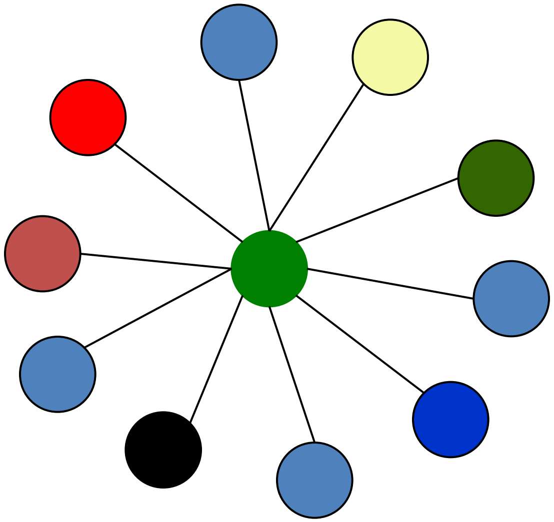
Economics of Networks I



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1. Introduction

- Economic activity takes place at the intersection of the global and the local
  - large firms, big government, and international markets
  - small and overlapping neighborhoods (community, friends and family, partnerships, suppliers, alliances).
- Game theory is well suited for the study of small exclusive groups, general equilibrium theory appropriate for large anonymous systems.
- Networks: span the large and details of the small, within a common framework.
- Important in the tool-kit of economists.

Introduction

- Language/concepts from graph theory, matrix/linear algebra.
- Two overarching themes:
  1. networks shapes human behavior
  2. individuals form links and create networks
- Through 1990's, focus on theoretical models.
- Last decade: tremendous growth in applied and empirical research.
- Close analogy with game theory in the 1980's and 1990's.
- Aim: Introduction to themes and methods

Theme 1: Networks shape behavior

- Two ingredients: the structure of connections and individual's information, actions and rewards.
- *Network:* A network g consists of a collection of nodes $N = \{1, 2, ...n\}$ with $n \geq 2$, and the links (g_{ij}), $i, j \in N$, between them. It is denoted by g.
- Undirected links (friendship, research collaboration, defence alliance).
- Directed links (investments, citation, loans, hyperlink, following a tweet).

Networks shape behavior

- Individuals located on nodes of a graph.
- A is a *neighbor* of B if they have a direct link.
- Externalities: positive and negative. Local and global.
- Strategic Structure: Complements and substitutes.
- Effects may differ between neighbors and non-neighbors.

Theme 2: Network Formation

- Significant network effects motivate a study of origins of network.
- Economic approach: individuals/firms/nations compare the costs and benefits of forming links.
- Links create externalities.

Network Formation

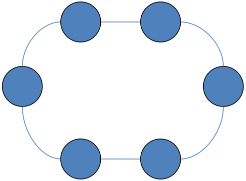
- *Two-sided or bilateral linking:* A link between two players requires approval of both players.
  - Solution concept pairwise stability.
- *Unilateral linking:* an individual chooses links with others on his/her own.
  - Network formation as a noncooperative game.

Nodes and links

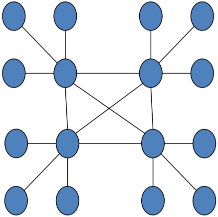
- Set of nodes, $N = \{1, 2, , 3, ...n\}$, where $n \geq 2$.
- Denote by $g_{ij} \in \mathbb{R}_+$ a relationship between two nodes i and j.
- Nodes and links defines a network g.
- $N_i(g) = \{j | g_{ij} > 0\}$ is the *neighbors* of i.
- Binary link $g_{ij} \in \{0, 1\}$; $\eta_i(g) = |N_i(g)|$ is degree of i.

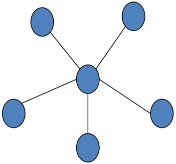
Small Networks

Ring



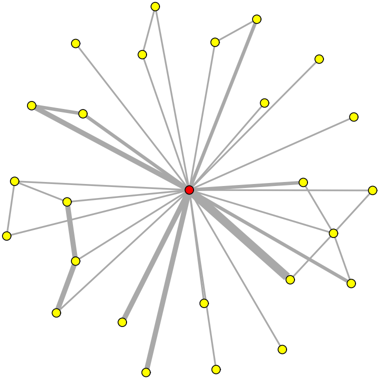
Core-periphery network





Star Network

Weighted Networks



The study of large Network

- Networks with thousands or millions of nodes: World wide web or the diffusion of ideas in a community of scholars.
- Rely on aggregate statistics: e.g., distribution of degree, distance, centrality.

