^o, which was shown to be achievable with tax t. The same outcome can also be achieved by issuing x^o licenses. This simple and direct argument shows there is equivalence with certainty.

In practice, it is more likely that the government must take decisions before the actual costs and benefits of an externality are known for sure. Such uncertainty brings with it the question of timing: Who chooses what and when? The natural sequence of events is the following. The government must make its policy decision (the quantity of licenses or the tax rate) before costs and benefits are known. In contrast, the economic agents can act after the costs and benefits are known. For example, in the case of pollution by a firm, the government may not know the cost of reducing pollution for sure when it sets the tax rate but the firm makes its abatement decision with full knowledge of the cost.

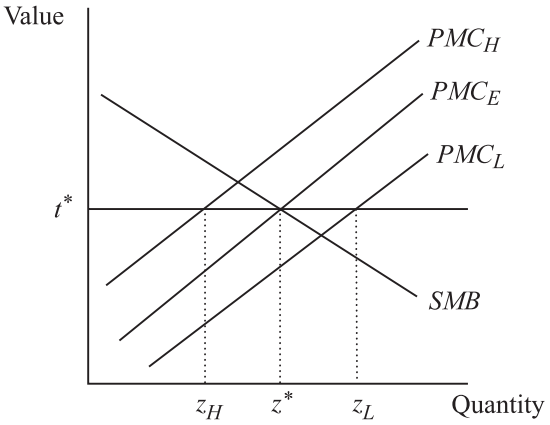


Figure 7.9
Uncertain costs

The effect of this difference in timing is to break the equivalence between the two policies. This can be seen by considering figure 7.9, which illustrates the pollution abatement problem for an uncertain level of cost. In this case the level of private marginal cost takes one of two values, PMC_L and PMC_H, with equal probability. Benefits are known for sure. When the government chooses its policy, it is not known whether private marginal cost is high or low, so it must act on the expected value, PMC_E. This leads to pollution abatement z^* being required (which can be supported by licenses equal in quantity to present pollution less z^*) or a tax rate t^*.

Under the license scheme, the level of pollution abatement will be z^* for sure—there is no uncertainty about the outcome. With the tax, the level of abatement will depend on the realized level of cost since the firm chooses after this is known. Therefore, if the cost turns out to be PMC_L, the firm will choose abatement level z_L. If its is PMC_H, abatement is z_H. This is shown in figure 7.9. Two observations emerge from this. First, the claim that licenses and taxation will not be equivalent when there is uncertainty is confirmed. Second, when cost is realized to be low, taxation leads to abatement in excess of z^*. The converse holds when cost is high.

The analysis of figure 7.9 may be taken as suggesting that licenses are better, since they do not lead to the variation in abatement that is inherent in taxation. However, it should also be realized that the choices made by the firm in the tax case are responding to the actual cost of abatement, so there is some justification

for what the firm is doing. In general, there is no simple answer to the question of which of the two policies is better.

7.7 Internalization

Consider the example of a beekeeper located next door to an orchard. The bees pollinate the trees and the trees provide food for the bees, so a positive production externality runs in both directions between the two producers. According to the theory developed above, the producers acting independently will not take account of this externality. This leads to too few bees being kept and too few trees being planted.

The externality problem could be resolved by using taxation or insisting that both producers raise their quantities. Although both these would work, there is another simpler solution. Imagine the two producers merging and forming a single firm. If they were to do so, profit maximization for the combined enterprise would naturally take into account the externality. By so doing, the inefficiency is eliminated. The method of controlling externalities by forming single units out of the parties affected is called *internalization*, and it ensures that private and social costs become the same. It works for both production and consumption externalities whether they are positive or negative.

Internalization seems a simple solution, but it is not without its difficulties. To highlight the first of these, consider an industry in which the productive activity of each firm causes an externality for the other firms in the industry. In this situation the internalization argument would suggest that the firms become a single monopolist. If this were to occur, welfare loss would then arise due to the ability of the single firm to exploit its monopoly position, and this may actually be greater than the initial loss due to the externality. Although this is obviously an extreme example, the internalization argument always implies the construction of larger economic units and a consequent increase in market power. The welfare loss due to market power then has to be offset against the gain from eliminating the effect of the externality.

