Delhi School of Economics
Course 001: Microeconomic Theory
Problem Set 5

1. Consider the following lotteries

$$\begin{aligned} A &= \text{Rs. } 100 \text{ with prob } \frac{1}{3}, \text{ Rs. } 50 \text{ with prob } \frac{1}{3}, \text{ Rs. } 20 \text{ with prob } \frac{1}{3} \\ B &= \text{Rs. } 100 \text{ with prob } \frac{3}{4}, \text{ Rs. } 20 \text{ with prob } \frac{1}{4} \\ C &= \text{Rs. } 100 \text{ with prob } \frac{1}{4}, \text{ Rs. } 20 \text{ with prob } \frac{3}{4} \\ D &= \text{Rs. } 50 \text{ with prob } 1 \end{aligned}$$

and the following preference orderings

$$\begin{aligned} \text{Preference ordering 1} &: B \succ D \succ A \succ C \\ \text{Preference ordering 2:} &\quad C \succ A \succ D \succ B \\ \text{Preference ordering 3:} &\quad B \succ D \succ C \succ A \end{aligned}$$

   (a) Which of these preference orderings cannot be represented by a von Neumann utility function?

   (b) Which of these preferences cannot be represented by a von Neumann utility function if we make the additional assumption that more money is preferred to less?

2. A risk averse farmer has preferences described by the utility function: $u(y) = \log y$, where y is the market value of his crop (and hence his income) in rupees. In a normal season, $y = 100$ but in case there is a flood, Rs 75 worth of crops will be destroyed and the farmer will be left with only $y = 25$. It is known that the probability of a flood is $\frac{1}{2}$.

   (a) Derive the coefficient of absolute risk aversion and the certainty equivalent of the lottery faced by the farmer.

   (b) An insurance company is selling crop insurance at a premium p, i.e., the farmer can insure any amount x (up to Rs 75) of the crop value by paying Rs p per rupee insured. Derive the farmer's demand function for insurance, $x(p)$.

   (c) The insurance company has administrative costs of Rs 5 *per client*. This amount is independent of how much insurance the client buys. The government wants to regulate the insurance market by setting the insurance premium p so that the company just breaks even, i.e., earns zero expected profit. What value of p should the government choose? (It is enough to derive an equation in p whose solution will give us the answer. You do not have to compute an exact numerical value).

3. An agent facing retirement has preferences captured by the Von Neumann Morgenstern utility function $u(c) = \sqrt{c}$, where c is consumption after retirement. The agent has a total saving of w, which he can hold partly in the form of a risky asset and partly in the form of a safe asset (e.g. cash). If the agent invests x in the risky asset ($0 \leq x \leq w$), he receives $2x$ with probability α and 0 with probability $1 - \alpha$ upon retirement. If he invests y in the safe asset ($x + y = w$), he receives y for sure upon retirement. Retirement consumption c is the sum of the returns from the safe and risky assets.

   (a) Solve for this agent's optimal portfolio choice, i.e., how he will allocate his savings w across safe and risky assets.

   (b) For what values of α is the entire saving invested in the safe asset? For what values of α is it invested entirely in the risky asset?

   (c) Suppose there is a financial derivative market which offers the opportunity to bet at fair odds against the event that the risky asset will produce positive returns, i.e., if the agent bets an amount z, he must pay z if the return is positive but will receive pz if the return is 0, where p is such that the issuer of the bet makes zero expected profit. Calculate the amount invested in the risky asset, the amount placed on bet and the increase in consumer utility from having access to betting.

4. Consider another portfolio choice problem. An agent with wealth w must allocate amount $x \in [0, w]$ to asset A and the remaining to asset B. Asset A either doubles the amount investmented or reduces it to zero, each outcome being equally likely. Asset B either trebles the amount invested in it, or reduces it to zero, again both outcomes being equally likely. The returns from assets A and B are statistically uncorrelated. The agent's consumption c is the value of his portfolio after all returns are realized, and his VNM utility function is $u(c) = \ln c$.

   (a) Solve for the optimal portfolio choice.

   (b) Give some intuition why the agent will choose to invest in both assets even though one has higher expected returns than the other.

5. Monty Hall, an American TV personality, used to offer contestants the following choice on his televised game show. The contestant would be positioned before three closed doors, one of which has a valuable prize behind it (e.g., a car), while the other two are empty. The contestant is then asked to choose one door, and if the prize object happens to be behind it, he will win it. However, after a choice was made, Monty would typically pull a stunt. He would open one of the remaining two doors, reveal it to be empty, and then ask the contestant if he wants to stick with his choice or switch to the other closed door. Does switching increase the chance of winning? Carefully spell out your assumptions.