Problem Set 1, MIEG, Winter Term, 2015

Osborne, An introduction to Game Theory
Chapter 2: 27.1, 31.1, 33.1, 34.1, 44.1
Chapter 3: 59.2, 60.1, 60.2, 63.1, 67.1, 69.1, 74.1, 75.2
Chapter 4: 120.2, 120.3, 128.1, 141.1, 141.3

1. Suppose that in a strategic form game, all players have strongly dominant strategy. Prove that this game has unique Nash equilibrium. Do we get the same result if all players have weakly dominant strategy?

2. A two person strategic form game (with finite number of actions for each player) is called 'strictly competitive' if $u_1(a_1, a_2) = -u_2(a_1, a_2)$ for all $a_1 \in A_1$ and $a_2 \in A_2$.
a) Check that 'Matching coin' game is a strictly competitive game.
An action $a_1^* \in A_1$ is a 'maxminimizer' for player 1 if

$$\min_{a_2 \in A_2} u_1(a_1^*, a_2) \geq \min_{a_2 \in A_2} u_1(a_1, a_2) \text{ for all } a_1 \in A_1$$

Similarly, an action $a_2^* \in A_1$ is a 'maxminimizer' for player 2 if

$$\min_{a_1 \in A_1} u_1(a_1, a_2^*) \geq \min_{a_1 \in A_1} u_1(a_1, a_2) \text{ for all } a_2 \in A_2$$

In words, a maxminimizer for player i is an action that maximizes the payoff that player i can guarantee (irrespective of j's action).
b) Find the maxminimizer of 'Matching coin' game.
c) Suppose that a strictly competitive game has a Nash equilibrium $(\bar{a}_1, \bar{a}_2)$. Show that $\bar{a}_i$ is a maxminimizer of player i.
d) Give an example of strictly competitive game, which has pure strategy Nash equilibrium.

3. [Based on Exercise 34.1 from Osborne]
a) Does this game have any mixed strategy Nash equilibrium?
b) Which strategy profiles survive iterative elimination of strictly dominated strategies?

4. Army A has a single plane with which it can strike one of the three possible targets. Army B has one anti-aircraft gun that can be assigned to to one of the tagets. The value of target k is v_k with $v_1 > v_2 > v_3 > 0$. Army A can destroy a target only if the target is undefended and A attacks it. Army A wishes to maximize the value of the damage and army B wishes to minimize it. Find all Nash equilibria of this game.

5. Two roommates each need to choose to clean their apartment, and each can choose an amount of time $t_i \geq 0$ to clean. If their choices are t_i and t_j, then player i's payoff is given by $[(10 - t_j)t_i - t_i^2]$. This payoff function implies that the more one roommate cleans, the less valuable is cleaning for the other roommate.

a) What is the best response correspondence of each player i?

b) Which choices survive iterative elimination of strictly dominated strategies?

c) Find Pure strategy Nash equilibrium.

6. Find all (Pure and mixed) Nash equilibria of the following game

	M	N
A	9,1	2,4
B	0,3	3,2
C	5,4	0,3

7. Suppose that two friends are dividing a prize worth 100. Each of two players announces an integer between 0 and 100. If $a_1 + a_2 \leq 100$, where a_i is the announcement of player i, then each player i receives a_i. If $a_1 + a_2 > 100$ then the minimum of the two claims get the claimed amount and the other receives the residual. If $a_1 + a_2 > 100$ and both claims are the same then the prize is equally divided. Use iterative elimination of weakly dominated strategies to solve this game.