Topic: Sovereign Debt
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Empirical Facts

- Default happens with regularity throughout history
  - Some countries "graduate" but rare...
- Default often occurs in bad times, but with exceptions
  - Coincide with financial crisis
  - Capital flight
- Defaults involve a heterogeneous pattern of haircuts
  - Difference in promised payments between old and new bond offerings in exchange.
  - Losses of 30-40% on average. (1990s, 2000s)
  - Haircut increases with the size of debt at the time of default (at the extreme)

Empirical Facts

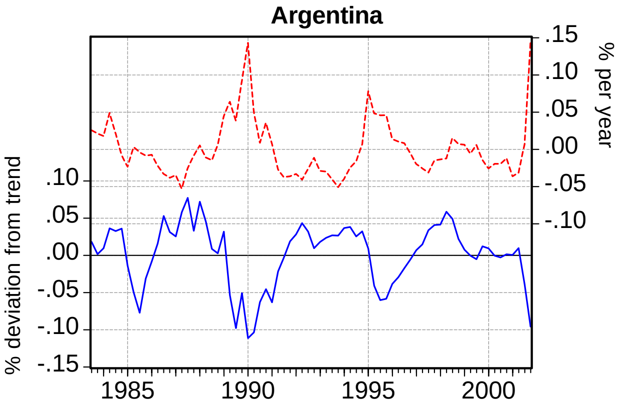
- Default generates a period of lengthy renegotiation
  - Bank-debt and bond renegotiations from 1989 through 2005.
  - Restructurings are a time-consuming process, taking eight years on average.
- Sovereign bond spreads
  - Emerging market bond yields from 1990 to 2009.
  - During crisis the yield curve “inverts”.
  - maturity of newly issued bonds shorten during crises.
  - emerging market bond yields exhibit significant co-movement.

Empirical Facts

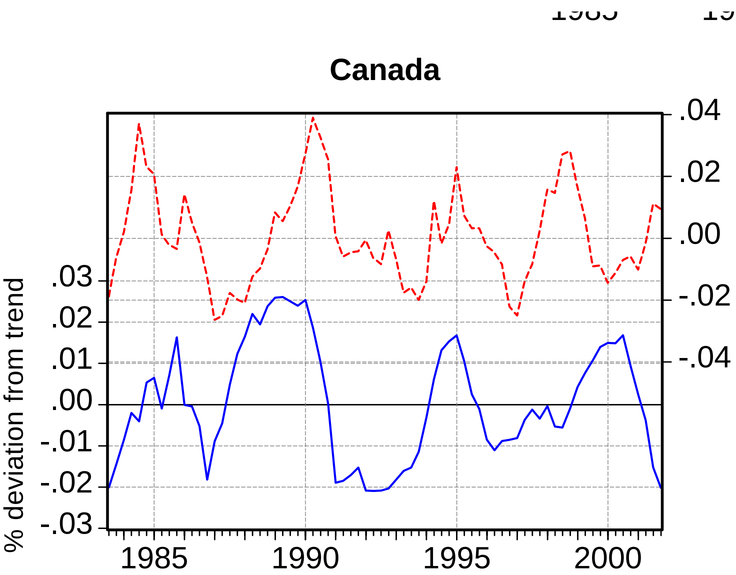
- Debt overhang and growth
  - "allocation puzzle": countries with above average growth rates are net exporters of capital on average.
  - Pattern driven by government net foreign assets.
  - Emerging market growth lower when external debt-to-GDP ratios exceed 60 percent, and both advanced and emerging market economies under perform when public debt-to-GDP ratios exceed 90 percent.