Laptop supply chain

2. Games on Networks

- Individuals located on nodes of a network. Choose actions and their rewards depend on these actions along with the actions of others on the network.
  - The effect of player 1's action on player 2's payoff depends on where the two players are located in a network.

Games on Networks

- Two basic building blocks: one, formal description of the pattern of relationships among individual entities and two, the externalities that an individual's actions create for other individuals. We ask:
  1. What are the effects of network location on individual behavior?
  2. How can external actors exploit networks to influence behavior?

Games on Networks: Background

- Pure local effects:
  - Local public goods: game of substitutes, Bramoulle and Kranton, 2007
  - Schooling and crime: game of complements, Ballester, Calvo-Armengol and Zenou, 2006.
- Local interactions and markets:
  - Coppetition: Firms collaborate with partners but compete in market, Goyal and Moraga, 2001.
  - Status seeking: individuals compare with neighbors, Goyal and Ghiglino, 2010.
- Games on large random graphs: Galeottti, Goyal, Jackson, Fernando-Vega, Yariv, 2010.

Production and Exchange

- In standard Walrasian model, individuals are anonymous, trade with everyone and at common price.
- Terms of trade often differ and are not uniform.
- What is the relation between networks and pricing, allocation of surplus and aggregate efficiency? What types of networks will be formed?

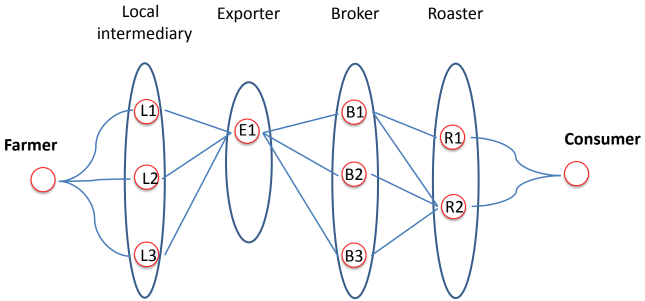
Production and Exchange

- Given a network nodes determine prices.
- Pricing protocol: posted prices, bargaining, auctions.
- Network and pricing protocol defines a game on a network.
- Study equilibrium of this game.
- Early work focused on buyer-seller graphs: Kirman (1988), Kranton and Minehart (2001), Corominas-Bosch (2006), Lever-Guzman (2011).
- Recent work on intermediation: focus of my talk.

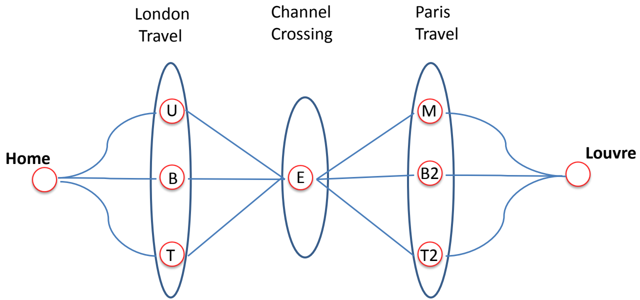
Intermediaries: A Network Approach

- Supply, service and trading chains are a defining feature of the modern economy. In agriculture, in transport and communication, in international trade, in markets for bribes, and in finance.
- The routing of economic activity, the earnings of individuals and resilience of economy depend on them.
- Examples: laptop, commodities and transport.

Coffee supply chain



Transport Network: London to Paris



Posted prices in networks
Choi, Galeotti and Goyal, 2017

- There is a source $\mathcal{S}$ and a destination $\mathcal{D}$.
- A path between the two is a sequence of interconnected nodes.
- The passage from source to destination generates *value*, 1.
- Intermediaries simultaneously *post a price*; the prices determine the cost for every path between $\mathcal{S}$ and $\mathcal{D}$.
- The tourist moves along a least cost path.
- We have defined a game in posted prices on a network.

