Delhi School of Economics
Course 001: Microeconomic Theory
Solutions to Problem Set 4

1. (a) The monopoly price and quantity are:

$$q_m = \arg\max_q q(\alpha - \beta q) - aq^2 = \frac{\alpha}{2(a+\beta)}$$
$$p_m = \frac{\alpha(2a+\beta)}{2(\alpha+\beta)}$$

Competitive quantity q^* is given by (remember $p = MC$):

$$\alpha - \beta q^* = 2aq^* \Rightarrow q^* = \frac{\alpha}{2a+\beta}$$

Deadweight loss is the area between demand and MC curve from monopoly to competitive output levels:

$$\int_{q_m}^{q^*} [\alpha - \beta q - 2aq]\, dq$$

(b) Clearly the government should choose T to siphon off all the profits generated and leave the monopolist indifferent between staying in the market and quitting. Therefore

$$T = q[\alpha - \beta q] - aq^2 - tq \tag{1}$$

The monopolist's choice is

$$q_m(t) = \arg\max_q q(\alpha - \beta q) - aq^2 - tq - T = \frac{\alpha - t}{2(a+\beta)} \tag{2}$$

Clearly, the government can choose t to induce any q it wishes. Let us formulate the government's problem as a choice of q. Keep in mind that net tax revenue will be transfered to consumers, so it must be added to consumers' surplus. Then the problem becomes

$$\max_q CS(q) = \int_0^q [\alpha - \beta x] dx - pq + T + tq$$

Now since the monopolist is pushed down to zero profit, we have

$$pq = aq^2 + tq + T$$

Using this above, the government solves

$$\max_q CS(q) = \int_0^q [\alpha - \beta x] dx - aq^2$$

Therefore it is optimal for the government to induce a q that maximizes social surplus! This is given by

$$\alpha - \beta q^* = 2aq^* \Rightarrow q^* = \frac{\alpha}{2a+\beta} \tag{3}$$

Let t^* be the optimal tax rate. We must have $q_m(t^*) = q^*$. Using (2) and (3), we obtain

$$t^* = -\frac{\alpha\beta}{2a+\beta}$$

The optimal tax is a subsidy—a government working in consumers' interest must subsidize the monopolist! That encourages the monopolist to produce more. Since the resultant profits can always be squeezed out of him with a poll tax and returned to consumers, the government chooses to remove the quantity distortion with a suitable subsidy.

2. (a) The monopsonist's problem is

$$\max_q u(q) - q(a + bq)$$

The FOC gives the optimum choice q^*:

$$u'(q^*) = a + 2bq^* \tag{4}$$

(b) The competitive output level q_c is where demand price (marginal utility) is equal to supply price (marginal cost) i.e.,

$$u'(q_c) = a + bq_c \tag{5}$$

Subtracting (4) from (5):

$$u'(q_c) - u'(q^*) = b(q_c - q^*) - bq^*$$

Suppose $q_c \leq q^*$. Then the RHS is negative. However the LHS is positive since $u''(.) < 0$. This is a contradiction hence $q_c > q^*$.

(c) Using (4), we get for log utility:

$$q^* = \frac{\sqrt{a^2 + 8b} - a}{4b}$$

and for quadratic utility:

$$q^* = \frac{\alpha - a}{2(b + \beta)}$$

(d) The deadweight loss is given by

$$\int_{q^*}^{q_c} [\alpha - 2\beta q - a - bq] dq$$

3. The monopolist can adopt one of two strategies: set a price of v_H and sell only to H-type consumers, or set a price of v_L and sell to a mix of H and L-types. If v_L is the optimum price, the optimum quantity is given by the MR = MC rule, i.e.,

$$2aq = v_L \Rightarrow q = \frac{v_L}{2a}$$

However, there is always the option of keeping the price at v_H and selling only to H-type consumers. The above output level is optimal only as long as it yields higher profits, i.e., if

$$v_L\left(\frac{v_L}{2a}\right) - a\left(\frac{v_L}{2a}\right)^2 \geq \theta v_H - a\theta^2$$

Let $\underline{a}$ be the value of a for which the above inequality is binding (it is the solution to a quadratic equation). Then, the monopolist will serve some L-type consumers if and only if $a \leq \underline{a}$. When it is optimal to sell only to H-types (but not all of them), the MR = MC rule implies

$$2aq = v_H \Rightarrow q = \frac{v_H}{2a}$$

This is the true optimum as long as its value is less than the number of H consumers, θ. In other words, for

$$a \geq \frac{v_H}{2\theta} = \overline{a}$$

Therefore, the optimum price-output pair is as follows:

$$(p^*, q^*) = \begin{cases} \left(v_H, \frac{v_H}{2a}\right) & \text{if } a \geq \overline{a} \\ (v_H, \theta) & \text{if } \underline{a} < a < \overline{a} \\ \left(v_L, \frac{v_L}{2a}\right) & \text{if } a \leq \underline{a} \end{cases}$$

4. (a) The proportion of high types for each individual good is λ (calculate the marginal distribution). The optimum price is either v_H (extracts all the surplus from high types but loses the market for the low types) or v_L (extracts all the surplus from the low types but not the high types). Therefore the monopolist will set a high price of v_H if $\lambda v_H > v_L$, otherwise a low price of v_L.

