Asymmetric Information: Lecture 3

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Second Context

Two parties: Monopolist and a consumer

- **Principal**: The monopolist
- **Agent**: A consumer

Payoff functions:

- **Principal**: $T(q) - cq$, where c is MC, $T(q)$ is the price charged for q units.
- **Agent**: $\theta_i u(q) - T$, $u'(q) > 0$ and $u''(q) < 0$.
- $\theta \in \{\theta_1, ..., \theta_n\}$ is the type of consumer.
- The Principal faces uncertainty about θ

First Best

For θ_i type of consumer, the social planner will solve

$$\max_q \{\theta_i u(q) - cq\}, i.e.,$$

$$\theta_i u'(q_i^*) = c.$$

Clearly $q_1^* < q_2^* < ... < q_n^*$.
The FB can be achieved when the principal can observe θ.

Symmetric Information I

Suppose,

- the principal can observe θ.
- for type θ_i, principal offers (q_i, T_i).

Clearly, for type θ_i, the principal will solve

$$\max_{T_i, q_i}\{T_i - cq_i\}$$

s.t. $\theta_i u(q_i) - T_i \geq \bar{u}_i (= 0), \;\; i = 1, ..., n, \qquad (IR)$
At the optimum IR binds for each type, i.e.,

$$T_i = \theta_i u(q_i), \;\; \forall i = 1, ..., n.$$

Question

Why there are no ICs when θ is observable to P?

Symmetric Information II

Therefore, the principal's problem is equivalent to:

$$\max\{\theta_i u(q_i) - cq_i\}$$

the f.o.c. is

$$\theta_i u'(q_i) = c, \;\; \forall i = 1, ..., n.$$

That is,

- the FB consumption and production levels are achieved
- the principal is able to discriminate perfectly
- $T_i = \theta_i u(q_i)$ appropriates all the surplus from all types.

Second Best I

Suppose,

- $\theta \in \{\theta_1, \theta_2\}$, and $\theta_1 < \theta_2$.
- principal knows only the probability distribution of θ, and $Pr(\theta = \theta_1) = \nu$.
- principal offers a menu of $\{(q_i, T(q_i))\}$ to consumers, $i = 1, 2$. That is, offers $\{(q_1, T_1), (q_2, T_2)\}$

However,

- The consumer chooses the pair that maximizes her utility.
- That is, θ_i type consumer chooses (q_i, T_i), if

$$q_i = argmax_{q_j \in \{q_1, q_2\}} \{\theta_i u(q_j) - T_j\}, \quad i = 1, 2 \qquad (IC)_i$$

Second Best II

Principal solves:

$$\max_{(q_1,T_1),(q_2,T_2)} \{\nu[T_1 - cq_1] + (1-\nu)[T_2 - cq_2]\}$$

s.t. IRs and ICs:

$$\begin{aligned} \theta_1 u(q_1) - T_1 &\geq 0, \quad (1) \\ \theta_2 u(q_2) - T_2 &\geq 0. \quad (2) \end{aligned}$$

$$\begin{aligned} \theta_1 u(q_1) - T_1 &\geq \theta_1 u(q_2) - T_2 \quad (3) \\ \theta_2 u(q_2) - T_2 &\geq \theta_2 u(q_1) - T_1 \quad (4) \end{aligned}$$

Now you can easily show that:

- (4) and (1) imply that (2) holds.
- (4) and (1) will both bind.

Second Best III

Now, ignore (4), and (4) and (1) to get rid of T_1 and T_2. So, Principal's problem becomes:

$$\max_{(q_1,q_2)} \{\nu[\theta_1 u(q_1) - cq_1] + (1-\nu)[\theta_2 u(q_2) - cq_2] - (1-\nu)(\theta_2 - \theta_1)u(q_1)\}$$

q_2^{SB} and q_1^{SB} respectively solve:

$$\theta_2 u'(q_2^{SB}) = c \quad (5)$$

$$\theta_1 u'(q_1^{SB}) = \frac{c}{1-K}, \quad (6)$$

where $K = \frac{1-\nu}{\nu}\frac{\theta_2-\theta_1}{\theta_1} > 0$.

$$q_2^{SB} = q_2^* \ \& \ q_1^{SB} < q_1^*.$$

Second Best IV

Again,

- Allocations are monotonic - more efficient type consumes more
- efficient allocation at the top, but inefficient for the low type
- allocative inefficiency, $q_2^* - q_2^{SB}$, increases with $\Delta\theta = \theta_2 - \theta_1$ and with $\frac{1-\nu}{\nu}$.
- *Ceteris paribus* the rent yielded to the efficient type increases with $\Delta\theta = \theta_2 - \theta_1$.