Pricing in a network
Choi, Galeotti and Goyal, 2017

- There is a source $\mathcal{S}$ and a destination $\mathcal{D}$.
- There are n traders located in a network g that connects $\mathcal{S}$ and $\mathcal{D}$.
- Traders simultaneously post prices: cost of a 'path' between s and b is sum of prices of traders on the path.
- The surplus between b and s is 1.
- Pick cheapest path if it is less than 1 (randomize).
- Seller and buyer split residual surplus equally.

Benchmark Model
Choi, Galeotti and Goyal 2017

- Given g and p, let $\mathcal{Q}^* = \{q \in \mathcal{Q} : c(q,p) = c(p), c(p) \leq 1\}$ be the set of feasible least cost paths.
- The expected payoff to intermediary is

$$\Pi_i(\mathbf{p}) = \begin{cases} 0 & \text{if } i \notin q \text{ for all } q \in \mathcal{Q}^* \text{ or } c^*(\mathbf{p}) > 1 \\ \frac{\eta_i}{|\mathcal{Q}^*|} p_i & \text{otherwise,} \end{cases} \tag{1}$$

where η_i is the number of paths in $\mathcal{Q}^*$ that contain trader i.

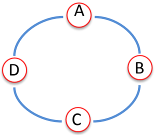
Price formation: Examples

- Network with two paths each has a single node: Bertrand competition, price equal to 0.
- Line Network: Nash Bargaining. Variety of possible outcomes.
- Rich Strategic Structure: prices on same path are substitutes, prices on distinct paths are complements.

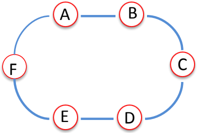
Market Power and Critical Nodes

- A node is said to be *critical* if it lies on all paths between $\mathcal{S}$ and $\mathcal{D}$.
- Choi et al (2014 show that existence of critical node is sufficient for full extraction by intermediaries.
- There are however multiple equilibrium and in some non-critical traders make large profits.
- Theory and experiments: *The existence of critical nodes is necessary and sufficient for extraction of surplus by intermediaries.*

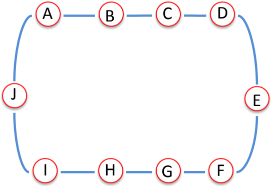
Experiment: Rings and Ring with Hubs



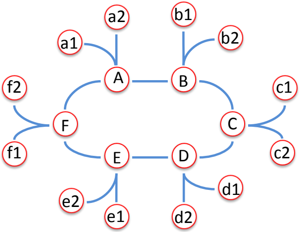
RING 4



RING 6



RING 10



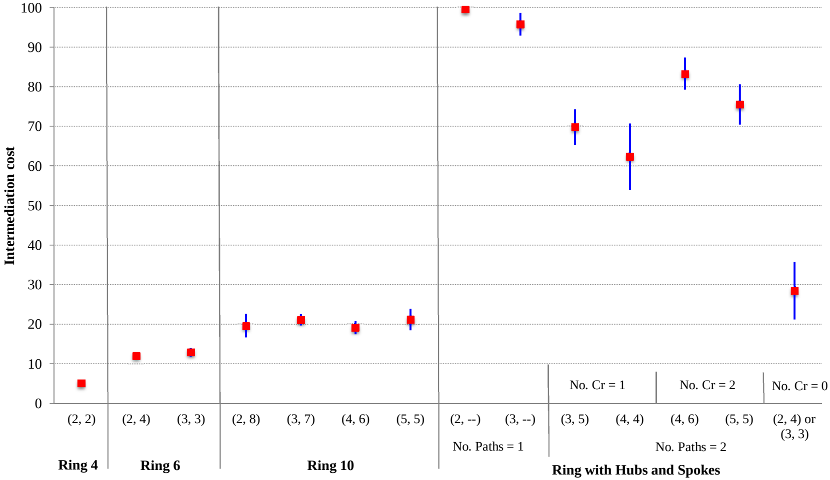
RING with HUBS & SPOKES

Finding 1: Efficiency is remarkably high in all networks

Network	minimum distance of buyer-sell pair				
	All ($\geq$ 2)	2	3	4	5
Ring 4	1.00 (480)	1.00 (480)	--	--	--
Ring 6	1.00 (420)	1.00 (289)	1.00 (131)	--	--
Ring 10	1.00 (240)	1.00 (49)	1.00 (87)	1.00 (69)	1.00 (35)
Ring with Hubs and Spokes	0.95 (420)	1.00 (126)	0.94 (155)	0.90 (109)	0.90 (30)

Note. The number of group observations is reported in parentheses.