(b) See below. I'll leave the exact details of pricing the bundle optimally to you.

(c) Since the value of the bundled good to a consumer can only take three values—$2v_H$, $2v_L$ or $(v_H + v_L)$—these are the only candidates for optimum price. Charging $2v_H$ or $2v_L$ essentially replicates some pricing strategy for the unbundled goods and therefore cannot yield a profit level that is higher than what could be earned by selling them separately. Let us then calculate the profit obtained when the bundled good is sold at a price $(v_H + v_L)$. This is given by

$$\left[1 - (1 - \lambda)^2\right](v_H + v_L)$$

To see why, note that all consumers with valuations (v_H, v_H), (v_H, v_L) and (v_L, v_H) will buy the bundled good at the offered price. Add up the numbers in all the cells except the bottom right to get the mass of buyers.

Now the bundling strategy is optimum if it yields higher profits than separate sales, i.e.,

$$\left[1 - (1 - \lambda)^2\right](v_H + v_L) > \max\{2\lambda v_H, 2v_L\}$$

This can be rewritten as

$$\left[\frac{1 + (1 - \lambda)^2}{\lambda(2 - \lambda)}\right] v_L < v_H < \left(\frac{2 - \lambda}{\lambda}\right) v_L$$

It is easy to verify that the upper bound is higher than the lower bound for $\lambda < 1$ and hence the set of parameters for which bundling is optimum is non-empty.

5. This question is about monopoly and regulation.

(a) First, find the supply from the competitive fringe at any price p set by the dominant firm. Using the rule $p = MC$, each firm supplies $y(p) = p$, and so the competitive fringe as a whole supplies $Y(p) = 50p$. Subtracting this from total demand, we get the dominant firm's "residual demand" function

$$Q = 1000 - 100p$$

The firm then solves

$$\max_p p(1000 - 100p) \Rightarrow p^* = 5$$

Its optimum quantity is 500, and its maximized profit is 2,500. Note that the competitive fringe produces an additional quantity of 250 at this price, so the total quantity consumed by buyers is 750. Also the profit of each firm in the fringe is $py(p) - \frac{1}{2}y(p)^2 = \frac{25}{2}$.

(b) (i) In a perfectly competitive market (removing the dominant firm), equilibrium implies demand and supply are equal, i.e.,

$$1000 - 50p = 50p \Rightarrow p_c = 10, q_c = 500$$

(ii) Under pure monopoly (no competitive fringe), the firm solves

$$\max_p p(1000 - 50p) \Rightarrow p_m = 10, q_m = 500$$

It may seem curious that the outcome is the same under pure monopoly and pure competition, but the puzzle is easily resolved if you observe that the monopolist has a cost advantage over the competitive fringe. The usual monopolistic distortion is exactly cancelled by this cost advantage.

(c) Comparing, we can see that the competitive fringe softens the impact of monopoly—it produces lower prices, more output and higher consumers' surplus. Calculation should show that it also produces higher social surplus.

(d) First, the government should only allow the most cost efficient firm to produce, i.e., allow the dominant firm and ban all the fringe firms. However, this will lead to the usual monopolistic distortion, so to prevent that, the government needs to introduce price regulation and force the dominant firm to sell at its marginal cost, i.e., 0. To keep the firms (both dominant and fringe) at the same level of profits as in part (a), they have to be paid 2,500 and 25/2 respectively. Since there are 50 fringe firms, the total compensation payments amount to $2,500 + 50\left(\frac{25}{2}\right) = 3,125$. When $p = 5$, consumers' surplus was $\frac{1}{2} \times (20 - 5) \times 750 = 5,625$. When $p = 0$, consumers' surplus is $\frac{1}{2} \times 20 \times 1000 = 10,000$. The increase in consumers' surplus is 4,375. Even after paying off the firms to the tune of 3,125 by taxing consumers, they are still left with a gain of 4,375 - 3,125 = 1,250. This illustrates how suitable regulation can bring about a Pareto improvement.