Note that

$$T_1 = \theta_1 u(q_1) \ \& \ T_2 = \theta_2 u(q_2) - \Delta\theta u(q_1)$$

Second Best V

Examples of Contracts:

$$
\begin{aligned}
C1: &\quad (q_1^*, \theta_1 u(q_1^*)), (q_2^*, \theta_2 u(q_2^*)) \\
C2: &\quad (0,0), (q_2^*, \theta_2 u(q_2^*)) \\
C3: &\quad (q_1^*, \theta_1 u(q_1^*)), (q_1^*, \theta_1 u(q_1^*)) \\
C4: &\quad (q_1^{SB}, \theta_1 u(q_1^{SB})), (q_2^{SB}, \theta_2 u(q_2^{SB}) - \Delta\theta u(q_1)),
\end{aligned}
$$

where q_1^{SB} and q_2^{SB} are as above.

Question

- *What are the actions available to agents under each of the above contracts?*
- *What are the outcomes of the above contracts?*
- *For P, which of the above contracts is the best?*

More General Schemes I

Question

Can the principal do better for herself by offering more general/complicated contracts?

Suppose: Principal offers wider choice set $[q, T_i(q)]$, for $i = 1, 2$, where $q \in Q \subset \Re_+$ and $T_i(q)$ is some function

$$T_i : Q \mapsto \Re_+.$$

Principal solves:

$$\max_{(T_1(q), T_2(q))} \{\nu[T_1(q_1) - cq_1] + (1 - \nu)[T_2(q_2) - cq_2]\}$$

Principal can offer even a wider choice set $[q, T(q)]$, where $q \in Q \subset \Re_+$ and $T(q)$ is **any** function, i.e.,

$$T : Q \mapsto \Re_+.$$

More General Schemes II

Now, Principal solves:

$$\max_{(q_1,T_1),(q_2,T_2)} \{\nu[T_1 - cq_1] + (1-\nu)[T_2 - cq_2]\}$$

Under this approach, the agent of type θ_i will choose

$$q_i = q(\theta_i) = \arg\max_{q \in Q}\{U(\theta_i, q, T(q)) \equiv \arg\max_{q \in Q}\{\theta_i u(q) - T(q)\}$$

Question

Does this more general scheme lead to a different outcome? Is the outcome better for the Principal?

Monotonicity of Allocations I

Question

- *In above examples, what accounts for the monotonicity of allocations?*
- *What are the common features of the payoff functions considered so far?*
- *Do the above results hold for any 'plausible' payoff function of the agents?*

Suppose, the agent's utility function is given by

$$\varphi(\theta, x, t) : X \times \Theta \mapsto \mathfrak{R}$$

where $X, \Theta \subset \mathfrak{R}$.
E.g.,

$$\begin{aligned}
\varphi(\theta, x, t) &= \theta u(x) - t \\
\varphi(\theta, x, t) &= u(\theta, x) - t
\end{aligned}$$

Monotonicity of Allocations II

Definition

Single Crossing Property (Simpler form): $\varphi(\theta, x, t)$ satisfies SCP if $\varphi_x(\theta, x, t)$ and $\varphi_t(\theta, x, t)$ exist, and for all $x \in X$:

$$\text{Either } \frac{\partial}{\partial\theta}\left[\frac{\varphi_x(\theta, x, t)}{\varphi_t(\theta, x, t)}\right] > 0, \text{ or } \frac{\partial}{\partial\theta}\left[\frac{\varphi_x(\theta, x, t)}{\varphi_t(\theta, x, t)}\right] < 0.$$

If the payoff functions of the agents satisfy SCP, then the IC for different types cross only once.

Proposition

If the payoff functions of the agents satisfy SCP, then second best allocations will be monotonic.