Lecture 1: Hidden Action

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Contexts

Two players: Principal and Agent

Example

- Principal as a firm and Agent as a worker
- Principal as the owner(s) and Agent as the Manager
- Principal as a landlord and Agent as a Tenant
- Principal as a client and Agent as a Professional service provide

Production Technology I

- $q =$ output; $q = q(e, \epsilon)$;
- $e =$ effort level opted by the agent
- $\epsilon =$ a random variable, a noise term;
- $q \in \{q_L, q_H\}$. Let $\{q_L, q_H\} = \{0, 1\}$;
- $p(e) = Pr(q = 1|e)$ is the probability of the realized output being 1;
- $1 - p(e) = Pr(q = 0|e)$ is the probability of the realized output being 0;

$$p'(.) > 0, \ p''(.) < 0, \ p'(0) > 1;$$

- $w =$ wage paid by the principal to the agent; $w(.) = w(q)$.

Production Technology II

Payoffs:

- Principal: $V(q-w)$, $V' > 0$, $V'' \leq 0$;
- Agent: $u(w,e) = u(w) - \psi(e)$, where $u' > 0$, $u'' \leq 0$, $\psi'(e) > 0$, $\psi''(e) \geq 0$;
- $\psi(e)$ is the cost of effort e to the agent.
- Let $\psi(e) = e$. Therefore,

$$u(w,e) = u(w) - \psi(e) = u(w) - e.$$

Assumptions:

- The payoffs from the outside option are zero for both the parties;
- q and w are contractible;
- e may or may not be contractible.

Complete Information (FB): No Hidden Action I

Assume that there is no informational asymmetry w.r.t. the effort level opted by the agent. Formally, assume that e is contractible. Let,

$$w_1 = w(1) \text{ and } w_0 = w(0).$$

In that case, the Principal will solve:

$$\max_{w_0, w_1, e} \{p(e)V(1 - w_1) + (1 - P(e))V(-w_0)\}$$

s.t

$$p(e)u(w_1) + (1 - P(e))u(w_0) - e \geq \bar{u} = 0. \tag{IR}$$

where $\bar{u} = 0$ denotes the outside payoff for the agent.

Complete Information (FB): No Hidden Action II

Letting λ denote the lagrangian multiplier, the *foc* w.r.t. w_1 and w_0 are given by (1) and (2), respectively:

$$\begin{aligned} -p(e)V'(1-w_1)+\lambda p(e)u'(w_1) &= 0 \quad (1)\\ (1-p(e))V'(-w_0)+\lambda(1-p(e))u'(w_0) &= 0 \quad (2) \end{aligned}$$

(1) and (2) together imply

$$\frac{V'(1-w_1)}{u'(w_1)} = \lambda = \frac{V'(-w_0)}{u'(w_0)} \quad (3)$$

So, $\lambda > 0$, i.e., IR will bind. foc w.r.t e is

$$p'(e)[V(1-w_1)-V(-w_0)]+\lambda p'(e)[u(w_1)-u(w_0)]-\lambda = 0 \quad (4)$$

So the optimum (FB) contract will be $\{(w_0, w_1), e^*\}$, where w_1 and w_0 solve (1) and (2) respectively, and e^* solves (4).

Complete Information (FB): No Hidden Action III

Note: (3) gives the risk sharing rule for the SB.

When e is contractible,

- deviation from the e^* can be detected.
- e^* can be implemented by imposing heavy penalties if the agent does not put in e^*.
- The Principle can extract entire surplus from production.

Complete Information (FB): No Hidden Action IV

Special Cases:

Case One: *Risk-neutral Principal:* In this case, assuming $V(x) = x$, $V'(1 - w_1) = V'(-w_0)$ is constant. So, (3) implies

$$\frac{1}{u'(w_1)} = \lambda = \frac{1}{u'(w_0)}, i.e.,$$

$$w_1 = w_0 = w^*.$$

Since IR binds, w^* is such that

$$u(w^*) = e^*. \tag{5}$$

In view of this (4) can be reduced to

$$p'(e^*) = \frac{1}{u'(w^*)}. \tag{6}$$

Complete Information (FB): No Hidden Action V

Case Two: *Risk-neutral Agent:* In this case, assuming $u(x) = x$, $u'(w_1) = u'(w_0)$ is constant. So, (3) implies

$$V'(1 - w_1) = \lambda = V'(-w_0), i.e.,$$

$$1 - w_1 = -w_0, \ i.e., \ w_1 - w_0 = 1.$$

In view of this (4) can be written as

$$p'(e^*) = 1. \tag{7}$$

Under this contract, the P's payoff is

$$p(e^*)V(1 - w_1) + (1 - P(e^*))V(-w_0), i.e.,$$

$$p(e^*)V(-w_0) + (1 - P(e^*))V(-w_0) = V(-w_0).$$

And, Since IR binds, the expected cost of inducing e^* is simply

$$p(e^*)w_1 + (1 - P(e^*))w_0 = e^*. \tag{8}$$

Second Best: Hidden Information I

When effort is not be observed by the principal,

- There is informational asymmetry w.r.t. the effort level opted by the agent.
- Moreover, effort is not contractible.
- So wage cannot be a function of the effort.
- Assuming that q is verifiable, wage can depend on the output level q.