The second difficulty is that the economic agents involved may simply not wish to be amalgamated into a single unit. This objection is particularly true when applied to consumption externalities. That is, if a household generates an externality for their neighbor, it is not clear that they would wish to form a single household unit, particularly if the externality is a negative one.

In summary, internalization will eliminate the consequences of an externality in a very direct manner by ensuring that private and social costs are equated. However, it is unlikely to be a practical solution when many distinct economic agents contribute separately to the total externality and it has the disadvantage of leading to increased market power.

7.8 The Coase Theorem

After identifying externalities as a source of market failure, this chapter has taken the standard approach of discussing policy remedies. In contrast to this, there has developed a line of reasoning that questions whether such intervention is necessary. The focal point for this is the Coase theorem, which suggests that economic agents may resolve externality problems themselves without the need for government intervention. This conclusion runs against the standard assessment of the consequences of externalities and explains why the Coase theorem has been of considerable interest.

The Coase theorem asserts that if the market is allowed to function freely then it will achieve an efficient allocation of resources. This claim can be stated formally as follows.

Theorem 3 (Coase theorem) In a competitive economy with complete information and zero transaction costs, the allocation of resources will be efficient and invariant with respect to legal rules of entitlement.

The legal rules of entitlement, or property rights, are of central importance to the Coase theorem. Property rights are the rules that determine ownership within the economy. For example, property rights may state that all agents are entitled to unpolluted air or the right to enjoy silence (they may also state the opposite). Property rights also determine the direction in which compensation payments will be made if a property right is violated.

The implication of the Coase theorem is that there is no need for policy intervention with regard to externalities except to ensure that property rights are clearly defined. When they are, the theorem presumes that those affected by an externality will find it in their interest to reach private agreements with those causing it to eliminate any market failure. These agreements will involve the payment of compensation to the agent whose property right is being violated. The level of

compensation will ensure that the right price emerges for the externality and a Pareto-efficient outcome will be achieved. These compensation payments can be interpreted in the same way as the personalized prices discussed in section 7.5.

As well as claiming the outcome will be efficient, the Coase theorem also asserts the equilibrium will be invariant to the how property rights are assigned. This is surprising since a natural expectation would be, for example, that the level of pollution under a polluter-pays system (i.e., giving property rights to pollutees) will be less than that under a pollutee-pays (i.e., giving property rights to the polluter). To show how the invariance argument works, consider the example of a factory that is polluting the atmosphere of a neighboring house. When the firm has the right to pollute, the householder can only reduce the pollution by paying the firm a sufficient amount of compensation to make it worthwhile to stop production or to find an alternative means of production. Let the amount of compensation the firm requires be C. Then the cost to the householder of the pollution, G, will either be greater than C, in which case they will be willing to compensate the firm and the externality will cease, or it will be less than C and the externality will be left to continue. Now consider the outcome with the polluter pays principle. The cost to the firm for stopping the externality now becomes C and the compensation required by the household is G. If C is greater than G, the firm will be willing to compensate the household and continue producing the externality; if it is less than G, it stops the externality. Considering the two cases, it can be seen the outcome is determined only by the value of G relative to C and not by the assignment of property rights, which is essentially the content of the Coase theorem.

There is a further issue before invariance can be confirmed. The change in property rights between the two cases will cause differences in the final distribution of income due to the direction of compensation payments. Invariance can only hold if this redistribution of income does not cause a change in the level of demand. This requires there to be no income effects or, to put it another way, the marginal unit of income must be spent in the same way by both parties.

When the practical relevance of the Coase theorem is considered, a number of issues arise. The first lies with the assignment of property rights in the market. With commodities defined in the usual sense, it is clear who is the purchaser and who is the supplier and, therefore, the direction in which payment should be transferred. This is not the case with externalities. For example, with air pollution it may not be clear that the polluter should pay, with the implicit recognition of the right to clean air, or whether there is a right to pollute, with clean air something

that should have to be paid for. This leaves the direction in which payment should go unclear. Without clearly specified property rights, the bargaining envisaged in the Coase theorem does not have a firm foundation: neither party would willingly accept that they were the party that should pay.

