Hidden Information

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Contexts

Two players: Principal and Agent

Example

- Principal (an insurance company) and Agent an insuree
- Principal (a firm) and Agent a worker/manager
- Principal (a litigant) and Agent a lawyer
- Principal (a government) and Agent a firm
- Principal (a monopolist) and Agent a consumer
- Principal (a landlord) and Agent a tenant
- Principal (consumers) and Agent a firm

Assumption: The payoffs from the outside option are zero for both the parties.

Production Technology I

Cost of production $C(q,\theta) = \theta q + F$, where

$$\theta \in \{\theta_1, ..., \theta_n\},\ s.t.\ \theta_1 < \theta_2 ... < \theta_n$$

and

$$Pr(\theta = \theta_i) = \nu_i.$$

Let $\theta \in \{\theta_1, \theta_2\}$ and $Pr(\theta = \theta_1) = \nu$.

The (money) value of q units of output to the principal is

$V(q)$, where $V'(q) > 0$ and $V''(q) < 0$.

Therefore, for given $\theta \in \{\theta_1, \theta_2\}$, the FB is a solution to

$$\max_q \{V(q) - C(q,\theta)\}, i.e.,\ \max_q \{V(q) - \theta q - F\}$$

$$V'(q) = \theta$$

Production Technology II

Let q_i^* solve

$$V'(q) = \theta_i.$$

Clearly $q_1^* > q_2^*$.

Assumption: $W(q_i^*) = V(q_i^*) - \theta_i q_i - F \geq 0$ for all $i = 1, 2$. Indeed

$$W(q_2^*) = V(q_2^*) - \theta_2 q_2^* - F \geq 0 \Rightarrow W(q_1^*) = V(q_1^*) - \theta_1 q_1^* - F > 0.$$

Let $F = 0$.

First Best: Symmetric Information

Let

$$t_i^* = \theta_i q_i^* .$$

When there is no informational asymmetry about θ, the Principal will make the following *take it or leave it* offers:

- If $\theta = \theta_1$ the agent is offered (q_1^*, t_1^*), i.e., t_1^* for production level q_1^*; and
- If $\theta = \theta_2$ the agent is offered (q_2^*, t_2^*), i.e., t_2^* for production level q_2^*

Since each agent gets non-negative utility from the offer, they accept it.

The first best payoff for the principal is

- $V(q_1^*) - \theta_1 q_1^*$, if $\theta = \theta_1$;
- $V(q_2^*) - \theta_2 q_2^*$, if $\theta = \theta_2$;
- Ex-ante payoff is
  $V^* = W^* = \nu(V(q_1^*) - \theta_1 q_1^*) + (1-\nu)(V(q_2^*) - \theta_2 q_2^*), i.e.,$

the principal is able to appropriate the entire surplus from the trade. So the job delegation is costless for the principal.

Second Best: Hidden Information I

Assumption: The principal does not observe θ. However, know the distribution: knows that

$$Prob(\theta = \theta_1) = \nu.$$

Let

$$\Delta\theta = \theta_2 - \theta_1.$$

$\Delta\theta > 0$ is a measure of spread of uncertainty about agent type.

Contractible variables are q and monetary transfers from principle to the agent t.

Definition

Contracts: A contract is a feasible, observable and verifiable allocation (q, t). The set of contracts is

$$\mathcal{A} = \{(q, t) : q \in \mathcal{R}_+, t \in \mathcal{R}\}.$$

Second Best: Hidden Information II

Proposition

When principal does not observe θ, contract $((q_1^*, t_1^*), (q_2^*, t_2^*))$ cannot be implemented.

Note under contract $\{(q_1^*, t_1^*), (q_2^*, t_2^*)\}$,

$$U_1^* = t_1^* - \theta_1 q_1^* = 0 \text{ and } U_2^* = t_2^* - \theta_2 q_2^* = 0, i.e.,$$

truth telling means that each agent's payoff is equal to zero, i.e., the outside (reservation) payoff. However,

$$\theta_2 > \theta_1 \Rightarrow t_2^* - \theta_1 q_2^* > t_2^* - \theta_2 q_2^* = 0.$$

So

$$t_2^* - \theta_1 q_2^* > t_1^* - \theta_1 q_1^* = 0, \ i.e.,$$

type 1 agent is better of mimicking as type 2.