Co-movement of GDP and interest rates

Figure 1. Output and Interest Rates in Emerging Economies



Co-movement of GDP and interest rates



Business Cycle Moments

TABLE 1A. BUSINESS CYCLES IN EMERGING AND DEVELOPED ECONOMIES (STANDARD DEVIATIONS)

	% Standard Deviation			% Standard Deviation / % Standard Deviation of GDP				
	GDP	R	NX	PC	TC	INV	EMP	HRS
				Emerging Economies				
Argentina	4.22	3.87	1.42	1.08	1.17	2.95	0.39	0.57
	(0.36)	(0.52)	(0.11)	(0.05)	(0.03)	(0.13)	(0.07)	(0.08)
Brazil	1.76	2.34	1.40	1.93	1.24	3.05	0.89	1.95
	(0.23)	(0.26)	(0.45)	(0.38)	(0.23)	(0.26)	(0.13)	(0.33)
Korea	3.54	1.42	3.58	1.34	2.05	2.20	0.59	0.71
	(0.50)	(0.23)	(0.55)	(0.07)	(0.18)	(0.16)	(0.07)	(0.05)
Mexico	2.98	2.64	2.27	1.21	1.29	3.83	0.43	0.33
	(0.36)	(0.38)	(0.28)	(0.08)	(0.06)	(0.17)	(0.09)	(0.08)
Philippines	1.44	1.33	3.31	0.93	2.78	4.44	1.34	NA
	(0.17)	(0.13)	(0.45)	(0.11)	(0.44)	(0.43)	(0.33)	
Average	2.79	2.32	2.40	1.30	1.71	3.29	0.73	0.89
				Developed Economies				
Australia	1.19	2.00	1.02	0.84	1.20	4.13	1.13	1.40
	(0.09)	(0.17)	(0.08)	(0.07)	(0.08)	(0.22)	(0.10)	(0.14)
Canada	1.39	1.54	0.76	0.74	0.84	2.91	0.75	0.82
	(0.08)	(0.12)	(0.06)	(0.05)	(0.05)	(0.18)	(0.04)	(0.04)
Netherlands	0.93	0.93	0.67	1.17	1.44	2.66	1.27	NA
	(0.06)	(0.12)	(0.07)	(0.08)	(0.12)	(0.22)	(0.14)	
New Zealand	1.99	1.92	1.31	0.82	0.86	3.32	1.15	1.28
	(0.18)	(0.19)	(0.13)	(0.08)	(0.09)	(0.34)	(0.10)	(0.12)
Sweden	1.35	1.92	0.86	1.01	1.67	4.18	1.24	2.94
	(0.14)	(0.26)	(0.09)	(0.10)	(0.22)	(0.34)	(0.13)	(0.17)
Average	1.37	1.66	0.92	0.92	1.08	3.44	1.11	1.61

Aguiar and Gopinath (JIE, 2006)

Incomplete Market Models

- Eaton and Gersovitz (1981)
- Bonds only (non state-contingent)
- Government cannot commit to repay.
- Dynamic business cycle model with default in equilibrium.
- Shocks to the Endowment process.
    - Aguiar - Gopinath (2006)
    - Arellano (2008)
    - Chatterjee, Dean, Makoto and Rios-Rull (2002)

Model

- Representative agent.
- Endowment economy.
- Borrow and lend for consumption smoothing purposes.
- Bonds only.

Model

- Each period can decide whether to repay or default.
- Cost to Default
  - Autarky: Fully excluded from Financial Markets with exogenous re-entry possibility (λ).
  - If redeemed, all past debt is forgiven and the economy starts off with zero net assets.
  - Default Penalty. Lose a fraction of output per period (δ). Rose (2002, trade losses).
- Benefit to Default: Higher consumption in the default period

Model

- Preferences
$$u = \frac{c^{1-\gamma}}{1-\gamma}. \tag{1}$$
- Technology
$$y_t = e^{z_t}\Gamma_t. \tag{2}$$
- Asset: International Bond a_t.

Model

- Transitory shock, z_t, follows an $AR(1)$ around a long run mean μ_z

$$z_t = \mu_z(1-\rho_z) + \rho_z z_{t-1} + \varepsilon_t^z \tag{3}$$

$|\rho_z| < 1$, $\varepsilon_t^z \sim N(0, \sigma_z^2)$,

- Trend:

$$\begin{aligned} \Gamma_t &= g_t \Gamma_{t-1} && (4) \\ \ln(g_t) &= (1-\rho_g)(\ln(\mu_g) - c) + \rho_g \ln(g_{t-1}) + \varepsilon_t^g && (5) \end{aligned}$$

$|\rho_g| < 1$, $\varepsilon_t^g \sim N(0, \sigma_g^2)$, and $c = \frac{1}{2}\frac{\sigma_g^2}{1-\rho_g^2}$.

