Delhi School of Economics
Course 001: Microeconomic Theory
Problem Set 3

1. Find the cost and supply functions for the following production functions, where labour (L) and capital (K) are the two inputs:
   (a) (Cobb Douglas) $q = L^{\alpha}K^{\beta}$ $(\alpha + \beta \leq 1)$
   (b) (Linear) $q = aL + bK$ $(a, b > 0)$
   (c) (Leontief) $q = \min\left\{\frac{L}{a}, \frac{K}{b}\right\}$

2. Consider the CES production function (a and ρ are constants):
$$f(L, K) = \left[aL^{\frac{1}{\rho}} + (1-a)K^{\frac{1}{\rho}}\right]^{\rho}$$
   (a) What is the returns to scale for this production function?
   (b) Define the elasticity of substitution as follows:
$$\sigma_{LK} = \frac{d\ln\left(\frac{K}{L}\right)}{d\ln\left(\frac{f_L}{f_K}\right)}$$
   Calculate σ_{LK} for the above production function and show that it is a constant.
   (c) Show that for special values of the parameter ρ, the CES production reduces to the Cobb Douglas, linear and Leontief forms.
   (d) Derive the cost function and supply function for a firm that has the CES technology.

3. Derive the supply function for a price taking firm when
   (a) the firm's production function is $y = l^{\frac{1}{4}}k^{\frac{1}{2}}$, and the price of inputs l and k are w and r respectively.
   (b) the firm can produce output in two different factories with individual cost functions $c_1(y_1) = ay_1^2$ and $c_2(y_2) = by_2^2$, and the firm's total output is $y = y_1 + y_2$.

4. A perfectly competitive firm can buy labour (l) and capital (k) at prices w and r, and sell output (q) at price p. The firm can produce output in two different factories with production functions as follows:
$$\begin{aligned} q_1 &= \min\left\{\sqrt{l_1}, \sqrt{k_1}\right\} \\ q_2 &= \min\left\{2\sqrt{l_2}, 2\sqrt{k_2}\right\} \end{aligned}$$
   (a) Do these production functions display constant, increasing or decreasing returns to scale? What is the degree of homogeneity of each function?
   (b) Derive the firm's supply function and input demand functions for labour and capital.
   (c) Suppose the firm has 1 unit of capital purchased in the past. The firm cannot buy additional capital or sell any existing capital on the market, but can allocate the 1 unit across the two factories in any way it likes. Derive the supply function and the demand for labour under this restriction.

5. The market for bhel puri is perfectly competitive. Each firm has a cost function given by
$$c(q) = 8 + 2q^2$$
   where q is the output produced by the firm. Any firm which stops production has no cost. The inverse demand function is given by
$$Q = 20 - p$$

(a) Suppose the number of firms in the market is fixed at 4. Find the equilibrium price and quantity.

(b) Now suppose there is free entry and exit in this market. New entrants will have the same cost function. What is the equilibrium price, quantity and number of firms in the long run?

(c) The government imposes a sales tax of 2 per unit on producers. Find the effect on market price, quantity and number of firms in the long run. What is the deadweight loss due to the tax?