Microeconomic Theory: Lecture 4
Monopoly

Parikshit Ghosh

Delhi School of Economics

Summer Semester, 2014

The Monopolist's Problem

- The monopolist realizes his quantity choice affects market price through the demand function.
- Monopolist's problem can be described either as one of choosing optimal price or optimal quantity.
- Inverse demand function $p(q)$ and cost function $c(q)$.
- Maximizing profits:

$$\max_q p(q)q - c(q)$$

- First order condition:

$$\underbrace{p(q^m) + qp'(q^m)}_{\text{marginal revenue}} = \underbrace{c'(q^m)}_{\text{marginal cost}}$$

Monopolist's Optimum: Another Look

- The FOC can be rewritten as

$$p_m \left[1 + \frac{q_m}{p_m} . \frac{dp}{dq}\right] = c'(q_m)$$

$$\text{or,} \quad p_m \left(1 - \frac{1}{e(q_m)}\right) = c'(q_m)$$

- $e(q)$ is the elasticity of demand at quantity q. The monopolist never operates on the inelastic part of the demand curve.
- Yet another way to write it:

$$\frac{p_m - c'(q_m)}{p_m} = \frac{1}{e(q_m)}$$

- The mark-up is equal to the inverse of demand elasticity.

Comparison With Competitive Markets

- Competitive output level:

$$p(q^*) = c'(q^*)$$

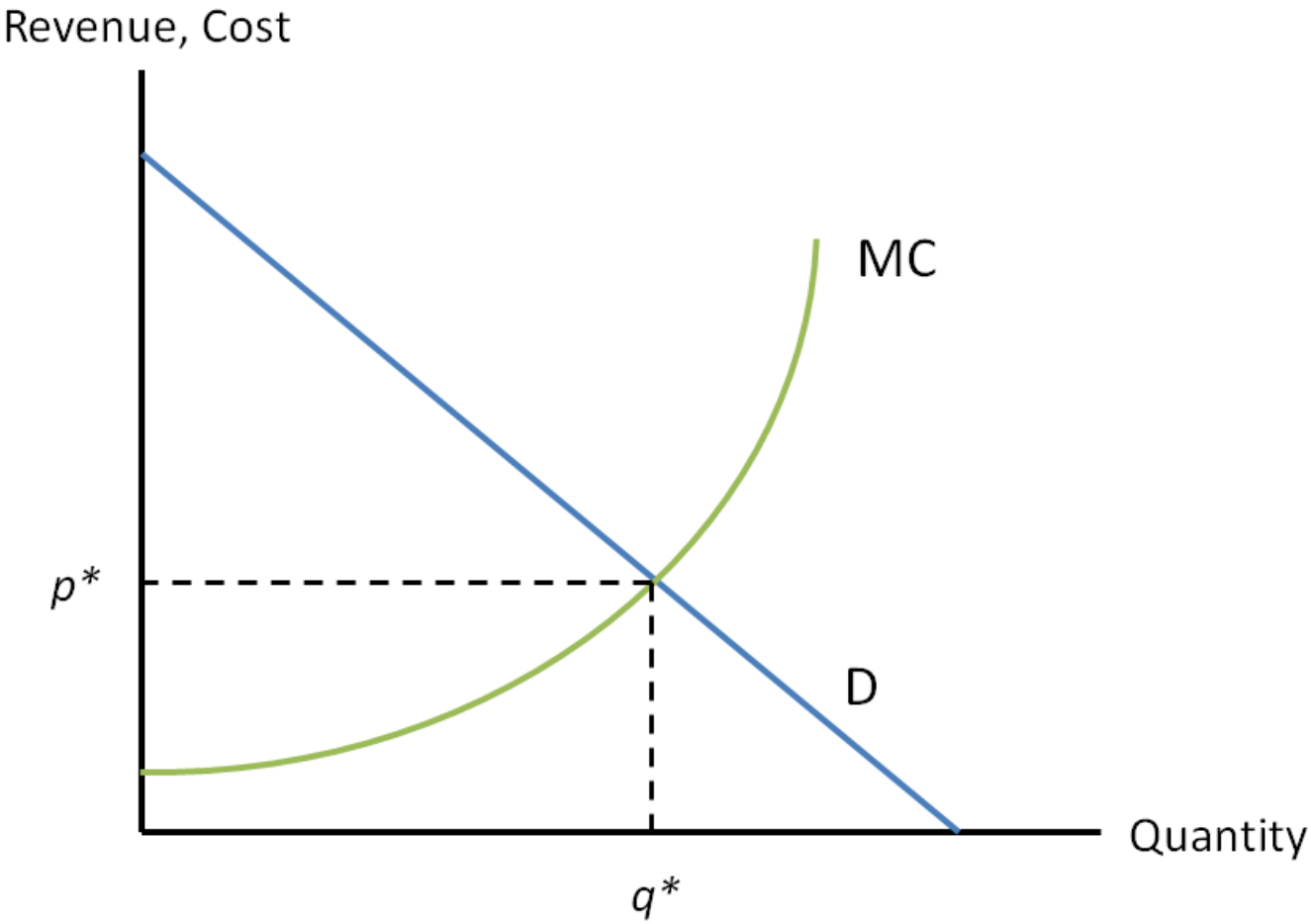
- The monopolist produces less than the competitive market.
- Suppose $q^m \geq q^* \Rightarrow p(q^m) \leq p(q^*)$ and $c'(q^m) \geq c'(q^*)$.
- Subtracting the f.o.c's yields a contradiction:

$$\underbrace{p(q^m) - p(q^*)}_{-} + \underbrace{qp'(q^m)}_{-} = \underbrace{c'(q^m) - c'(q^*)}_{+}$$

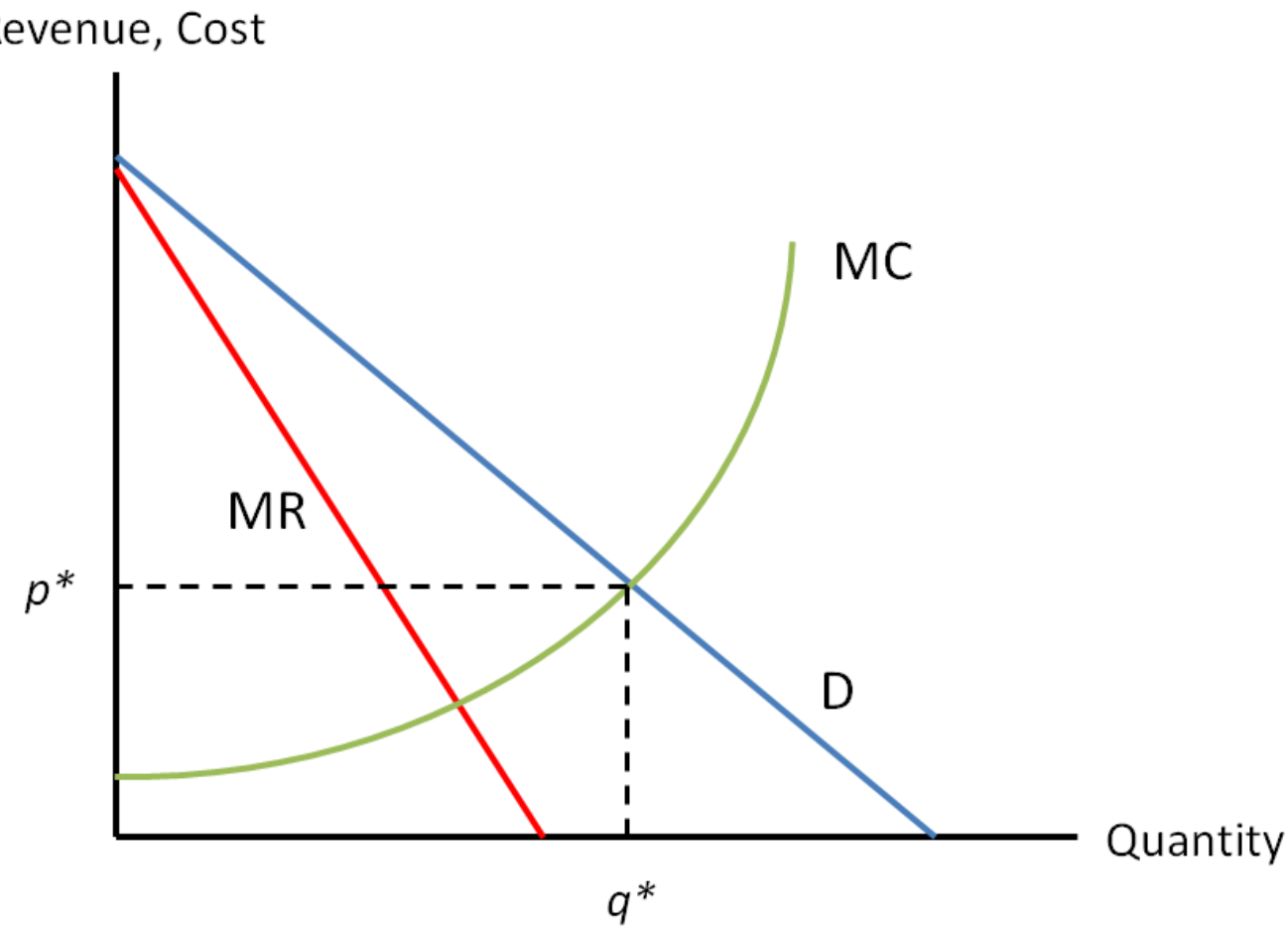
$$- \quad - \quad = \quad +$$

- Unlike a competitive firm, when a monopolist raises output, he earns lower revenue on previous units.