Finding 2: Distribution of surplus is extremal



Finding 3: Criticality yields large payoffs

Network	(#Cr,#Paths, d(q),d(q'))	Rounds		
		1 ~ 20	21 ~ 41	41 ~ 60
Ring with Hubs and Spokes	(1, 2, 3, 5)	0.56 (20)	0.68 (26)	0.72 (25)
	(1, 2, 4, 4)	0.48 (16)	0.56 (13)	0.67 (10)
	(2, 2, 4, 6)	0.73 (16)	0.77 (19)	0.80 (24)
	(2, 2, 5, 5)	0.65 (8)	0.67 (8)	0.74 (11)

Notes. The number in a cell is the average fraction of costs charged by critical traders. The number of observations is reported in parentheses. #Cr denotes the number of critical intermediaries, #Paths denotes the number of paths connecting buyer and seller, d(q) denotes the length of path q beween buyer and seller.

General observations

- Result is sharp, but ‘criticality’ is too extreme.
  - Node lying on most paths is same as node lying on only one path.
  - All critical paths have equal status. Upstream/downstream?
  - Full information on value; discontinuous demand.
- Very active field of research: bargaining and auctions. Kotowski and Leister (2015)), Condorelli and Galeotti (2016), Gofman (2011), Manea (2017), Acemoglu and Ozdagler (2007), Blume et al. (2007) and Gale and Kariv (2009).

Intermediaries: Network formation

- Key idea: Given the potentially large rewards of being critical, firms and individuals will make investments in connections to make themselves critical.
- However, these efforts will face counter-efforts from other nodes who would not like to keep intermediation rents down.
- What is the outcome of these pressures?

3. Network formation

- The strategic aspect of link formation arises from the observation that links between a pair of individuals influences the payoffs of others, i.e., generates externalities.
- A game of network formation specifies a set of players, the link formation actions available to each player and the payoffs to each player from the networks that arise out of individual linking decisions.
- Network formation is a complicated problem. How should payoffs be allocated across nodes, who should decide on links?

Network formation: Background

- The origins of an economic approach Boorman (1975), Aumann and Myerson (1988) and Myerson (1991).
- Boorman (1975): individuals allocate time to links. Larger resources on a link make it stronger. But more links of others means lower probability of receiving job information. Thus linking creates externalities.
- Group formation a central concern in economics: traditional approach of *coalitions*
- Systematic approach to network formation: Bala and Goyal (2000) and Jackson and Wolinsky (1996).
- For a survey that covers coalitions and networks, Bloch and Dutta (2011).

Forming links to becoming Critical

- Consider n individuals; every pair has a value 1.
- Two linked individuals split surplus equally. If they are linked via others then the division of surplus depends on the competition between these 'intermediaries'.
- There are three forces:
  1. individuals form links to have a path for trade
  2. form links to become critical for trade between others
  3. individuals circumvent intermediation through direct links.

Network formation
Goyal and Vega-Redondo, 2007

- Players announce intention to form links.
- A link formed costs $c > 0$ to each player.
- For $k \in N_i(g)$, define $C(j,k;g)$ as the set of critical players for j and k and let $c(j,k;g) = |C(j,k;g)|$.
- Payoffs of i are given by:

$$\Pi_i(s_i, s_{-i}) = \sum_{j \in N_i(g)} \frac{1}{c(i,j;g)+2} + \sum_{j,k \in N} \frac{I_{\{i \in C(j,k)\}}}{c(j,k;g)+2} - \eta_i^d(g)c,$$

where $I_{\{i \in E(j,k)\}} \in \{0,1\}$ is indicator function and $\eta_i^d(g)$ is the number of links of i.