If the exchange of commodities would lead to mutually beneficial gains for two parties, the commodities will be exchanged unless the cost of doing so outweighs the benefits. Such transactions costs may arise from the need for the parties to travel to a point of exchange or from the legal costs involved in formalizing the transactions. They may also arise due to the search required to find a trading partner. Whenever they arise, transactions costs represent a hindrance to trade and, if sufficiently great, will lead to no trade at all taking place. The latter results in the economy having a missing market.

The existence of transactions costs is often seen as the most significant reason for the nonexistence of markets in externalities. To see how they can arise, consider the problem of pollution caused by car emissions. If the reasoning of the Coase theorem is applied literally, then any driver of a car must purchase pollution rights from all of the agents that are affected by the car emissions each time, and every time, that the car is used. Obviously this would take an absurd amount of organization, and since considerable time and resources would be used in the process, transactions costs would be significant. In many cases it seems likely that the welfare loss due to the waste of resources in organizing the market would outweigh any gains from having the market.

When external effects are traded, there will generally only be one agent on each side of the market. This thinness of the market undermines the assumption of competitive behavior needed to support the efficiency hypothesis. In such circumstances the Coase theorem has been interpreted as implying that bargaining between the two agents will take place over compensation for external effects and that this bargaining will lead to an efficient outcome. Such a claim requires substantiation.

Bargaining can be interpreted as taking the form of either a cooperative game between agents or as a noncooperative game. When it is viewed as cooperative, the tradition since Nash has been to adopt a set of axioms that the bargain must satisfy and to derive the outcomes that satisfy these axioms. The requirement of Pareto-efficiency is always adopted as one of the axioms so that the bargained agreement is necessarily efficient. If all bargains over compensation payments were placed in front of an external arbitrator, then the Nash bargaining solution would have some force as descriptive of what such an arbitrator should try and

achieve. However, this is not what is envisaged in the Coase theorem, which focuses on the actions of markets free of any regulation. Although appealing as a method for achieving an outcome agreeable to both parties, the fact that Nash bargaining solution is efficient does not demonstrate the correctness of the Coase theorem.

The literature on bargaining in a noncooperative context is best divided between games with complete information and those with incomplete information, since this distinction is of crucial importance for the outcome. One of the central results of noncooperative bargaining with complete information is due to Rubinstein who considers the division of a single object between two players. The game is similar to the fund-raising game presented in the public goods chapter. The players take it in turns to announce a division of the object, and each period an offer and an acceptance or rejection are made. Both players discount the future, so they are impatient to arrive at an agreed division. Rubinstein shows that the game has a unique (subgame perfect) equilibrium with agreement reached in the first period. The outcome is Pareto-efficient.

The important point is the complete information assumed in this representation of bargaining. The importance of information for the nature of outcomes will be extensively analyzed in chapter 9, and it is equally important for bargaining. In the simple bargaining problem of Rubinstein the information that must be known are the preferences of the two agents, captured by their rates of time discount. When these discount rates are private information, the attractive properties of the complete information bargain are lost, and there are many potential equilibria whose nature is dependent on the precise specification of the structure of bargaining.

In the context of externalities it seems reasonable to assume that information will be incomplete, since there is no reason why the agents involved in bargaining an agreement over compensation for an external effect should be aware of each other's valuations of the externality. When they are not aware, there is always the incentive to try to exploit a supposedly weak opponent or to pretend to be strong and make excessive demands. This results in the possibility that agreement may not occur even when it is in the interests of both parties to trade.