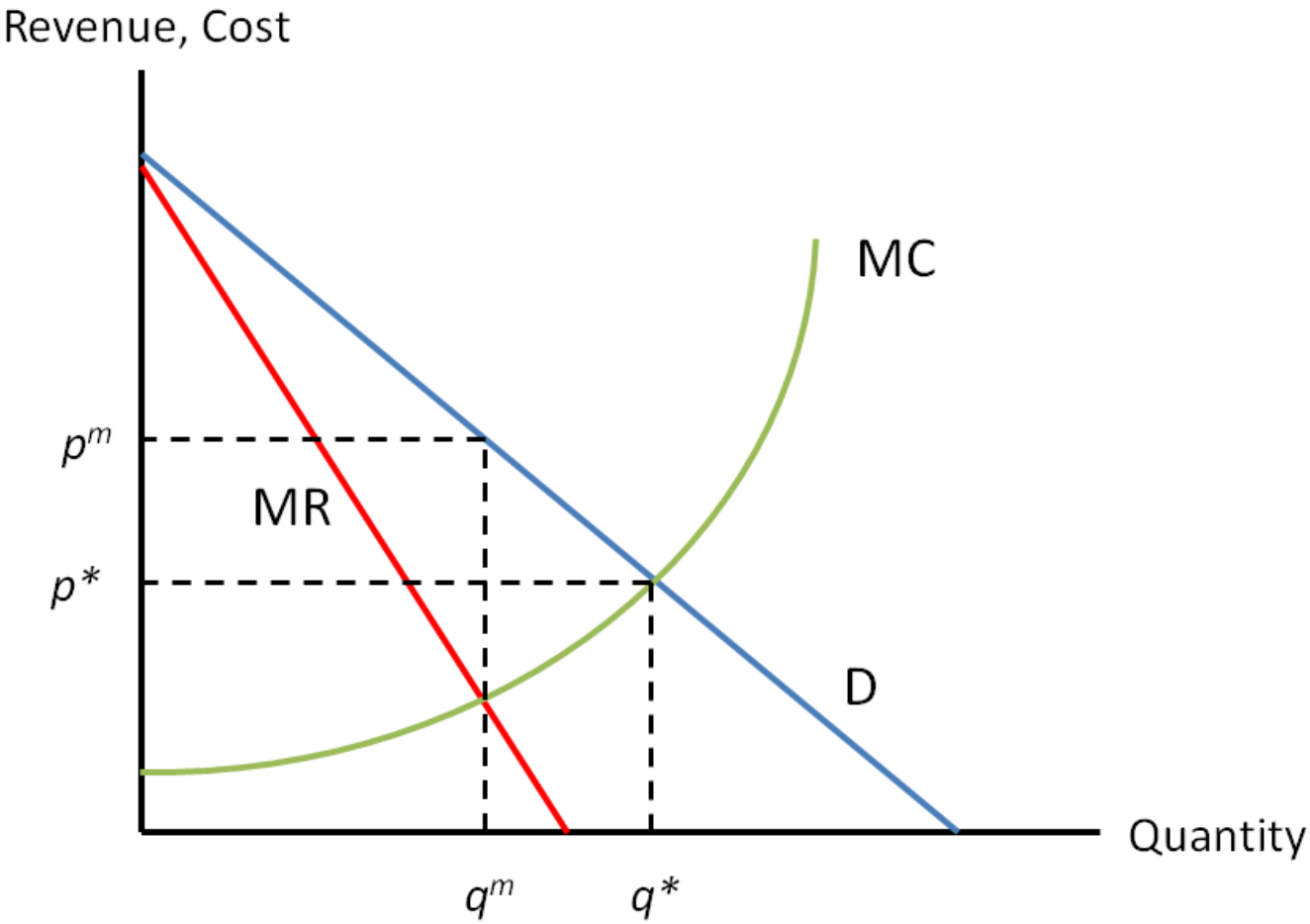
Monopoly in Pictures



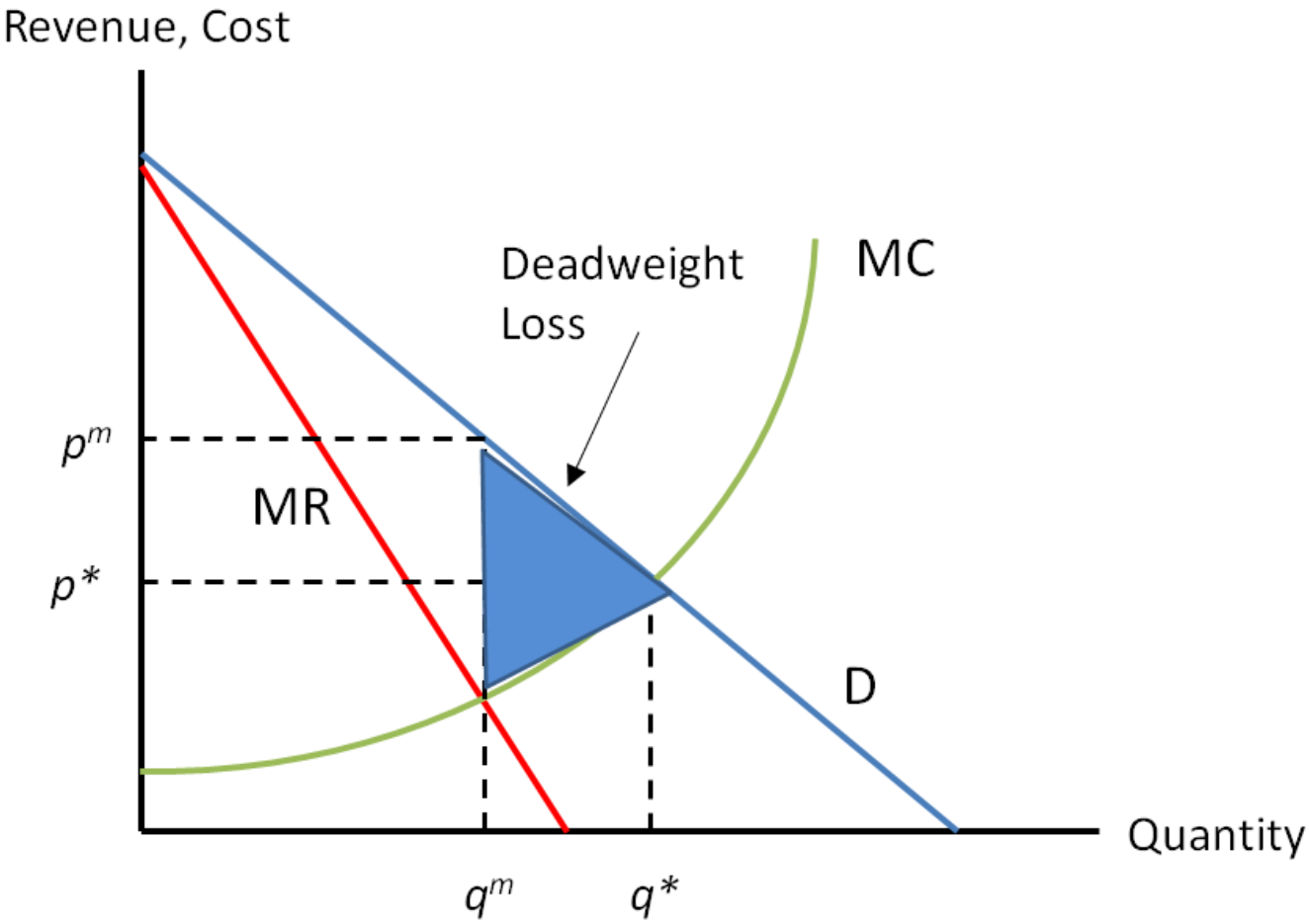
Monopoly in Pictures



Monopoly in Pictures



Monopoly in Pictures



Price Discrimination: First Degree

- Each consumer buys 0 or 1 unit and is willing to pay up to v.
- There is a continuum of consumers whose v follow a distribution with c.d.f $F(v)$.
- The monopolist can charge each consumer his personal v.
- Must choose a cutoff v^* above which to sell:

$$\max_{v^*} \int_{v^*}^{\infty} vf(v)dv - c\left(1 - F(v^*)\right)$$

- First-order condition (using Leibnitz Rule) implies absence of inefficiency:

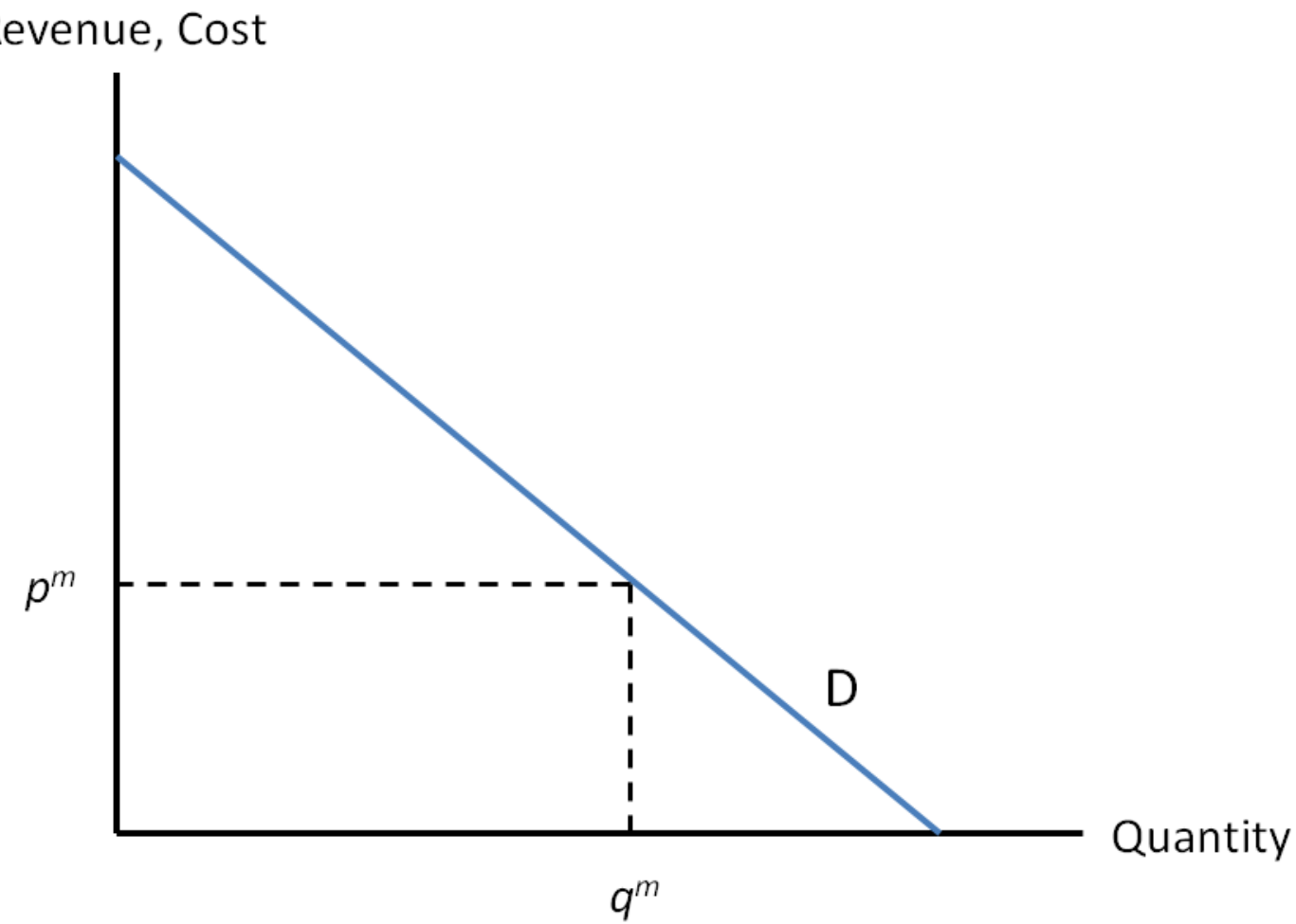
$$-v^* f(v^*) + c'(1 - F(v^*))f(v^*) = 0$$

$$\underbrace{v^*}_{\text{price to marginal customer}} = \underbrace{c'(1 - F(v^*))}_{\text{marginal cost}}$$

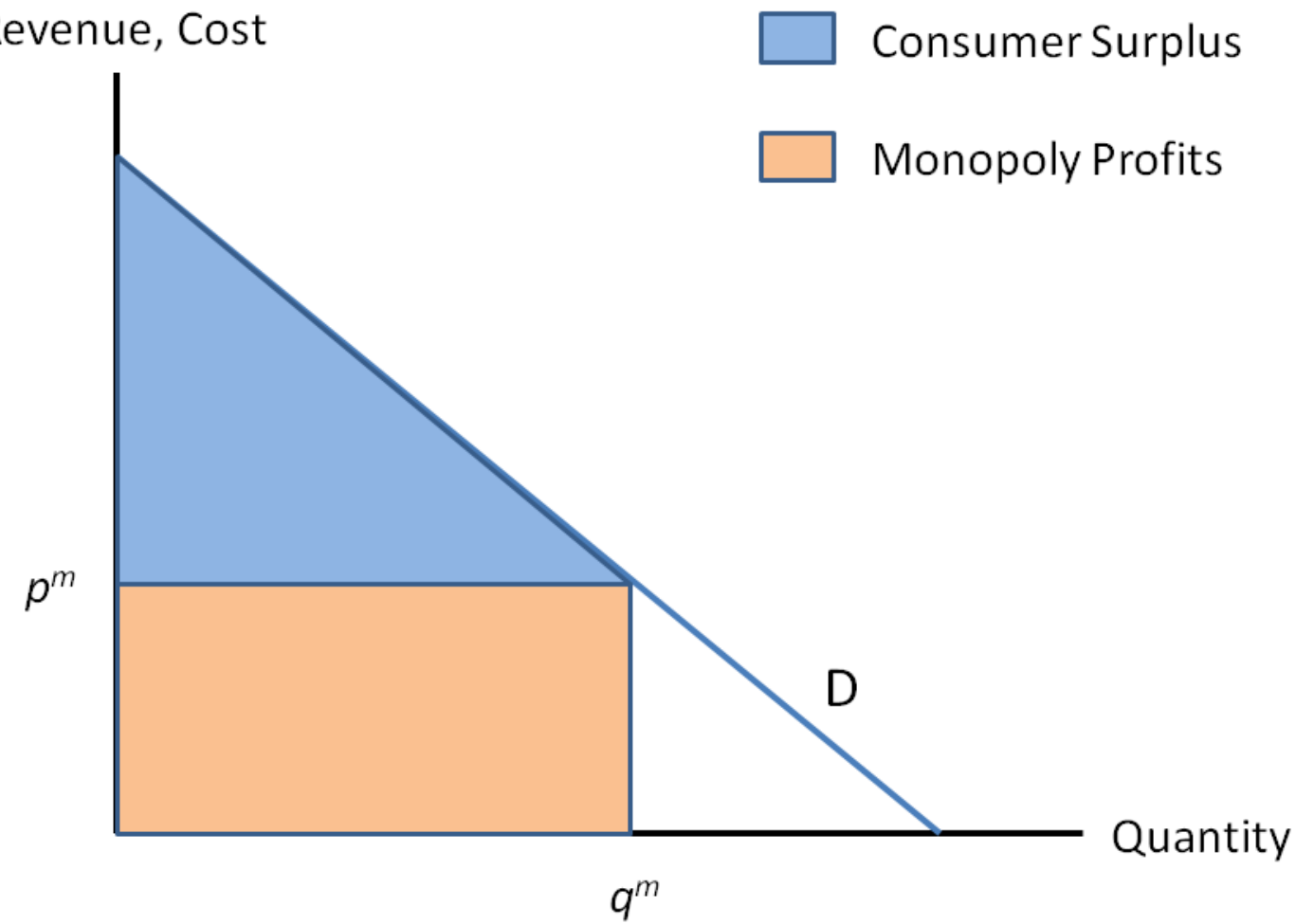
Price Discrimination: Second and Third Degree

- Second degree price discrimination arises when the monopolist can charge different prices for different quantities.
- E.g., bulk discounts, multi-packs, frequent-flyer miles, buy-one-get-50%-off on next purchase, etc.
- A way to extract consumer's surplus from a single consumer.
- Third degree price discrimination arises when observably different groups are charged different prices.
- E.g., student/senior citizen discounts, country specific prices.
- A cruder form of first degree price discrimination—using group identity as a predictor of individual traits.
- A common instrument of price discrimination: screening.