Second Best: Hidden Information III

Definition

A menu of contracts $\{(q_1, t_1), (q_2, t_2)\}$ is incentive compatible if

$$t_1 - \theta_1 q_1 \geq t_2 - \theta_1 q_2 \tag{1}$$
$$t_2 - \theta_2 q_2 \geq t_1 - \theta_2 q_1 \tag{2}$$

Definition

A menu of contracts $\{(q_1, t_1), (q_2, t_2)\}$ is incentive compatible and (rationality) feasible if it satisfies (1) and (2) and

$$t_1 - \theta_1 q_1 \geq 0 \tag{3}$$
$$t_2 - \theta_2 q_2 \geq 0 \tag{4}$$

Second Best: Hidden Information IV

Example

Consider contract $\{(q_2^*, t_2^*), (q_2^*, t_2^*)\}$. You can check that this contract

- is incentive compatible, and
- results in pooling of types.

Example

Consider contract $\{(q_1^*, t_1^*), (0, 0)\}$. You can check that this contract

- is incentive compatible, but
- results in shutting down of inefficient types.

Second Best: Hidden Information V

Remark

Properties of Incentive Compatible contracts:

- Monotonicity of Output: Adding (1) and (2) gives us

$$(\theta_2 - \theta_1)q_1 \geq (\theta_2 - \theta_1)q_2, i.e.,$$

$$q_1 \geq q_2.$$

  In fact, any pair (q_1, q_2) is implementable iff $q_1 \geq q_2$.

- (1) and (4) imply that as long as $q_2 > 0$,

$$t_1 - \theta_1 q_1 > 0, i.e.,$$

  if inefficient type is required to produce, the payoff of the efficient type will be positive.

Second Best: Hidden Information VI

The principal's optimization problem is

$$\max_{(t_1,q_1),(t_2,q_2)} \{\nu(V(q_1)-t_1)+(1-\nu)(V(q_2)-t_2)\}$$

s.t., (1)-(4).
Let $U_1 = t_1 - \theta_1 q_1$ and $U_2 = t_2 - \theta_2 q_2$. We rewrite (1)-(4) as

$$\begin{aligned} U_1 &\geq U_2 + \Delta\theta q_2 && (5) \\ U_2 &\geq U_1 - \Delta\theta q_1 && (6) \end{aligned}$$

$$\begin{aligned} U_1 &\geq 0 && (7) \\ U_2 &\geq 0 && (8) \end{aligned}$$

Second Best: Hidden Information VII

Now the principal's optimization problem

$$\max_{(t_1,q_1),(t_2,q_2)} \{\nu(V(q_1) - t_1) + (1-\nu)(V(q_2) - t_2)\}$$

can be rewritten as

$$\max_{(U_1,q_1),(U_2,q_2)} \{\nu(V(q_1) - \theta_1 q_1) + (1-\nu)(V(q_2) - \theta_2 q_2) - (\nu U_1 + (1-\nu)U_2)\}$$

s.t., (5)-(8).

$$\underbrace{\nu(V(q_1) - \theta_1 q_1) + (1-\nu)(V(q_2) - \theta_2 q_2)}_{allocative\ efficiency} - \underbrace{(\nu U_1 + (1-\nu)U_2)}_{information\ rent}$$

Second Best: Hidden Information VIII

Consider a contract

$$\{(\theta_1 q_1^* + \Delta\theta q_2^*, q_1^*), (\theta_2 q_2^*, q_2^*)\}, \, i.e.,$$

$$\{(U_1 = \Delta\theta q_2^*, q_1^*), (U_2 = 0, q_2^*)\}.$$

- It is incentive feasible and implements the FB.
- But, will principal offer this contract?