To see this most clearly, consider the following bargaining situation. There are two agents: a polluter and a pollutee. They bargain over the decision to allow or not the pollution. The pollutee cannot observe the benefit of pollution B but knows that it is drawn from a distribution $F(B)$, which is the probability that the benefit is less or equal to B. On the other hand, the polluter cannot

observe the cost of pollution C but knows that it is drawn from a distribution $G(C)$. Obviously the benefit is known to the polluter and the cost is known to the pollutee. Let us give the property rights to the pollutee so that he has the right to a pollution-free environment. Pareto-efficiency requires that pollution be allowed whenever $B \geq C$. Now the pollutee (with all the bargaining power) can make a take-it-or-leave-it offer to the polluter. What will be the bargaining outcome?

The pollutee will ask for compensation $T > 0$ (since $C > 0$) to grant permission to pollute. The polluter will only accept to pay T if his benefit from polluting exceeds the compensation he has to pay, so $B \geq T$. Hence the probability that the polluter will accept the offer is equal to $1 - F(T)$, that is, the probability that $B \geq T$. The best deal for the pollutee is to ask for compensation that maximizes her expected payoff defined as the probability that the offer is accepted times the net gain if the offer is accepted. Therefore the pollutee asks for compensation T^*, which solves

$$\max_{\{T\}}[1 - F(T)][T - C]. \tag{7.21}$$

Clearly, the optimal value, T^*, is such that

$$T^* > C. \tag{7.22}$$

But then bargaining can result (with strictly positive probability) in an inefficient outcome. This is the case for all realizations of C and B such that $C < B < T^*$, which implies that the offer is rejected (since the compensation demanded exceeds the benefit) and thus pollution is not allowed, while Pareto-efficiency requires permission to pollute to be granted (since its cost is less than its benefit).

The efficiency thesis of the Coase theorem relies on agreements being reached on the compensation required for external effects. The results above suggest that when information is incomplete, bargaining between agents will not lead to an efficient outcome.

7.9 Nonconvexity

One of the basic assumptions that supports economic analysis is that of convexity. Convexity gives indifference curves their standard shape, so consumers always prefer mixtures to extremes. It also ensures that firms have non-increasing returns so that profit-maximization is well defined. Without convexity, many problems

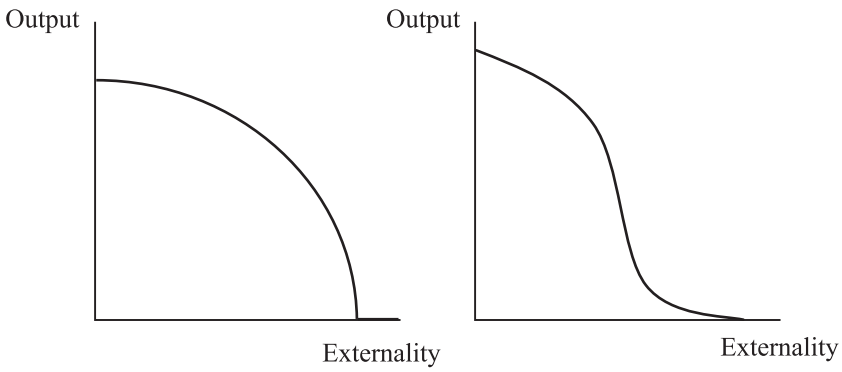


Figure 7.10
Nonconvexity

arise with the behavior of the decisions of individual firms and consumers, and with the aggregation of these decisions to find an equilibrium for the economy.

Externalities can be a source of nonconvexity. Consider the case of a negative production externality. The left-hand part of figure 7.10 displays a firm whose output is driven to zero by an externality regardless of the level of other inputs. An example would be a fishery where sufficient pollution of the fishing ground by another firm can kill all the fish. In the right-hand part of the figure a zero output level is not reached but output tends to zero as the level of the externality is increased. In both situations the production set of the firm is not convex.

In either case the economy will fail to have an equilibrium if personalized taxes are employed in an attempt to correct the externality. Suppose that the firm were to receive a subsidy for accepting externalities. Its profit-maximizing choice would be to produce an output level of zero and to offer to accept an arbitrarily large quantity of externalities. Since its output is zero, the externalities can do it no further harm, so this plan will lead to unlimited profits. If the price for accepting externalities were zero, the same firm would not accept any. The demand for externalities is therefore discontinuous, and an equilibrium need not exist.