Model

- State of the economy:
    - Income (z and Γ)
    - Assets (a)
    - Credit rating (G or B)

Model

- V^B: Value function with bad credit rating

$$V^B(z_t, \Gamma_t) = u((1-\delta)y_t) + \lambda\beta E_t V(0, z_{t+1}, \Gamma_{t+1}) +$$
$$(1-\lambda)\beta E_t V^B(z_{t+1}, \Gamma_{t+1})$$

Model

- V^G: Value function with good credit rating.

$$V^G(a_t, z_t, \Gamma_t) = \max_{c_t} \{u(c_t) + \beta E_t V(a_{t+1}, z_{t+1}, \Gamma_{t+1})\}$$

$$s.t. \quad c_t = y_t + a_t - q_t a_{t+1}$$

- $$V = max(V^G, V^B)$$

- q is the price of a bond that pays one next period (inverse of interest rate)

Model

- International Investors: Risk neutral with outside option $r*$
- Default function

$$D(a_t, z_t, \Gamma_t) = \begin{cases} 1 & \text{if } V^B(z_t, \Gamma_t) > V^G(a_t, z_t, \Gamma_t) \\ 0 & \text{otherwise} \end{cases}$$

- Equilibrium price q

$$q(a_{t+1}, z_t, \Gamma_t) = \frac{E_t\{(1 - D_{t+1})\}}{1 + r^*}$$

- Euler equation:

$$E_t\left(\beta\frac{u'(c_{t+1})}{u'(c_t)}(1-D_{t+1})\right)=q_t+a_{t+1}\frac{\partial q_t}{\partial a_{t+1}}$$

- At the margin, additional borrowing/lending today affects future consumption only in non-default states.
- At the margin, the cost of additional debt has two components the interest rate and the change in interest rate.
- Govt. internalizes the effect of additional borrowing on r.

- Theorem: If shocks are i.i.d. then if do not default in state $z = z_1$ then will not default if $z = z_2 \geq z_1$
- This statement implies that

$$
\begin{aligned}
& \begin{bmatrix} u(z_2 + q_2 d_2 - d) \\ +\beta \mathbb{E}_{z_2} \max\{V^{ND}(d_2, z'), V^D(0, z')\} \end{bmatrix} - \begin{bmatrix} u(z_1 + q_1 d_1 - d) \\ +\beta \mathbb{E}_{z_1} \max\{V^{ND}(d_1, z'), V^D(0, z')\} \end{bmatrix} \\
> \quad & \begin{bmatrix} u(z_2) \\ +\beta \mathbb{E}_{z_2} V^D(0, z') \end{bmatrix} - \begin{bmatrix} u(z_1) \\ +\beta \mathbb{E}_{z_1} V^D(0, z') \end{bmatrix}
\end{aligned}
$$

- That is the change in the default value function is smaller than the change in the non-default value function.

- If we can show that

$$\begin{bmatrix} u(z_2 + q_1 d_1 - d) \\ +\beta \mathbb{E}_{z_2} \max\{V^{ND}(d_1, z'), V^D(0, z')\} \end{bmatrix} - \begin{bmatrix} u(z_1 + q_1 d_1 - d) \\ +\beta \mathbb{E}_{z_1} \max\{V^{ND}(d_1, z'), V^D(0, z')\} \end{bmatrix}$$
$$> \begin{bmatrix} u(z_2) \\ +\beta \mathbb{E}_{z_2} V^D(0, z') \end{bmatrix} - \begin{bmatrix} u(z_1) \\ +\beta \mathbb{E}_{z_1} V^D(0, z') \end{bmatrix}$$

- Then it follows from the optimality of d_2 that the first relation is true.
- If shocks are *iid* then the above simplifies to

$$u(z_2 + qd_1 - d) - u(z_1 + qd_1 - d) > u(z_2) - u(z_1)$$

- Since it must be that $qd_1 - d < 0$ for there to have been default in z_1, given the concavity of the utility function it must be true.
- Persistent shocks: q is no longer independent of z

- Numerical solution using discrete state space method (Problem set 2)
- Solution algorithm: See paper and problem set..