Definition: Pairwise Stable Network

- A network g is pairwise stable if
  - no individual has an incentives to delete any link
  - no pair of individuals wishes to form an additional link

A star is pairwise stable

- A star is pairwise stable so long as $1/6 < c < 1/2 + (n-2)/3$.
  - Center earns a payoff of $(n-1)[1/2 + ((n-2)/6) - c]$; has no incentive to delete a single link so long as $c < 1/2 + (n-2)/3$.
  - Two spokes have no incentive to form a link between them if $c > 1/6$,
  - No spoke has an incentive to delete a link if $c < 1/2 + (n-2)/3$.

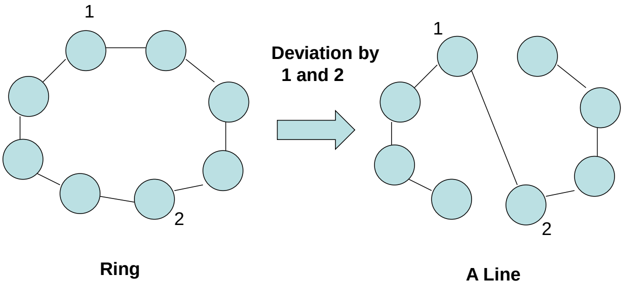
Cycle is pairwise stable

- Every player gets a payoff of $(n-1)/2 - 2c$.
- An additional link is clearly not profitable: it does not create any extra surplus while it increases costs.
- Deletion of one link not profitable: makes a neighbor critical for all transactions, lowers payoffs by at least $(n-2)/6$.
- This clearly exceeds the cost c, for large enough n.

Coordinated Deviations

- Two players choosing to add a link between themselves and delete a subset of links with others.
- Two players that far apart in the cycle, establish a direct link and simultaneously break one link each, they can produce a line and become central in it.

Example: Bilateral deviations away from cycle



Definition: Bilateral Equilibrium

A network g^* can be supported in a *bilateral equilibrium (SBE)* if the following conditions hold:

- There exists a strategy profile s^* which supports g^* as a bilateral equilibrium.
- For any $i \in N$, and every $s_i \in S_i$ such that $g(s_i, s^*_{-i}) \neq g(s^*)$:

$$\Pi_i(g(s^*)) > \Pi_i(g(s_i, s^*_{-i}))$$

- For any pair of players, $i, j \in N$ and every strategy pair (s_i, s_j) with $g(s_i, s_j, \mathbf{s}^*_{-i-j}) \neq g(\mathbf{s}^*)$,

$$\Pi_i(g(s_i, s_j, s^*_{-i-j)}) \geq \Pi_i(g(s^*_i, s^*_j, s^*_{-i-j}))$$
$$\Rightarrow \Pi_j(g(s_i, s_j, s^*_{-i-j})) < \Pi_j(g(s^*_j, s^*_j, s^*_{-i-j})).$$

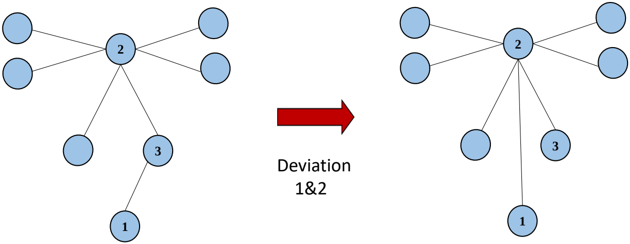
Theorem: Sustaining Structural Holes

The hub-spoke/star is the unique (non-empty) stable network.

Arguments in Proof

- One: Exploits access and intermediation advantages to show that an equilibrium network is either connected or empty.
- Two: Agglomeration pressures: a minimal network with long paths cannot be sustained. This is because players located at the ‘end’ of the network benefit from connecting to a *central player* in order to save on intermediation costs (cutting path lengths) while a central player is ready to incur the cost of an additional link because this enhances her intermediation payoffs because she shares the intermediation rents with fewer other players.