Contracts: A contract is a feasible, observable and verifiable allocation (q, w). The set of contracts is

$$\mathcal{A} = \{(q, w) : q \in \mathcal{R}_+, w(q) \in \mathcal{R}\}.$$

In our simple setting, the set of possible contracts is:

$$\mathcal{A} = \{(q, w) : q \in \{0, 1\}, w(q) \in \mathcal{R}\}.$$

Second Best: Hidden Information II

When effort is not be observed by the principal, the Principal's optimization problem is:

$$\max_{w_i,e}\{p(e)V(1-w_1)+(1-P(e))V(-w_0)\}$$

s.t

$$p(e)u(w_1)+(1-P(e))u(w_0)-e\geq\bar{u}=0. \qquad \text{(IR)}$$

$$e=\arg\max_{\hat{e}}\{p(\hat{e})u(w_1)+(1-P(\hat{e}))u(w_0)-\hat{e}\}. \qquad \text{(IC)}$$

In most cases, IC can be replaced by the following foc for the agent's optimization problem

$$p'(e)[u(w_1)-u(w_0)]=1 \qquad (9)$$

Clearly, $w_1>w_0$.

Second Best: Wealthy and Risk-neutral Agent I

Recall, when agent is risk-neutral, the optimum contract is such that $w_1 - w_0 = 1$ and it implements e^*, where e^* such that

$$p'(e^*) = 1.$$

Even when e is not observable, Principal offers a contract (w_0, w_1) such that $w_0 \& w_1$ solve

$$\begin{aligned} w_1 - w_0 &= 1 && (10) \\ p(e^*)w_1 + (1 - P(e^*))w_0 - e^* &= 0 && (11) \end{aligned}$$

Since the agent is risk-neutral, (9) can be written as

$$p'(e)[w_1 - w_0] = 1, i.e., \tag{12}$$

$$p'(e) = 1.$$

Second Best: Wealthy and Risk-neutral Agent II

Therefore, such a contract will induce e^* and the P is able to appropriate all the surplus.

$p(e^*)w_1 + (1 - P(e^*))w_0 - e^* = 0$ can be rewritten as $p(e^*)[w_1 - w_0] + w_0 - e^* = 0$, i.e.,

$$p(e^*) - e^* = -w_0 > 0. \tag{13}$$

The P's payoff from such a contract is

$$\begin{aligned} p(e^*)V(1 - w_1) + (1 - P(e^*))V(-w_0), i.e., & \\ p(e^*)V(-w_0) + (1 - P(e^*))V(-w_0) &= V(-w_0) \end{aligned}$$

Now, if the risk-neutral agent were to own the firm, he will solve

$$\max_e \{p(e)1 + (1 - p(e))0 - e\}$$

Second Best: Wealthy and Risk-neutral Agent III

So he will opt e that solves $p'(e) = 1$, i.e., e^* and his total profit will be $p(e^*) - e^* > 0$, by assumption. Therefore, from (13), we get

$$-w_0 > 0, \ i.e., \ w_0 < 0.$$

Remark

Under a contract (w_0, w_1) satisfying (10) and (11):

- The effort chosen is $e^* = e^{FB}$
- $w_0 < 0 < w_1$
- Total (monetary) surplus is $-w_0$.
- Principal's payoff is $V(-w_0)$, i.e., P appropriates all the entire surplus.

Second Best: Wealthy and Risk-neutral Agent IV

So, when the agent is risk-neutral and there are no wealth constraints, the P can induce the FB effort costlessly.

Exercise: Show that the contract (w_0, w_1) satisfying (10) and (11) is a unique solution to the Principal's optimization problem.

SB: Risk-neutral Agent with Wealth Constraints I

Suppose, due to wealth constraints $w_0 < 0$ is not possible, i.e., in a feasible contract $w_0 \geq 0$.

Question

Consider contract $(w_0, w_1) = (0, 1)$. *Under this contract*

- *What will be effort choice?*
- *Will P choose to offer it?*
- *What are the payoffs of A and P?*

SB: Risk-neutral Agent with Wealth Constraints II

Under this contract

- $w_1 - w_0 = 1$ holds
- So, from (9) and (12), the agent will still put in e^*.
- agent's payoff is $p(e^*) - e^* > 0$
- Principal's payoff is 0

So, P will not offer this contract.

P, instead, will solve

$$\max_{w_i, e}\{p(e)V(1 - w_1) + (1 - P(e))V(-w_0)\}$$

s.t

$$\begin{aligned} p(e)w_1 + (1 - P(e))w_0 - e &\geq 0 \qquad (IR) \\ p'(e)[w_1 - w_0] &= 1 \qquad (IC) \end{aligned}$$

SB: Risk-neutral Agent with Wealth Constraints III

So, P will choose $w_0 = 0$ (see the P's payoff function), i.e., ignoring IR, P will solve

$$\max_{w_1, e}\{p(e)V(1 - w_1)\}$$

s.t. $p'(e)w_1 = 1, \ i.e.,$

$$\max_{e}\{p(e)V(1 - \frac{1}{p'(e)})\}.$$

For a risk-neutral P, the e^{SB} solves

$$p'(e) = 1 - \frac{p(e)p^{''}(e)}{[p'(e)]^2}, \ i.e.,$$

$e^{SB} < e^*$.

SB: Risk-neutral Agent with Wealth Constraints IV

Ex: Prove that the P's payoff under wealth constraints (limited liability) are strictly less than his payoff without wealth constraints.

Ex: Prove that the (minimum) cost for P of inducing a given effort level say e^* is higher under wealth constraints than without wealth constraints.

Ex: Prove that under wealth constraints (limited liability), there is tradeoff between inducing the desired effort on one hand, and the information rent to be yielded to the the agent, on the other hand.