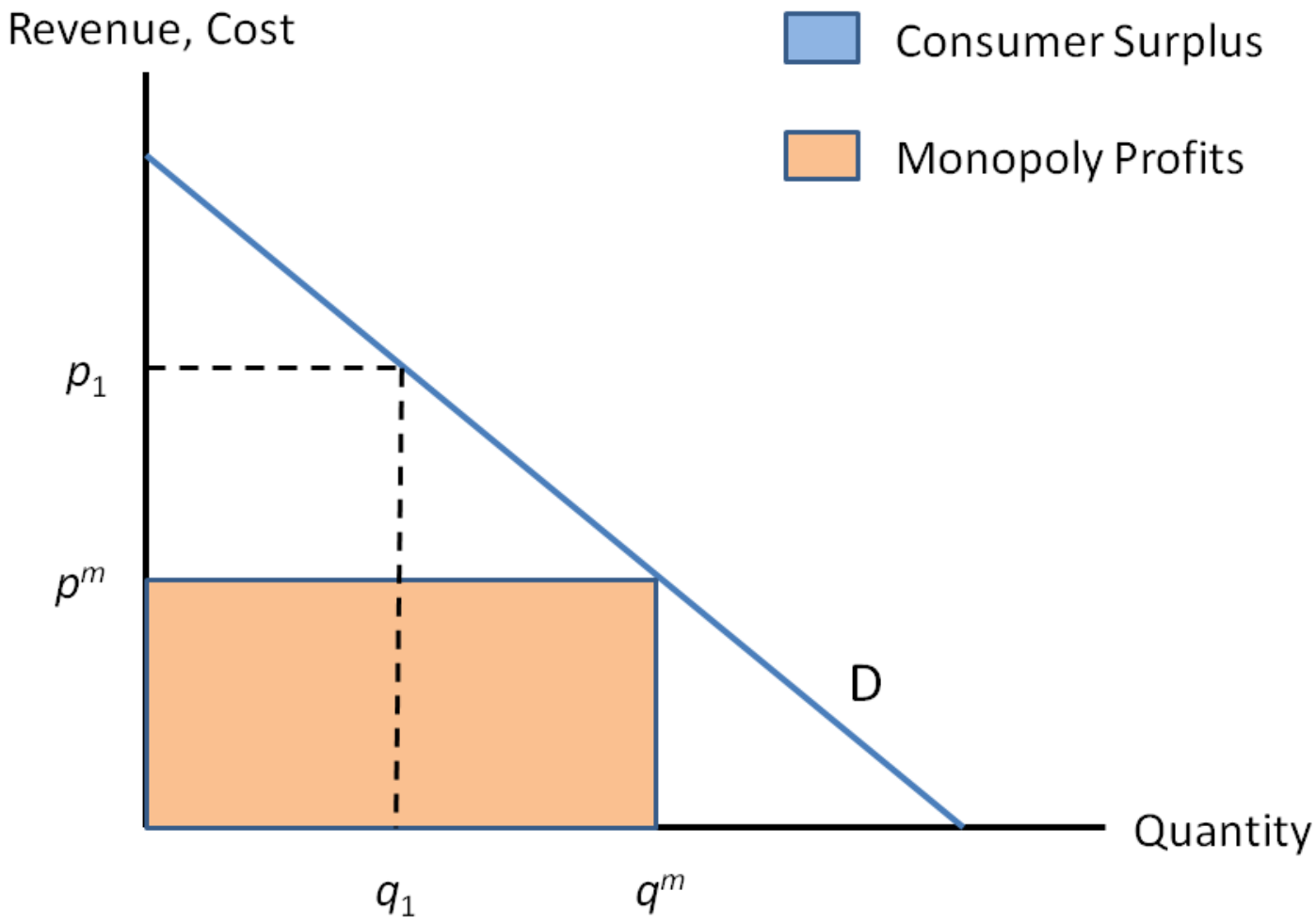
Second Degree Price Discrimination



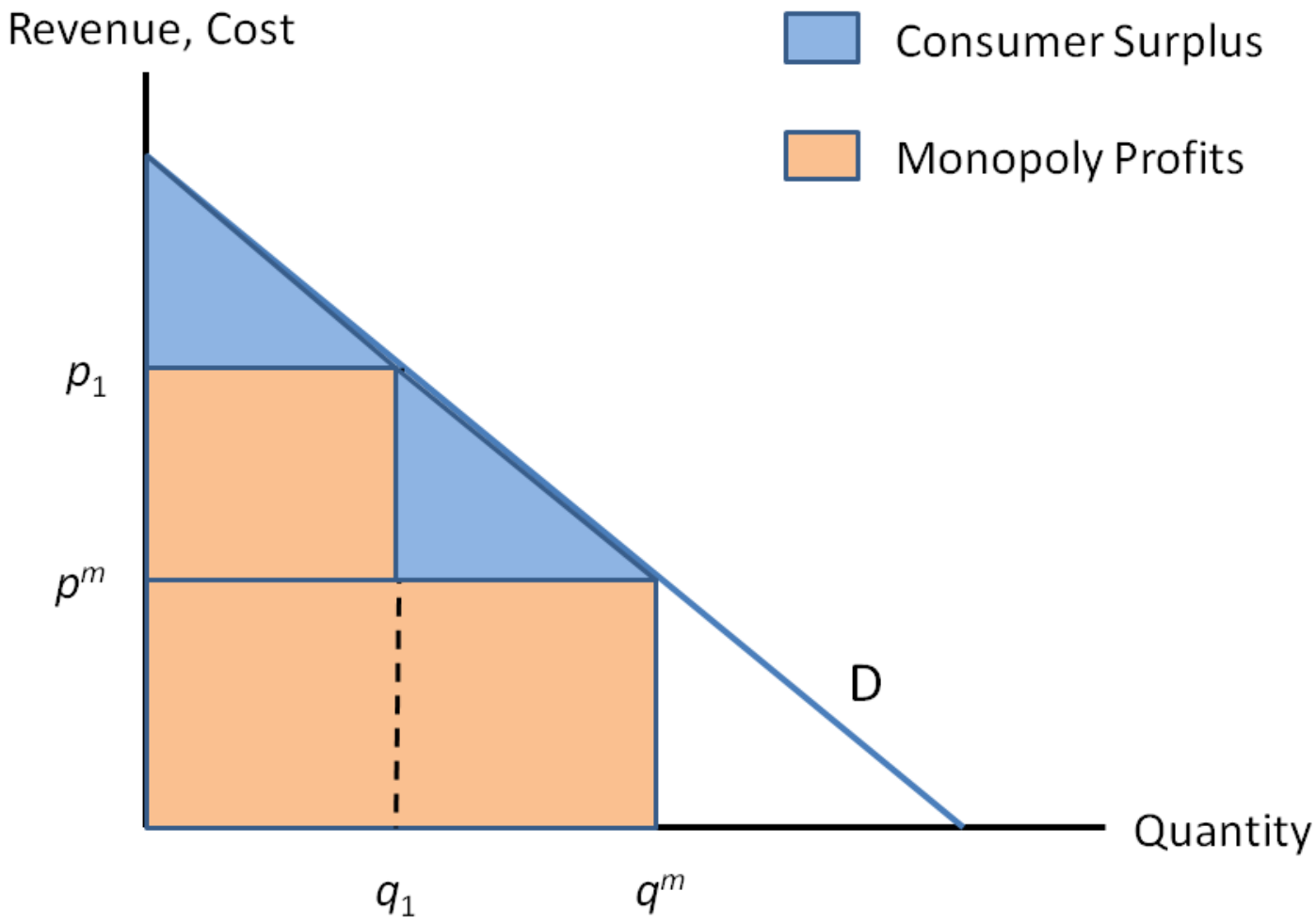
Second Degree Price Discrimination



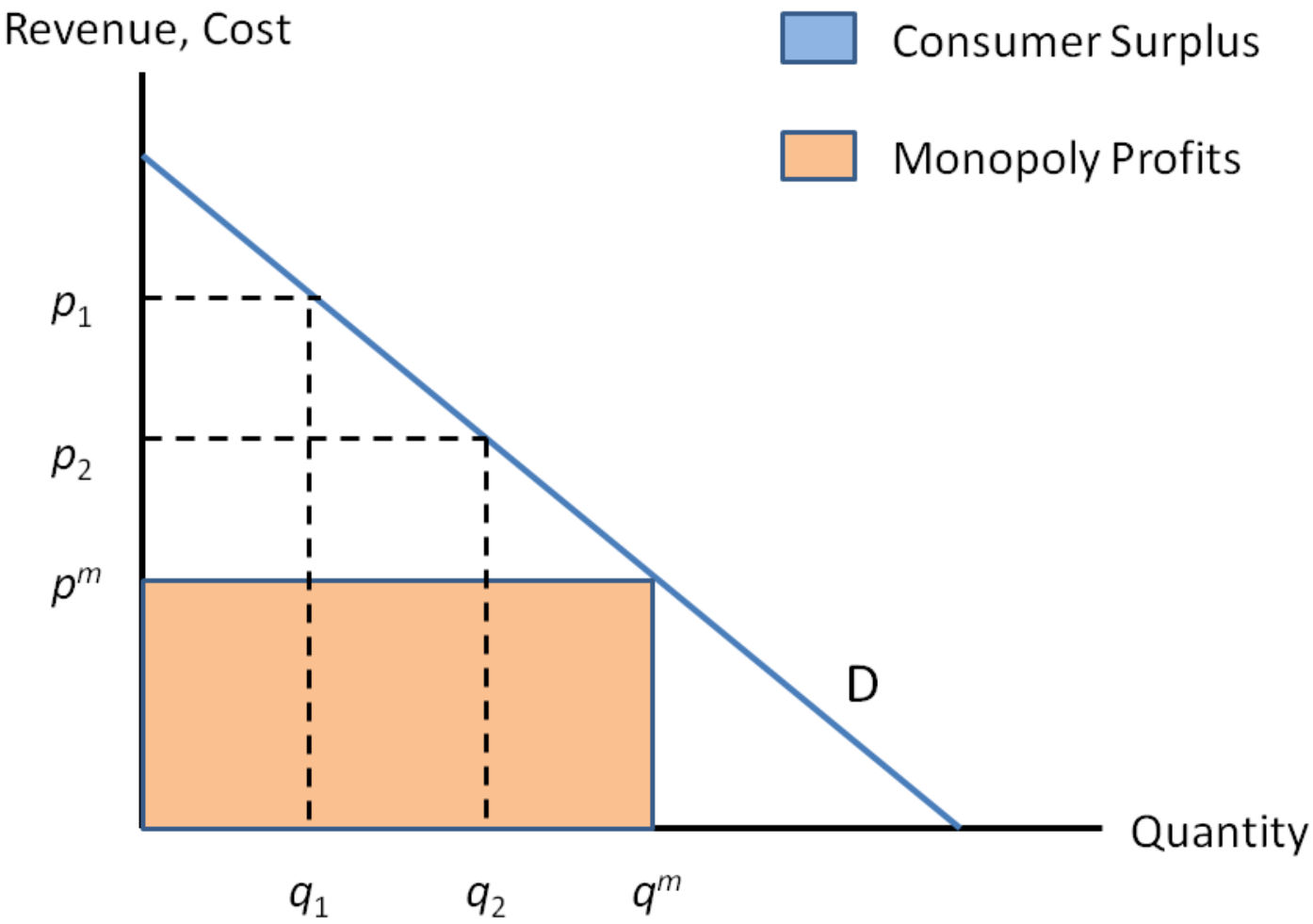
Second Degree Price Discrimination



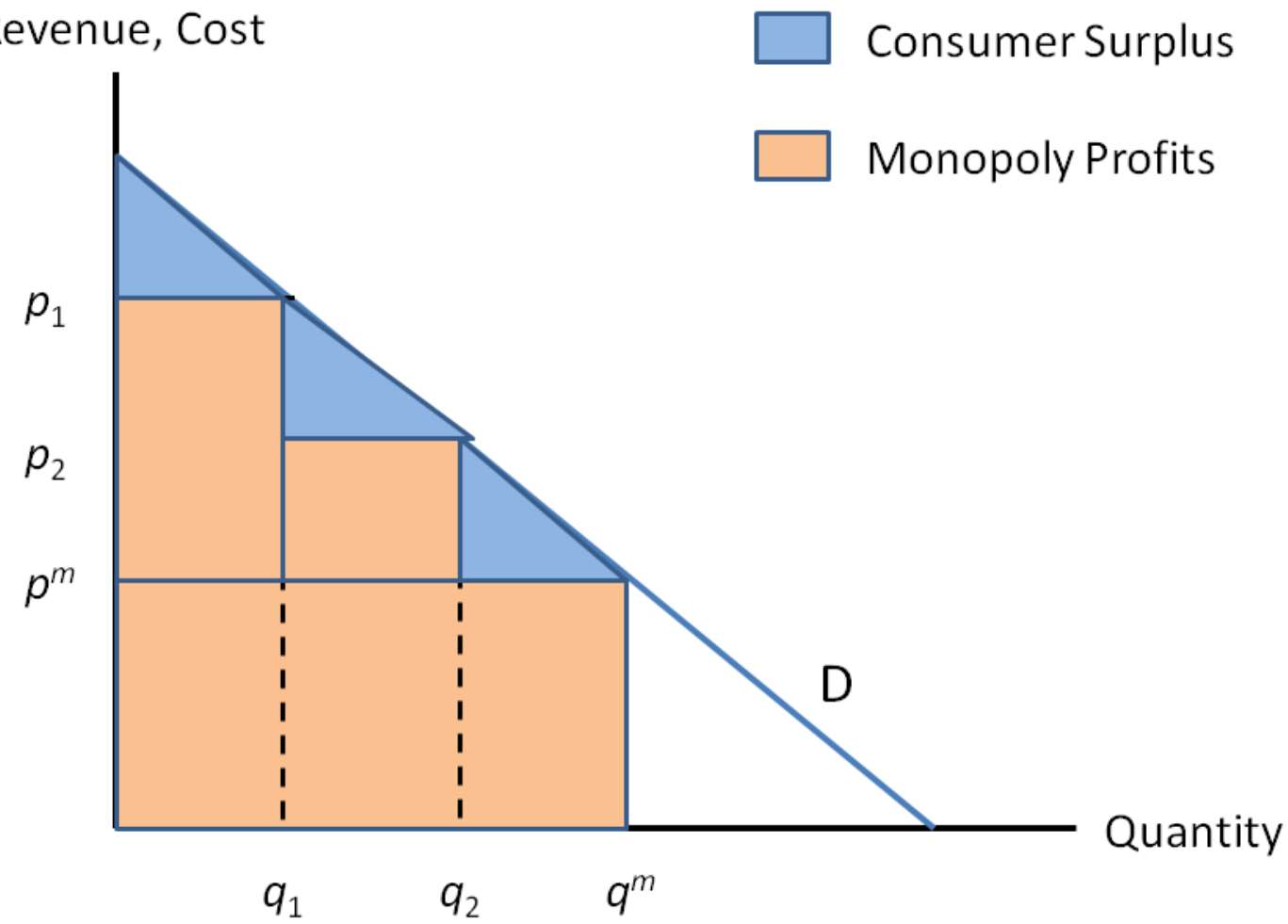
Second Degree Price Discrimination



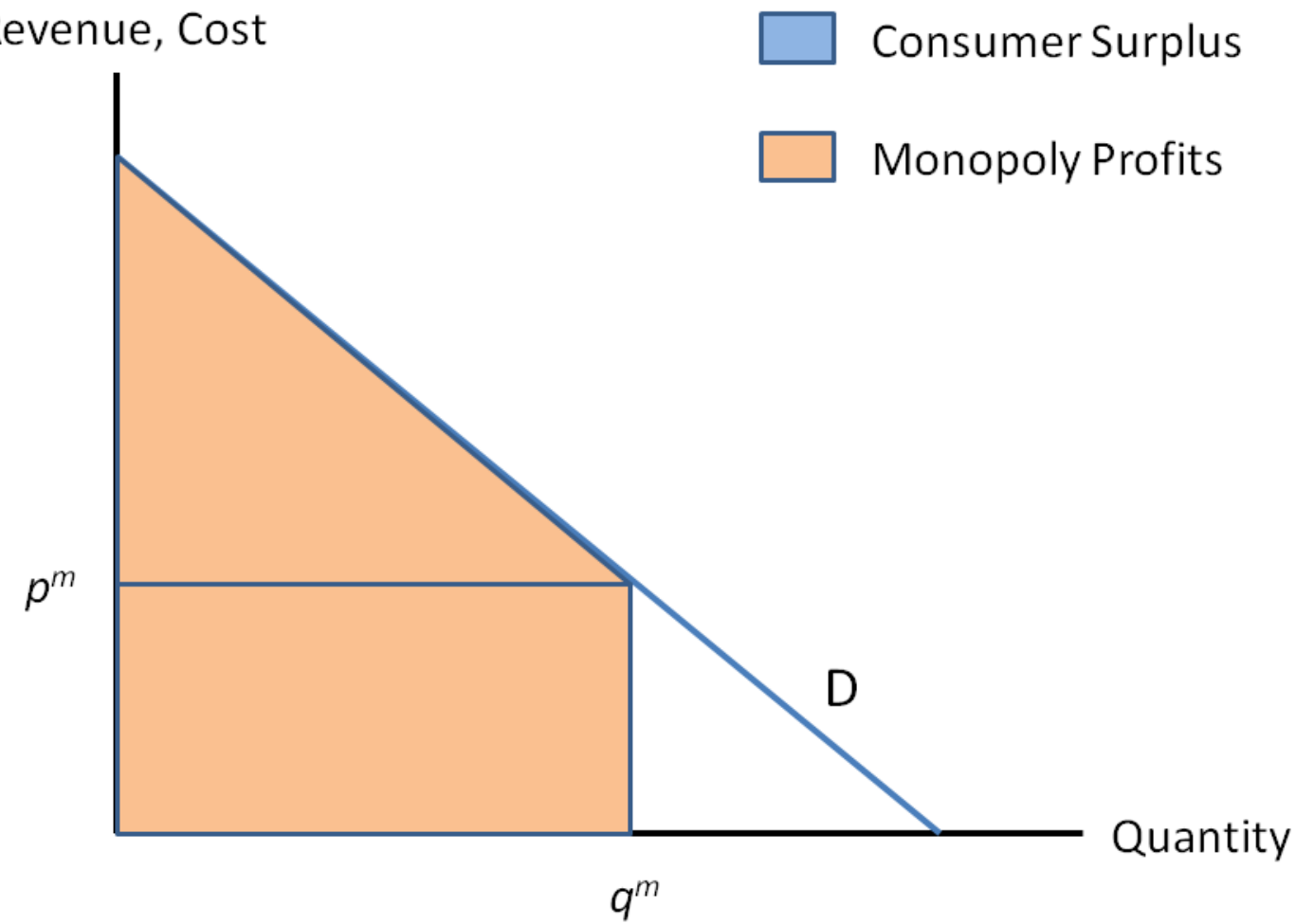
Second Degree Price Discrimination



Second Degree Price Discrimination



Second Degree Price Discrimination



Two Part Tariff

PURCHASE ADMISSION NOW
THE METROPOLITAN MUSEUM OF ART

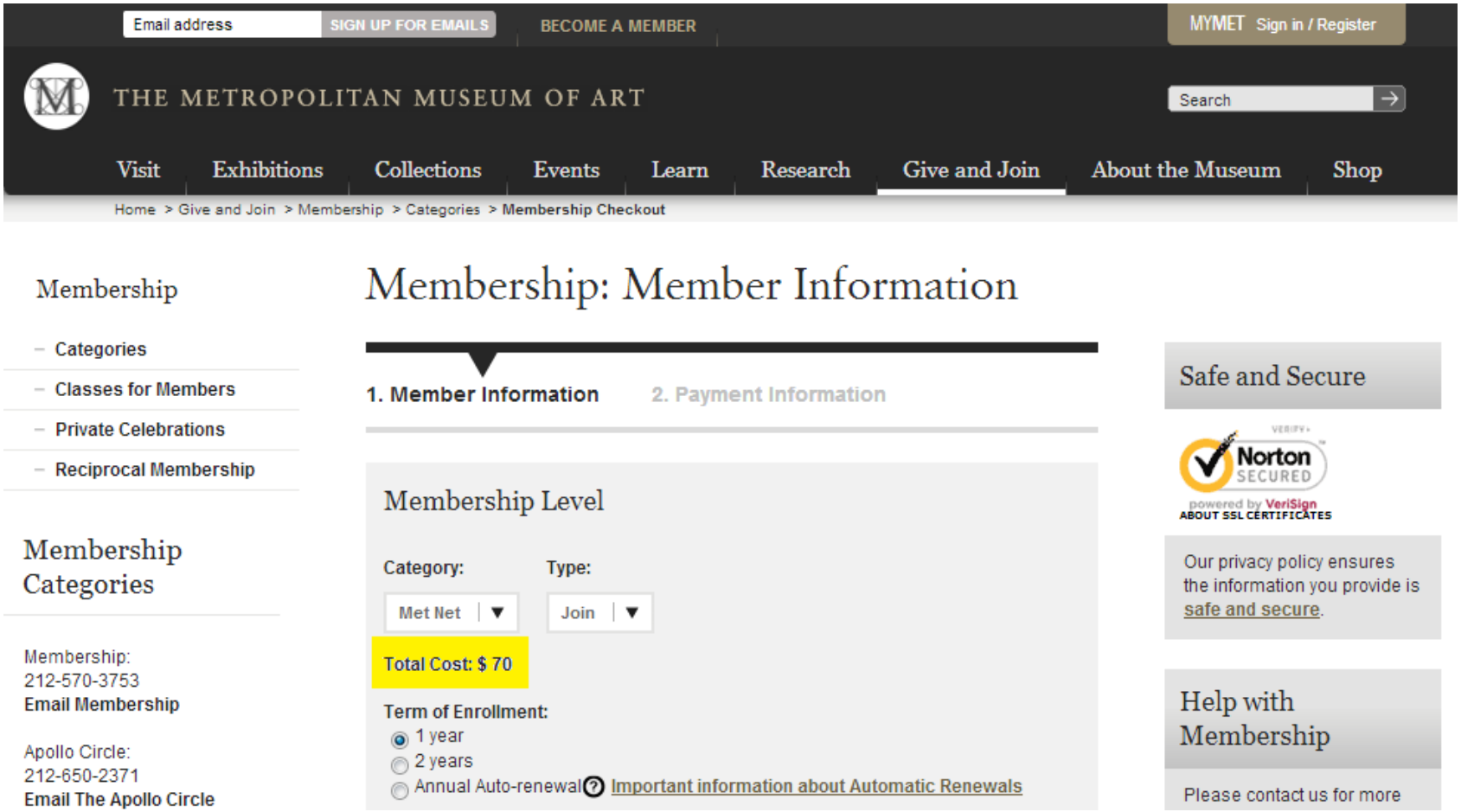
Step 1 of 4 - SELECT TICKETS

General Admission