There is also a second reason for nonconvexity with externalities. It is often assumed that once all inputs are properly accounted for, all firms will have constant returns to scale, since behavior can always be replicated. That is, if a fixed set of inputs (i.e., a factory and staff) produce output y, doubling all those inputs must produce output $2y$, since they can be split into two identical subunits (e.g., two factories and staff) producing an amount y each. Now consider a firm subject

to a negative externality, and assume that it has constant returns to all inputs including the externality. From the perspective of society, there are constant returns to scale. Now let the firm double all its inputs but with the externality held at a constant level. Since the externality is a negative one, it becomes diluted by the increase in other inputs, and output must more than double. The firm therefore faces private increasing returns to scale. With such increasing returns, the firm's profit-maximizing decision may not have a well-defined finite solution and market equilibrium may again fail to exist.

These arguments provide some fairly powerful reasons why an economy with externalities may not share some of the desirable properties of economies without. The behavior that follows from nonconvexity can prevent some of the pricing tools that are designed to attain efficiency from functioning in a satisfactory manner. At worst, nonconvexity can even cause there to be no equilibrium in the economy.

7.10 Conclusions

Externalities are an important feature of economic activity. They can arise at a local level between neighbors and at a global level between countries. The existence of externalities can lead to inefficiency if no attempt is made to control their level. The Coase theorem suggests that well-defined property rights will be sufficient to ensure that private agreements can resolve the externality problem. In practice, property rights are not well defined in many cases of externality. Furthermore the thinness of the market and the incomplete information of market participants result in inefficiencies that undermine the Coase theorem.

The simplest policy solution to the externality problem is a system of corrective Pigouvian taxes. If the tax rate is proportional to the marginal damage (or benefit) caused by the externality then efficiency will result. However, for this argument to apply when there are many consumers and firms requires that the taxes are so differentiated between economic agents that they become equivalent to a system of personalized prices. The optimal system then becomes impractical due to its information limitations. An alternative policy response is the use of marketable licenses that limit the emission of externalities. These have some administrative advantages over taxes and will produce the same outcome when costs and benefits are known with certainty. With uncertainty, licenses and taxes have different effects and combining the two can lead to a superior outcome.

Further Reading

The classic analysis of externalities is in:

Meade, J. E. 1952. External economies and diseconomies in a competitive situation. *Economic Journal* 62: 54–76.

The externality analysis is carried further in a more rigorous and complete analysis in:

Buchanan, J. M. and Stubblebine, C. 1962. Externality. *Economica* 29: 371–84.

A persuasive argument for the use of corrective taxes is in:

Pigou, A. C. 1918. *The Economics of Welfare*. London: Macmillan.

The problem of social cost and the bargaining solution with many legal examples is developed in:

Coase, R. H. 1960. The problem of social cost. *Journal of Law and Economics* 3: 1–44.

An illuminating classification of externalities and non-market interdependences is in:

Bator, F. M. 1958. The anatomy of market failure. *Quarterly Journal of Economics* 72: 351–78.

A comprehensive and detailed treatment of the theory of externalities can be found in:

Lin, S., ed. 1976. *Theory and Measurement of Economic Externalities*. New York: Academic Press.

The efficient noncooperative bargaining solution with perfect information is in:

Rubinstein, A. 1982. Perfect equilibrium in a bargaining model. *Econometrica* 50: 97–110.

The general theory of bargaining with complete and incomplete information and many applications is in:

Muthoo, A. 1999. *Bargaining Theory with Applications*. Cambridge: Cambridge University Press.

An extremely simple exposition of the conflict between individual motives and collective efficiency is in:

Schelling, T. 1978. *Micromotives and Macrobehavior*. New York: Norton.

The bandwagon effect and technology adoption is in:

Arthur, B. 1988. Self-reinforcing mechanisms in economics. In P. Anderson, K. Arrow, and D. Pines, eds., *The Economy as an Evolving Complex System*. New York: Addison-Wesley.