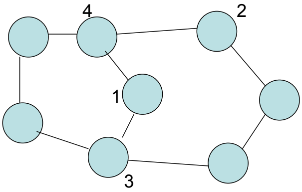
Agglomeration pressures



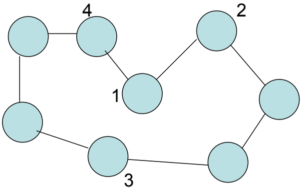
Arguments

- Three: show that a cycle or a hybrid cycle-star network is not sustainable.
- Four: rules out networks with two or more cycles.

Pressure toward a single cycle



Cycles with common players



Players 1 and 2 deviate and retain old payoffs

Inequality in networks

- The payoffs of hub player are:

$$(n-1)[1/2+((n-2)/6)-c]$$

- The spoke payoffs are:

$$[1/2+((n-2)/3-c]$$

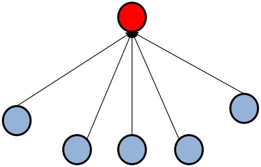
- The ratio is unbounded in n

Financial Intermediation

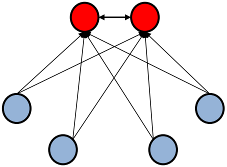
- Following the financial crises of 2008: renewed interest in financial contagion.
- Finding *Empirical networks exhibit a core-periphery structure: core of densely connected large banks and many small banks at the periphery.*
- Bech and Atalay (2010), Afonso and Lagos (2012), Van Lelyveld I., and t' Veld (2012).
- How can we account for such structures and what are their welfare properties?

Core-periphery Network

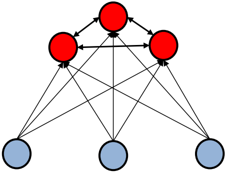
Periphery
Sponsored Star



Core-Periphery
(2 hubs)



Core-Periphery
(3 hubs)



Intermediaries: Heterogeneity and Rents

- Veld, van der Leij and Hommes (2014) extend network formation model: smoother competition between paths.
- Proposition: *With bank size heterogeneity core-periphery network is stable. The higher value banks constitute the core.*
- The model predicts core-periphery structure in the Dutch interbank market for reasonable parameter values.
- Farboodi (2014): heterogeneity in functions of bank.
- Proposition: *Rent seeking leads to core-periphery structure and excessive risk taking.*
- Related work: Acemoglu, Ozdaglar and Tahbaz-Salehi (2015), Cabrales, Gottardi and Vega-Redondo (2012).

General Remarks

- Gap in economics:
- Connections shape Behavior
  - Topology and content of interaction
  - Network statistics
- Individuals create networks
  - stable networks
  - welfare and inequality

References

- Goyal (2007), *Connections.*
- Goyal (2017), Networks and Markets in *Advances in Economics: Eleventh World Congress of Econometric Society.*

Related Material

- Bala and Goyal (2000), A non-cooperative model of network formation. *Econometrica*.
- Ballester, Calvo-Armengol, and Zenou (2006), Who's who in networks. Wanted: The Key Player, *Econometrica*.
- Bloch and Dutta (2011), Strategic Models of Network and Coalition Formation, in *Handbook of Social Economics*, edited by Benhabib, Bisin, Jackson.
- Bramoullé and Kranton (2007), Local public goods in Networks, *JET*.
- Choi, Galeotti and Goyal (2017), Trading in Networks: theory and experiments, *JEEA*.

References

- Galeotti, Goyal, Jackson, Vega-Redondo and Yariv (2010), Network Games, *Review of Economic Studies*.
- Galeotti and Goyal 2010, Law of the few. *American Economic Review*.
- Ghiglino and Goyal (2010), Keeping up with the neighbors, *JEEA*.
- Goyal and Moraga-Gonzalez (2001), R&D Networks, *Rand Journal*.
- Goyal and Vega-Redondo (2007), Structural Holes in Social Networks, *JET*.
- Jackson and Wolinsky (1996) A strategic model of social and economic networks, *JET*.
- Manea (2016), Models of Bilateral Trade in Networks, in *The Oxford Handbook of the Economics of Networks*, edited by Bramoulle, Galeotti and Rogers.