Advance tickets to The Metropolitan Museum of Art allow the visitor to avoid waiting in admission lines at the Museum. Tickets include admission to the Met's collection and all special exhibitions. Tickets also include same-day admission to The Metropolitan Museum of Art and The Cloisters museum and gardens. Tickets are valid for up to one year after the date of purchase.

Qty	Description	Price
0	ADULT GENERAL ADMISSION	$25.00
0	SENIOR GENERAL ADMISSION	$17.00
0	STUDENT GENERAL ADMISSION	$12.00
0	CHILDREN UNDER 12 WITH ADULT	FREE

Two Part Tariff



Two Part Tariff

- Suppose a single consumer with income y has quasi-linear utility: $u(q, m) = \phi(q) + m$.
- The monopolist can charge an entry fee (f) and a price (p) per unit of consumption.
- Consumer's optimum quantity choice (if she subscribes):

$$\max_q \phi(q) + y - f - pq \Rightarrow q(p) = \left(\phi'\right)^{-1}(p)$$

- Consumer subscribes if (participation constraint):

$$\phi(q(p)) + y - f - pq(p) \geq y$$

Two Part Tariff

- The monopolist's problem:

$$\max_{f,p} pq(p) + f - c\left(q(p)\right)$$

$$\text{subject to } \phi(q(p)) + y - f - pq(p) \geq y$$

- Participation constraint must be binding at optimum (otherwise simply increase f).

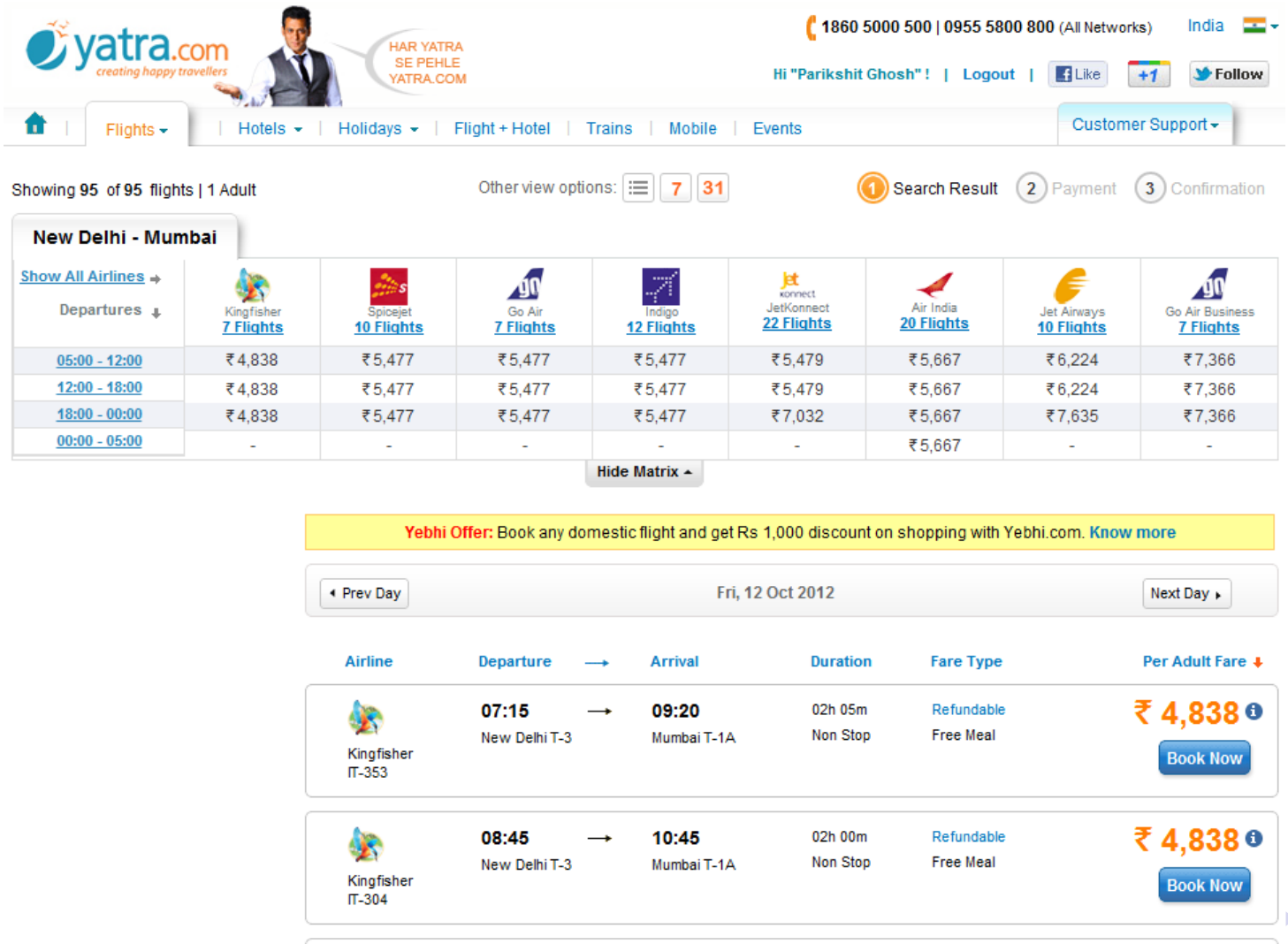
$$\max_{p} \phi(q(p)) - c\left(q(p)\right) \Rightarrow \phi'(\widehat{q}) = c'(\widehat{q})$$