Table 2A: Common Benchmark Parameter Values

Risk Aversion	γ	2
World Interest Rate	r^*	1%
Loss of Output in Autarky	δ	2%
Probability of Redemption	λ	10%
Mean (Log) Transitory Productivity	μ_z	$-\frac{1}{2}\sigma_z^2$
Mean Growth Rate	μ_g	1.006

Table 2B: Model Specific Benchmark Parameter Values

	Model I: Transitory Shocks	*Model II: Growth Shocks*	*Model II with Bail Outs*
σ_z	3.4%	0	0
ρ_z	0.90	NA	NA
σ_g	0	3%	3%
ρ_g	NA	0.17	0.17
β	0.8	0.8	0.95
Bail Out Limit	NA	NA	18%

Table 3: Benchmark Simulation Results

	Data	Model I (3A)	Model II (3B)	Model II with Bail Outs (3C)
$\sigma(y)$	4.08	4.32	4.45	4.43
$\sigma(c)$	4.85	4.37	4.71	4.68
$\sigma(TB/Y)$	1.36	0.17	0.95	1.10
$\sigma(R_s)$	3.17	0.04	0.32	0.12
$\rho(C,Y)$	0.96	0.99	0.98	0.97
$\rho(TB/Y,Y)$	-0.89	-0.33	-0.19	-0.12
$\rho(R_s,Y)$	-0.59	0.51	-0.03	-0.02
$\rho(R_s,TB/Y)$	0.68	-0.21	0.11	0.38
Rate of Default (per 10,000 quarters)	75	2	23	92
Mean Debt Output Ratio (%)		27	19	18
Maximum R_s *(basis points)*		23	151	57

Note: Simulation results reported are averages over 500 simulations each of length 500 (drawn from a stationary distribution). The simulated data is treated in an identical manner to the empirical data. Standard deviations are reported in percentages.

Sustaining debt in equilibrium

- Difficult to sustain debt in equilibrium without additional penalty (beyond reputation).
- Calculation a la Lucas (1987): i.i.d shocks
  - Autarky : No domestic savings and i.i.d shocks.
  - Financial Integration: Constant Consumption stream.
  - Suppose pay rB each period to maintain constant consumption.
  - How much is it worth to have perfect insurance vs. autarky?

Sustaining debt in equilibrium

- Stack the deck against autarky by assuming no domestic savings (capital or storage technology), that shocks are *iid*, and that autarky lasts forever.
- stack the deck in favor of financial integration by supposing that integration implies a constant consumption stream (perfect insurance)
- In order to maintain perfect consumption insurance, we suppose that the agent must make interest payments of rB each period.

Sustaining debt in equilibrium

- $Y_t = \overline{Y} e^{z_t} e^{-(\frac{1}{2})\sigma_z^2}$
- $z \sim N(0, \sigma_z^2)$ and *iid*, $\mu_g = 1$
- $EY_t = \overline{Y}$

$$V^B = E \sum_t \beta^t \frac{Y_t^{1-\gamma}}{1-\gamma} = \frac{(\overline{Y} e^{-(\frac{1}{2})\gamma\sigma_z^2})^{1-\gamma}}{(1-\gamma)(1-\beta)}. \tag{6}$$

$$V^G = E \sum_t \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} = \frac{(\overline{Y} - rB)^{1-\gamma}}{(1-\gamma)(1-\beta)}. \tag{7}$$

Sustaining debt in equilibrium

- The economy will not default as long as

$$V^G \geq V^B,$$

or

$$\frac{rB}{\overline{Y}} \leq 1 - \exp(-(\frac{1}{2})\gamma\sigma_z^2).$$

- The volatility of detrended output for Argentina is 4.08% (i.e. $\sigma_z^2 = 0.0408^2 = 0.0017$).
- For a coefficient of relative risk aversion of 2, this implies the maximum debt payments as a percentage of GDP is 0.17%.
- At a quarterly interest rate of 2%, debt cannot exceed 8.32% of output.

