Asymmetric Information: Lecture 2

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Incentive Compatible contracts: Properties

For Incentive Compatible contracts:

- Monotonicity of Output: Adding (1) and (2) gives us

$$(\theta_2 - \theta_1)q_1 \geq (\theta_2 - \theta_1)q_2, i.e.,$$

$$q_1 \geq q_2.$$

  In fact, any pair (q_1, q_2) is implementable iff $q_1 \geq q_2$.

- (1) and (4) imply that as long as $q_2 > 0$,

$$t_1 - \theta_1 q_1 > 0, i.e.,$$

  if inefficient type is required to produce, the payoff of the efficient type will be positive.

Second Best: Optimization Problem I

The principal's optimization problem is

$$\max_{(t_1,q_1),(t_2,q_2)} \{\nu(V(q_1)-t_1)+(1-\nu)(V(q_2)-t_2)\}$$

s.t. ICs,

$$\begin{aligned} t_1-\theta_1 q_1 &\geq t_2-\theta_1 q_2 \\ t_2-\theta_2 q_2 &\geq t_1-\theta_2 q_1 \end{aligned}$$

Let $U_1=t_1-\theta_1 q_1$ and $U_2=t_2-\theta_2 q_2$. That is,

$$\begin{aligned} U_1 &\geq U_2+\Delta\theta q_2 \qquad (1) \\ U_2 &\geq U_1-\Delta\theta q_1 \qquad (2) \end{aligned}$$

and IRs

$$\begin{aligned} U_1 &\geq 0 \qquad (3) \\ U_2 &\geq 0 \qquad (4) \end{aligned}$$

Second Best: Optimization Problem II

Now the principal's optimization problem

$$\max_{(t_1,q_1),(t_2,q_2)} \{\nu(V(q_1) - t_1) + (1-\nu)(V(q_2) - t_2)\}$$

can be rewritten as

$$\max_{(U_1,q_1),(U_2,q_2)} \{\nu(V(q_1) - \theta_1 q_1) + (1-\nu)(V(q_2) - \theta_2 q_2) - (\nu U_1 + (1-\nu)U_2)\}$$

$$\underbrace{\nu(V(q_1) - \theta_1 q_1) + (1-\nu)(V(q_2) - \theta_2 q_2)}_{allocative\ efficiency} - \underbrace{(\nu U_1 + (1-\nu)U_2)}_{information\ rent}$$

s.t., (1)-(4).

Second Best: Optimization Problem III

Consider a contract

$$\{(\theta_1 q_1^* + \Delta\theta q_2^*, q_1^*), (\theta_2 q_2^*, q_2^*)\}, \textit{i.e.},$$

$$\{(U_1 = \Delta\theta q_2^*, q_1^*), (U_2 = 0, q_2^*)\}.$$

Question

- *It is incentive feasible and implements the FB.*
- *But, will principal offer this contract?*

Second Best: Solution I

Remark

- (1) and (4) together imply (3). That is,

$$[U_1 \geq U_2 + \Delta\theta q_2] \& [U_2 \geq 0] \Rightarrow U_1 \geq 0.$$

$$[U_1 \geq U_2 + \Delta\theta q_2] \& [U_2 \geq 0] \& [q_2 > 0] \Rightarrow U_1 > 0.$$

- Under optimum contract (1) and (4) will both bind, i.e.,

$$U_1 = \Delta\theta q_2 \quad (5)$$
$$U_2 = 0 \quad (6)$$

Ignoring (2) for the time being, the principal's optimization problem becomes

Second Best: Solution II

$$\max_{(q_1,q_2)} \{\nu(V(q_1) - \theta_1 q_1) + (1-\nu)(V(q_2) - \theta_2 q_2) - \nu\Delta\theta q_2\}$$

the f.o.c are

$$V'(q_1) = \theta_1 \quad (7)$$

$$V'(q_2) = \theta_2 + \frac{\nu\Delta\theta}{1-\nu} \quad (8)$$

That is,

- $q_1^{SB} = q_1^*$ but $q_2^{SB} < q_2^*$.
- (2) is satisfied (you should verify)
- The SB transfers/wages are given by (5) and (6), i.e., $U_2^{SB} = U_2^* = 0$ and $U_1^{SB} > U_1^* = 0$, *i.e.*,

$$t_2^{SB} = \theta_2 q_2^{SB} \text{ and } t_1^{SB} = \theta_1 q_1^* + \Delta\theta q_2^{SB}.$$

Second Best: Solution III

Remark

- Allocations are monotonic - more efficient type produce more
- Efficient allocation for 'high type', but inefficient for the 'low type'
- Allocative **inefficiency** increases with $\Delta\theta$;
- The (information) rent yielded to the efficient type increases with $\Delta\theta$.

Moreover, (8) can be expressed as

$$(1-\nu)(V'(q_2^{SB})-\theta_2)=\nu\Delta\theta, i.e.,$$

at the SB marginal benefit (LHS) from increasing q_2 is equal to the marginal cost (RHS) of doing so.
Clearly, if V' is finite shutdown takes place for ν close to 1.

Second Best: Solution IV

More generally, shutting down of inefficient type is optimal for the principal if

$$\nu(V(q_1^*) - \theta_1 q_1^*) \geq [\nu(V(q_1^{SB}) - \theta_1 q_1^{SB} - \Delta\theta q_2^{SB}) + (1-\nu)(V(q_2^{SB}) - \theta_2 q_2^{SB})], i.e.,$$

$$\nu \Delta\theta q_2^{SB} \geq (1-\nu)(V(q_2^{SB}) - \theta_2 q_2^{SB}).$$

Market failure

Ex: Show that shutdown becomes more likely as the outside payoff (status quo utility level) goes up.

General cost function

Let $U = t - C(q, \theta)$. Suppose,

$$C_q > 0,\ C_\theta > 0,\ C_{qq} \geq 0.$$

Moreover,

$$(\forall q)(\forall \theta)[C_{q\theta} > 0].$$

Exercise

Show that:

$$q_1^{SB} = q_1^{FB} > q_2^{FB} > q_2^{SB}.$$