- If the market were competitive (price taking behaviour):

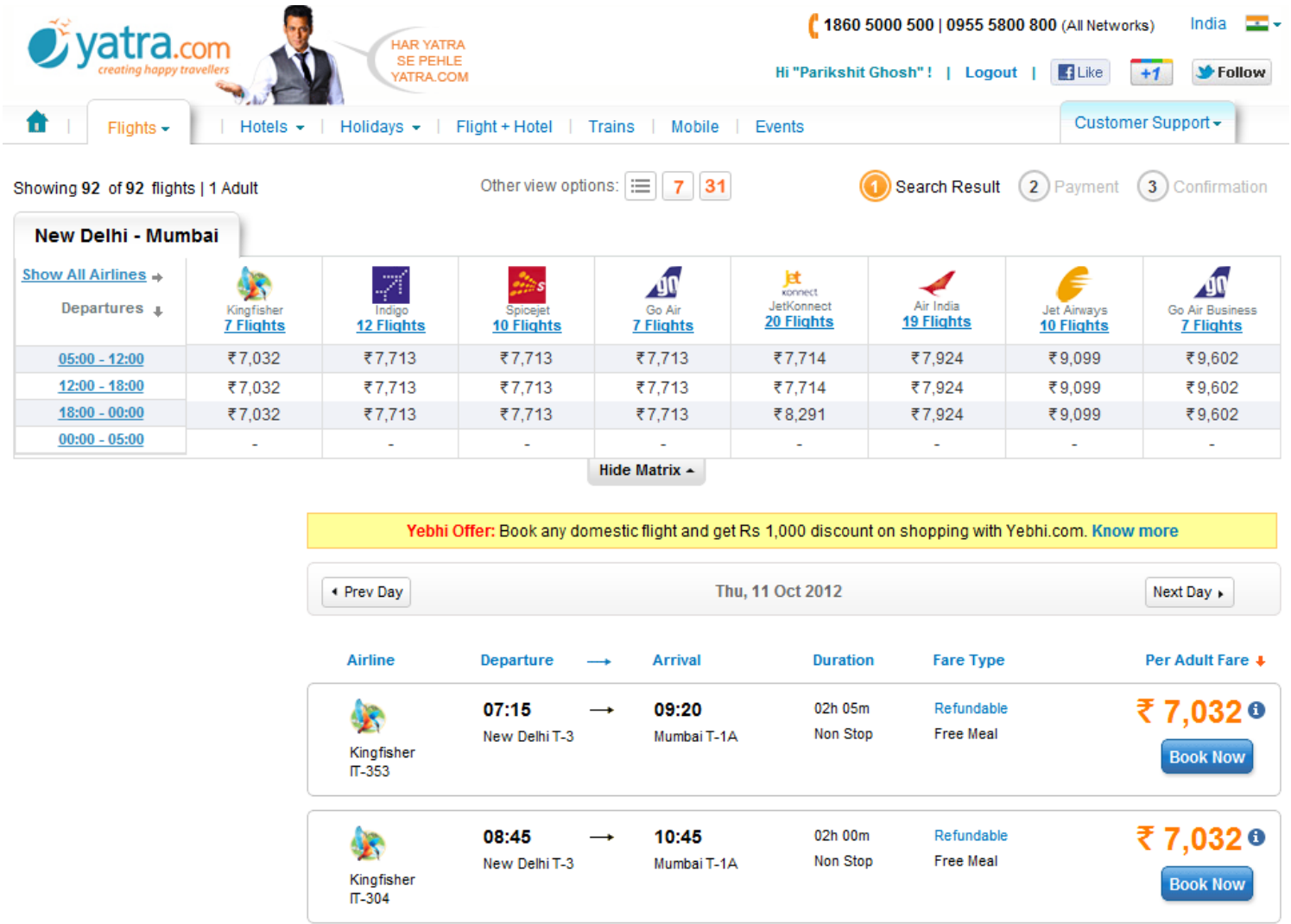
$$\phi'(q^*) = c'(q^*)$$

- Again, two-part tariffs remove the monopolistic distortion.

Airfares: More Than 2 Weeks



Airfares: Less Than 2 Weeks



Hardcover vs. Paperback: Prices

Hardcover vs. Paperback: Date and Sales

Product Details

Hardcover

Publisher: Alfrrd a Knopf Inc,2008 (January 1, 2008)

ASIN: B008254ZBA

Average Customer Review: ★★★★★ (1 customer review)

Amazon Best Sellers Rank: #488,973 in Books (See Top 100 in Books)

Product Details

Paperback

Publisher: Vintage Books / Random House (January 1, 2009)

ASIN: B00824X3U0

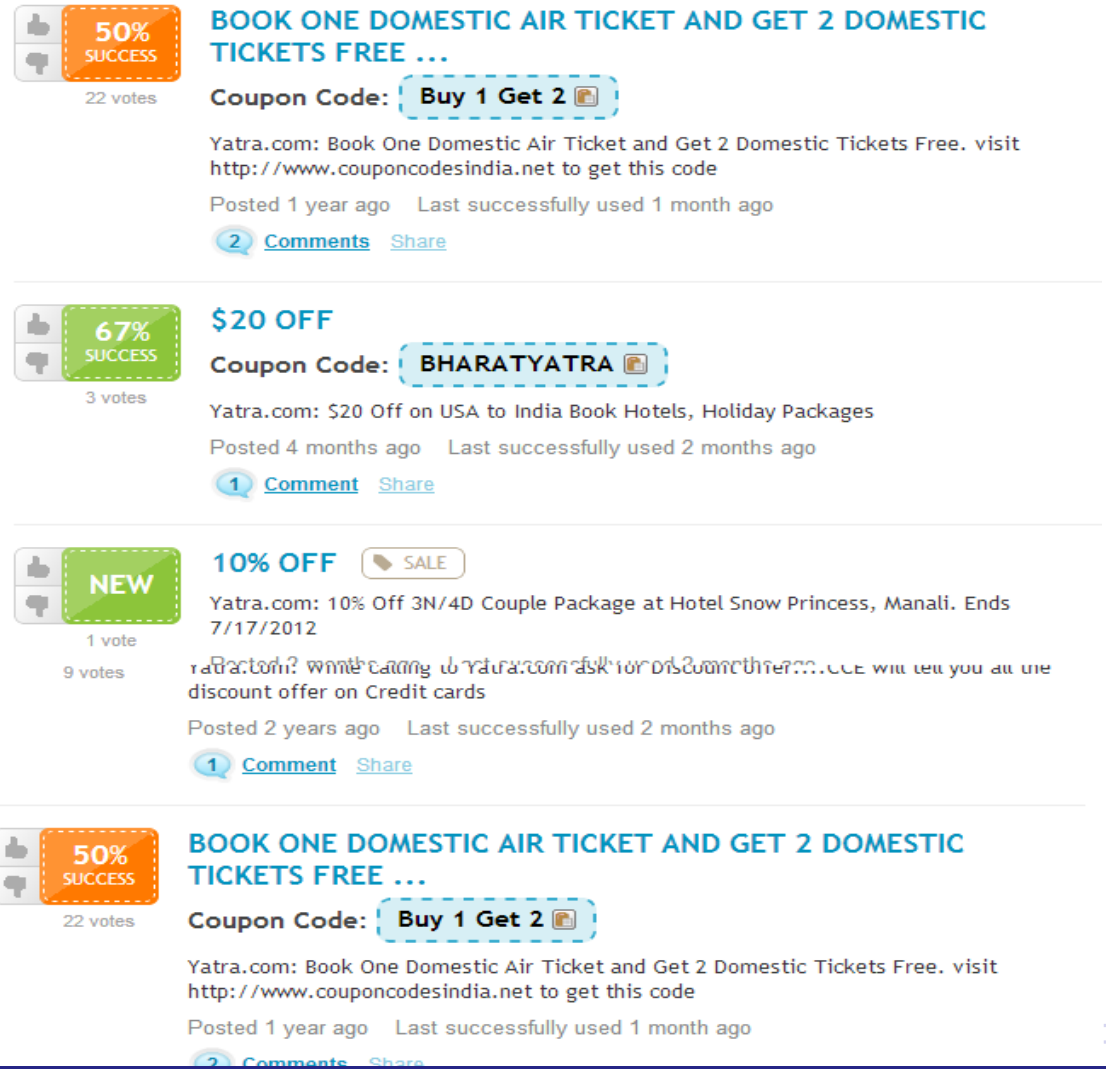
Product Dimensions: 8.4 x 5.8 x 0.7 inches