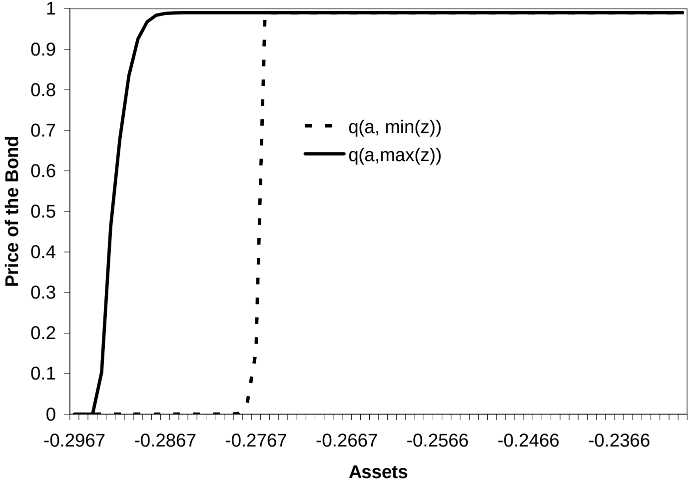
Sustaining debt in equilibrium

- Impose an additional loss of δ percent of output during autarky. $\frac{rB}{Y} \leq 1 - (1-\delta)\exp(-(\frac{1}{2})\gamma\sigma_z^2)$.
- If $\delta = 0.02$, we can support debt *payments* of 20% of GDP, which implies a potentially large debt to GDP ratio.

Why so few defaults in equilibrium?

- The interest rate schedule is very steep.
- The agent internalizes the effect of his borrowing on the interest rate he must pay. (consumer's euler equation)

Figure 3A: Model I



Why so few defaults in equilibrium?

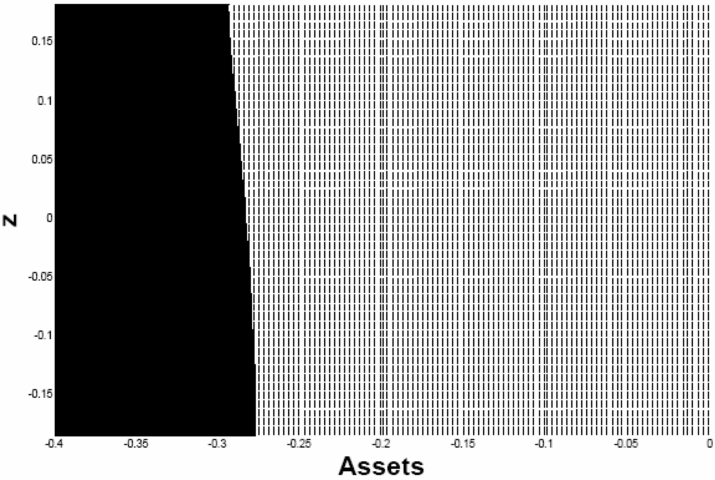
- Let $\overline{z}(\hat{a})$ denote the threshold endowment below which the agent defaults for the given asset level.
- Price function:

$$\hat{q}(\hat{a}_{t+1}) = \frac{(1 - Pr(z_{t+1} < \overline{z}(\hat{a}_{t+1})|z_t))}{1 + r*} = \frac{1 - F_t(\overline{z}(\hat{a}_{t+1})|z_t)}{1 + r*}$$

$$\hat{q}'(\hat{a}_{t+1}) = \frac{-f_t(\overline{z}(\hat{a}_{t+1}))}{1 + r*} \frac{d\overline{z}}{da}$$

Why so few defaults in equilibrium?

Figure 2: Default Region



Why so few defaults in equilibrium?

- The slope of $\bar{z}$

$$\hat{V}^G(\hat{a}, \bar{z}(\hat{a})) = \hat{V}^B(\bar{z}(\hat{a}))$$

$$\frac{d\bar{z}}{da} = \frac{-\frac{\partial \hat{V}^G}{\partial a}}{\frac{\partial \hat{V}^G}{\partial z} - \frac{\partial \hat{V}^B}{\partial z}}$$

Why so few defaults in equilibrium?