David, P. 1985. Clio and the economics of Qwerty. *American Economic Review* 75: 332–37.

A summary of the arguments on the Tragedy of the Commons appears first in:

Hardin, G. 1968. The Tragedy of the Commons. *Science* 162: 1243–48.

The nonconvexity problem with externalities was first pointed out in:

Starrett, D. 1972. Fundamental non-convexities in the theory of externalities. *Journal of Economic Theory* 4: 180–99.

Exercises

7.1. "Smoke from a factory dirties the local housing and poisons crops." Identify the nature of the externalities in this statement.

7.2. How would you describe the production function of a laundry polluted by a factory?

7.3. Let $U = [x_1]^{\alpha}[x_2 y]^{1-\alpha}$, where y is an externality. Is this externality positive or negative? How does it affect the demand for good 1 relative to the demand for good 2?

7.4. If the two consumers in the economy have preferences $U_1 = [x_1^1]^{\alpha}[x_2^1 x_1^2]^{1-\alpha}$ and $U_2 = [x_1^2]^{\alpha}[x_2^2 x_1^1]^{1-\alpha}$, show that the equilibrium is efficient despite the externality. Explain this conclusion.

7.5. Consider a group of n students. Suppose that each student i puts in h_i hours of work on her classes that involves a disutility of $\frac{h_i^2}{2}$. Her benefits depend on how she performs relative to her peers and take the form $u\left(\frac{h_i}{\bar{h}}\right)$ for all i, where $\bar{h} = \left(\frac{1}{n}\right)\sum_i h_i$ denotes the average number of hours put in by all students in the class and $u(\cdot)$ is an increasing and concave function.

a. Calculate the symmetric Nash equilibrium.

b. Calculate the Pareto-efficient level of effort.

c. Explain why the equilibrium involves too much effort compared to the Pareto-efficient outcome.

7.6. There is a large number of commuters who decide to use either their car or the tube. Commuting by train takes 70 minutes whatever the number of commuters taking the train. Commuting by car takes $C(x) = 20 + 60x$ minutes, where x is the proportion of commuters taking their cars, $0 \leq x \leq 1$.

a. Plot the curves of the commuting time by car and the commuting time by train as a function of the proportion of car users.

b. What is the proportion of commuters who will take their car if everyone is taking her decision freely and independently so as to minimize her own commuting time?

c. What is the proportion of car users that minimizes the total commuting time?

d. Compare this with your answer given in part b. Interpret the difference. How large is the deadweight loss from the externality?

e. Explain how a toll could achieve the efficient allocation of commuters between train and car and be beneficial for everyone.

7.7. Re-do the previous problem by replacing the train by a bus and assuming that commuting time by bus is increasing with the proportion of commuters using car (traffic congestion). Let the commuting time by bus be $B(x) = 40 + 20x$ and the commuting time by car be $C(x) = 20 + 60x$, where x is the proportion of commuters taking their car, $0 \leq x \leq 1$.

7.8. Consider a binary choice to allow or not the emission of pollutants. The cost to consumers of allowing the pollution is $C = 2{,}000$, but this cost is only observable to the consumers. The benefit for the polluter of allowing the externality is $B = 2{,}300$, and only the polluter knows this benefit. Clearly, optimality requires this externality is

allowed, since $B > C$. However, the final decision must be based on what each party chooses to reveal.

a. Construct a tax-subsidy revelation scheme such that it is a dominant strategy for each party to report truthfully their private information.

b. Show that this revelation scheme induces the optimal production of the externality.

c. Show that this revelation scheme is unbalanced in the sense that the given equilibrium reports the tax to be paid by the polluter is less than the subsidy paid to the pollutee.

7.9. How can licenses be used to resolve the Tragedy of the Commons?

7.10. If insufficient abatement is very costly, which of taxation or licenses is preferable?

7.11. Are the following statements true or false? Explain why.

a. If your consumption of cigarettes produces negative externalities for your partner (which you ignore), then you are consuming more cigarettes than is Pareto-efficient.

b. It is generally efficient to set an emission standard allowing zero pollution.

c. A tax on cigarettes induces the market for cigarettes to perform more efficiently.

d. A ban on smoking is necessarily efficient.

e. A competitive market with a negative externality produces more output than is efficient.

f. A snob effect is a negative (network) externality from consumption.