Shipping Weight: 12.6 ounces

Average Customer Review: ★★★★★ (1 customer review)

Amazon Best Sellers Rank: #163,351 in Books (See Top 100 in Books)

Cash Back Coupons



Rationing

Add to My Calendar Share

Alan Gilbert Conducts *Scheherazade*

Location: Avery Fisher Hall (Directions)
Price Range $30.00 - $90.00

Fri, Sep, 28, 2012
2:00 PM

Sat, Sep, 29, 2012
8:00 PM

Tue, Oct, 2, 2012 BUY TICKETS CREATE 3+
7:30 PM

Rationing

Student Rush

An Amazing Orchestra at an Amazing Price. What a Rush!

Students can purchase $13.50 rush tickets for select concerts up to 10 days before the concert at nyphil.org/studentrush. Concerts may also be available to purchase at the Avery Fisher Hall Box Office on the day of the performance only. Rush tickets are available on a first come, first serve basis, are subject to availability, and **are not available for Orchestra 1, First Tier Center and First Tier Front Boxes**.

Students High School and above MUST present a valid ID at the Avery Fisher Hall Box Office to purchase or pick up tickets. Two tickets maximum can be purchased under each valid ID.

*Senior tickets can only be purchased at the Avery Fisher Hall box office the day of the performance. For senior rush ticket availability please call 212-875-5656.

Price Discrimination by Screening

- Two types of consumers:
  - high value (value $= v_H$, proportion $= \alpha$).
  - low value (value $= v_L$, proportion $= 1 - \alpha$).
- Consumer's valuation is private information.
- Cost of production is 0.
- Uniform pricing strategy:
  - charge $p = v_H$ if $\alpha v_H \geq v_L$.
  - charge $p = v_L$ if$\alpha v_H < v_L$.
- Profit $= \max\{\alpha v_H, v_L\}$. Assume $\alpha v_H < v_L$.

Price Discrimination by Screening

- Monopolist can impose a burden B on consumers (delay, coupons, uncertainty, etc.).
- Cost of the burden is c_H for high value types and c_L for low value types ($c_H > c_L$).
- The burden reduces the willingness to pay of all customers. Its direct impact on profits is negative. Why will the monopolist hurt his own interests?
- By imposing the burden, the monopolist can gather valuable market information which allows him to price discriminate. This indirect benefit may compensate for the direct loss of lower reservation prices.
- The screening technique is useful in other contexts (employers seeking dedicated workers, governments targeting anti-poverty programmes at the poor).

Price Discrimination by Screening

- Monopolist offers a menu $(p_H, 0)$ and (p_L, B), satisfying:
  - **Self Selection Constraint:** H-type chooses $(p_H, 0)$, L-type chooses (p_L, B).

$$\begin{aligned} p_H &\leq p_L + c_H \quad \text{(IC-}H\text{)} \\ p_L + c_L &\leq p_H \quad \text{(IC-}L\text{)} \end{aligned}$$

  - **Participation Constraint:** Both types want to buy.

$$\begin{aligned} v_H - p_H &\geq 0 \quad \text{(PC-}H\text{)} \\ v_L - p_L - c_L &\geq 0 \quad \text{(PC-}L\text{)} \end{aligned}$$

- Monopolist solves: $\max_{p_H, p_L} \alpha p_H + (1-\alpha) p_L$ subject to these constraints.

Price Discrimination by Screening

- **Assumption A:** Relative to the L-type, the H-type gains more from the good than she suffers from the burden:

$$v_H - v_L \geq c_H - c_L$$

- **Step 1:** PC-H is implied by the other constraints and can be dropped.

$$\begin{aligned} v_L + (v_H - v_L) &\geq (p_L + c_L) + (c_H - c_L) && \text{(combining PC-}L\text{ and} \\ \text{or, } v_H &\geq p_L + c_H \\ v_H &\geq c_H && \text{(using IC-}H\text{)} \end{aligned}$$

Price Discrimination by Screening

- **Step 2:** PC-L must be binding at the optimum.
  - If not, increase both p_H and p_L by $\epsilon > 0$.
  - IC's continue to hold, profits have increased.
- **Step 3:** IC-H must be binding at the optimum.
  - Otherwise, increase p_H by $\epsilon > 0$.
  - The other remaining constraints continue to be satisfied.
- **Step 4:** Binding IC-H implies IC-L, so it can be dropped.

Price Discrimination by Screening

- Optimal solution:

$$\begin{aligned} p_L^* &= v_L - c_L \\ p_H^* &= v_L + c_H - c_L \end{aligned}$$

- Assume $v_H > v_L + c_H - c_L$.
- Profit from price discrimination:

$$\begin{aligned} \pi^* &= \alpha(v_L + c_H - c_L) + (1-\alpha)(v_L - c_L) \\ &= v_L + \alpha c_H - c_L \end{aligned}$$

- Price discrimination is better than uniform pricing if $\alpha c_H - c_L > 0$.

Horizontal vs. Vertical Mergers

- Downstream product X (furniture), upstream product Y (wood). Fixed coefficient technology (1:1).
- Each supplied by a separate monopolist, with marginal costs c_x and c_y.
- Inverse demand function for the final good: $p = p(x)$. Let $R(x) = p(x)x$ be downward sloping.
- Let the price charged by the upstream monopolist be q.
- Downstream problem:

$$\max_x R(x) - (c_x + q)x$$

- First order condition:

$$R'(x) = c_x + q$$

Upstream Problem

- The upstream monopolist takes into account the downstream demand function:

$$\max_y yq - c_y y \equiv \max_y y\left[R'(y) - c_x\right] - c_y y$$

- First order condition (replacing y by x^{**}):

$$R'(x^{**}) + x^{**}R''(x^{**}) = c_x + c_y$$

- Suppose the two monopolies merged. Then the firm will solve:

$$\max_x R(x) - (c_x + c_y)x$$

- First order condition shows vertical merger increases efficiency:

$$R'(x^*) = c_x + c_y \Rightarrow x^* > x^{**}$$

Centre and State Taxes

- Let $q = a - bp$ be the demand function.
- Supply is horizontal at some price $\widehat{p}$ (net of taxes).
- The Centre first chooses a tax t_c. Then the State chooses its own tax t_s.
- Both governments aim to maximize tax revenue.
- Centre and State do not coordinate when choosing their tax policies. They do not maximize total government revenue.
- This can be inefficient. Cumulative tax rates are too high.

State's Problem

- The State solves:

$$\max_{t_s} t_s \left[a - b(\widehat{p} + t_c + t_s) \right]$$

- State's FOC defines its reaction function $t_s(t_c)$:

$$a - b(\widehat{p} + t_c) = 2bt_s$$

- Differentiating w.r.t t_c:

$$t_s'(t_c) = -\frac{1}{2}$$

- If Central taxes are higher, State will lower its own taxes to some degree but not completely.

Centre's Problem

- Centre chooses t_c to solve:

$$\max_{t_c} t_c \left[a - b(\widehat{p} + t_c + t_s(t_c))\right]$$

- Replacing State's reaction function:

$$\begin{aligned} t_c^* &= \arg\max_{t_c} \frac{1}{2} t_c \left(a - b\widehat{p} - bt_c\right) \\ &= \frac{a - b\widehat{p}}{2b} \end{aligned}$$

- Plugging back:

$$t_s^* = \frac{a - b\widehat{p}}{4b}$$

Tax Harmonization

- Consider a harmonized single tax rate to maximize tax collection for Centre and State. It solves

$$\max_t t\left[a - b\left(\widehat{p} + t\right)\right]$$

- FOC is gives the optimal tax rate

$$t^* = \frac{a - b\widehat{p}}{2b}$$

- Since $t_s^* = t^*$, we have $t_s^* + t_c^* > t^*$.
- A harmonized single tax will
  - Decrease tax burden and increase consumer's surplus
  - Increase tax collection which can be suitably shared.