- $$\frac{d\bar{z}}{da} = \frac{-\frac{\partial \hat{V}^G}{\partial a}}{\frac{\partial \hat{V}^G}{\partial z} - \frac{\partial \hat{V}^B}{\partial z}}$$
- Suppose that z is a random walk. A shock to z today is expected to persist indefinitely and will have a large impact on expected lifetime utility. However, with a random walk income process there is limited need (up to the first order) to save out of additional endowment. This implies an additional unit of endowment will be consumed, leaving little difference between financial autarky and a good credit history.
- Suppose that z is *iid* over time. Then there is a stronger incentive to borrow and lend. However, the lack of persistence implies the impact of an additional unit of endowment today is limited to its effect on current endowment, resulting in a limited impact on the entire present discounted value of utility. That is, both $\Delta \widehat{V}^G$ and $\Delta \widehat{V}^B$ are relatively small and therefore so is the difference.

Why is it hard to match the facts?

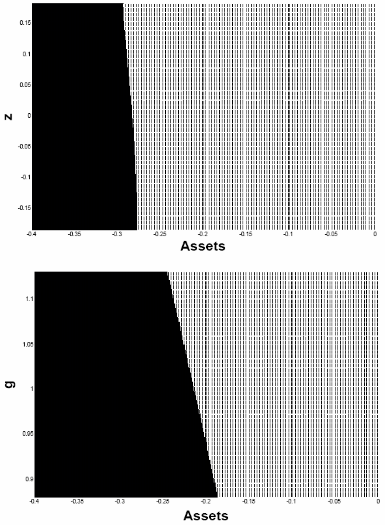
- The steepness of the interest rate schedule makes it challenging to even qualitatively match the positive correlation between interest rates and the current account.
- On the one hand, an increase in borrowing in good states (countercyclical current account) will, all else equal, imply a movement along the heuristic "loan supply curve" and a sharp rise in the interest rate.
- On the other hand, if the good state is expected to persist, this lowers the expected probability of default and is associated with a favorable shift in the interest rate schedule.
- To generate a positive correlation between the current account and interest rates we need the effect of the shift of the curve to dominate the movement along the curve.

How can trend shocks help?

- Shock to trend growth has a large impact on the two value functions (because of the shock's persistence) and on the *difference* between the two value functions.
- The latter effect arises because a positive shock to trend implies that income is higher today, but even higher tomorrow, placing a premium on the ability to access capital markets to bring forward anticipated income.
- The decision to default is relatively more sensitive to the particular realization of the shock and less sensitive to the amount of debt.
- Correspondingly, the interest rate function is less sensitive to the amount of debt held.

Trend shocks

Figure 2: Default Region



Trend shocks

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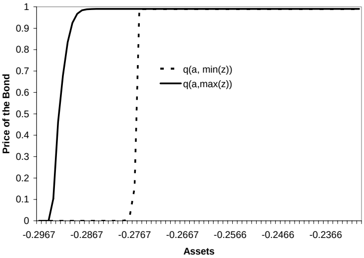
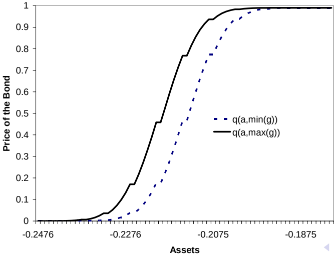


Figure 3B: Model II



Trend shocks

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- Arellano (2008 AER)
  - Only transitory shocks
  - Assumes a functional form for default output so that the slopes of the value functions are very different.

$$\begin{aligned} y^{def} &= \hat{y} \quad \text{if} \quad y > \hat{y} \\ y^{def} &= y \quad \text{if} \quad y \leq \hat{y} \end{aligned}$$

  - Greater success in matching the facts.

- Mendoza and Yue (2012 QJE): endogenize state-contingent output costs of default.

- To match empirical levels of debt to GDP plus frequency of default:
  - If default very attractive: low debts, no defaults
  - If default not very attractive: high debts, no default
  - State contingent penalty function helps:
    - Default gives state contingency which is useful in bad states.
    - Countries will use it if penalties not onerous in bad states.
    - To satisfy the lenders constraint need high penalties in good states.