7.12. Consider two consumers with utility functions

$$U^A = \log(x_1^A) + x_2^A - \left[\tfrac{1}{2}\right] \log(x_1^B), \quad U^B = \log(x_1^B) + x_2^B - \left[\tfrac{1}{2}\right] \log(x_1^A).$$

Both consumers have income M and the (before-tax) price of both goods is 1.

a. Calculate the market equilibrium.

b. Calculate the social optimum for a utilitarian social welfare function.

c. Show that the optimum can be sustained by a tax placed on good 1 (so the after-tax price becomes $1 + t$) with the revenue returned equally to the consumers in a lump-sum manner.

d. Assume now that preferences are given by

$$U^A = \rho^A \log(x_1^A) + x_2^A - \left[\tfrac{1}{2}\right] \log(x_1^B), \quad U^B = \rho^B \log(x_1^B) + x_2^B - \left[\tfrac{1}{2}\right] \log(x_1^A).$$

Calculate the taxes necessary to decentralize the optimum.

e. For preferences of part d and income $M = 20$, contrast the outcome when taxes can and cannot be differentiated between consumers.

7.13. A competitive refining industry releases one unit of waste into the atmosphere for each unit of refined product. The inverse demand function for the refined product is $p^d = 20 - q$, which represents the marginal benefit curve where q is the quantity consumed when the consumers pay price p^d. The inverse supply curve for refining is $MPC = 2 + q$, which represents the marginal private cost curve when the industry produces q units. The marginal external cost curve is $MEC = 0.5q$, where MEC is the marginal external cost when the industry releases q units of waste. Marginal social cost is given by $MSC = MPC + MEC$.

a. What are the equilibrium price and quantity for the refined product when there is no correction for the externality?

b. How much of the chemical should the market supply at the social optimum?

c. How large is the deadweight loss from the externality?

d. Suppose that the government imposes an emission fee of T per unit of emissions. How large must the emission fee be if the market is to produce the socially efficient amount of the refined product?

7.14. Discuss the following statement: "A tax is a fine for doing something right. A fine is a tax for doing something wrong."

7.15. Suppose that the government issues tradable pollution permits.

a. Is it better for economic efficiency to distribute the permits among polluters or to auction them?

b. If the government decides to distribute the permits, does the allocation of permits among firms matter for economic efficiency?

7.16. A chemical producer dumps toxic waste into a river. The waste reduces the population of fish, reducing profits for the local fishery industry by \$150,000 per year. The firm could eliminate the waste at a cost of \$100,000 per year. The local fishing industry consists of many small firms.

a. Apply the Coase theorem to explain how costless bargaining will lead to a socially efficient outcome, no matter to whom property rights are assigned (either to the chemical firm or the fishing industry).

b. Verify the Coase theorem if the cost of eliminating the waste is doubled to \$200,000 (with the benefit for the fishing industry unchanged at \$150,000).

c. Discuss the following argument: "A community held together by ties of obligation and mutual interest can manage the local pollution problems."

d. Why might bargaining not be costless?

7.17. It is often used as an objection to market-based policies of pollution abatement that they place a monetary value on cleaning up our environment. Economists reply that society implicitly places a monetary value on environmental cleanup even under command-and-control policies. Explain why this is true.

7.18. Use examples to answer whether the externalities related to common resources are generally positive or negative. Is the free-market use of common resources greater or less than the socially optimal use?

7.19. Why is there more litter along highways than in people's yards?

7.20. Evaluate the following statement: "Since pollution is bad, it would be socially optimal to prohibit the use of any production process that creates pollution."

7.21. Why is it not generally efficient to set an emissions standard allowing zero pollution?

7.22. Education is often viewed as a good with positive externalities.

a. Explain how education might produce positive external effects.

b. Suggest a possible action of the government to induce the market for education to perform more efficiently.