Lecture 5: Hidden Information

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Finite Types I

Baron and Myerson (1982, Econometrica)
Returning to n types, let

$$\theta \in \{\theta_1, ..., \theta_n\}, \ \ \theta_1 < \theta_2 < ... < \theta_n.$$

Let

$$Pr(\theta = \theta_i) = \nu_i, \ \ i = 1, 2, ..., n.$$

Payoff functions:

- Principal: $T(q) - C(q) = T(q) - cq$, where c is MC, $T(q)$ is the price charged for q units.
- Agent: $U(\theta_i, q, T) = \theta_i u(q) - T$, $u'(q) > 0$ and $u''(q) < 0$.

Finite Types II

The Principal's optimization problem is:

$$\max_{\{T(q)\}} \sum \{\nu_i [T(q_i) - cq_i]\}$$

s.t.

$$q_i = argmax_{q_j} \{\theta_i u(q_j) - T(q_j)\}, \quad \theta_i = \theta_1, ..., \theta_n, \qquad (IC)_i$$

and

$$\theta_i u(q_i) - T(q_i) \geq 0, \quad i = 1, ..., n, \qquad (IR)_i$$

Suppose, the outcome is (q_1, T_1) and (q_2, T_2))

Finite Types III

Consider two tariff/contract schemes; $T(q)$, and $T'(q)$.
In view of the revelation principle, we know that there exists (Θ, g) such that

- $g : \Theta \mapsto \mathcal{O}$; i.e., $(g(\theta_1), g(\theta_2)) = ((q_1, T_1), (q_2, T_2))$
- (q_1, T_1) and $(q_2, T_2))$ satisfy IR and ICs

Similarly, there exists (Θ, g') such that

- $g' : \Theta \mapsto \mathcal{O}$; i.e., $(g'(\theta_1), g'(\theta_2)) = ((q'_1, T'_1), (q'_2, T'_2))$
- (q'_1, T'_1) and $(q'_2, T'_2))$ satisfy IR and ICs

Finite Types IV

So, the principal can simply offer a menu of $\{(q_i, T(q_i))\} \equiv \{(q_i, T_i)\}$ that solves:

$$\max_{\{(q_i,T_i)\}} \{\nu_1(T_1 - cq_1) + \nu_2(T_2 - cq_2) + ... + \nu_n(T_n - cq_n)\}, i.e.,$$

$$\max_{\{(q_i,T_i)\}} \sum \{\nu_i[T_i - cq_i]\}$$

s.t.

$$\begin{aligned} \theta_i u(q_i) - T_i &\geq 0 \quad i = 1, ..., n && (1) \\ \theta_i u(q_i) - T_i &\geq \theta_i u(q_j) - T_j, \quad i, j = 1, ..., n. && (2) \end{aligned}$$

Finite Types V

Exercise: Given 1 and 2 prove that $IR_1 \Rightarrow IR_i$, $i > 1$, i.e.,

$$[\theta_1 u(q_1) - T_1 \geq 0] \Rightarrow (\forall i > 1)[\theta_i u(q_i) - T_i \geq 0].$$

Moreover 2, among others, implies the following inequalities

$$\theta_i u(q_i) - T_i \geq \theta_i u(q_j) - T_j \ \&$$
$$\theta_j u(q_j) - T_j \geq \theta_j u(q_i) - T_i, i.e.,$$

$$(\theta_i - \theta_j)[u(q_i) - u(q_j)] \geq 0. \tag{3}$$

In view of the assumption that $u'(.) > 0$, (3) implies

$$\theta_i > \theta_j \Rightarrow q_i \geq q_j. \tag{4}$$

Finite Types VI

Indeed, (4) is an implication of the *Spence Mirrlees single crossing condition*. That is,

- (4) will hold for every payoff function of agent that satisfies *SM single crossing condition*.
- In the present context, a payoff function $U(\theta, q, T)$ satisfies *SM single crossing condition* if it is s.t.

$$\frac{\partial}{\partial\theta}[-\frac{\frac{\partial U}{\partial q}}{\frac{\partial U}{\partial T}}] > 0. \tag{5}$$

In general, for $U(\theta, q, T)$ the *SM single crossing condition* holds if

$$(\forall(\theta, q, T) \in \Theta \times \mathcal{A})[\frac{\partial}{\partial\theta}[-\frac{\frac{\partial U}{\partial q}}{\frac{\partial U}{\partial T}}] > 0 \text{ or } < 0]. \tag{6}$$

Assumption (6), i.e., (5) has some interesting and useful implications.

Finite Types VII

- (6) implies Monotonicity of consumption
- (6) implies sufficiency of LDICs and LUICs.

By definition of ICs, we have

$$\begin{aligned} \theta_{i+1}u(q_{i+1}) - T_{i+1} &\geq \theta_{i+1}u(q_i) - T_i & (7) \\ \theta_i u(q_i) - T_i &\geq \theta_i u(q_{i-1}) - T_{i-1} & (8) \end{aligned}$$

(8) can be written as

$$\theta_i[u(q_i) - u(q_{i-1})] \geq T_i - T_{i-1}.$$

This, in view of $q_i \geq q_{i-1}$, i.e., $u(q_i) \geq u(q_{i-1})$, implies

$$\theta_{i+1}[u(q_i) - u(q_{i-1})] \geq T_i - T_{i-1}, i.e.,$$

Finite Types VIII

$$\theta_{i+1} u(q_i) - T_i \geq \theta_{i+1} u(q_{i-1}) - T_{i-1} \tag{9}$$

Now (7) and (9) give us

$$\theta_{i+1} u(q_{i+1}) - T_{i+1} \geq \theta_{i+1} u(q_{i-1}) - T_{i-1}. \tag{10}$$

Similarly, in view of $q_i \geq q_{i-2}$, we get

$$\theta_{i+1} u(q_{i+1}) - T_{i+1} \geq \theta_{i+1} u(q_{i-2}) - T_{i-2}.$$

In general,

$$\theta_i u(q_i) - T_i \geq \theta_i u(q_{i-k}) - T_{i-k} \tag{11}$$

for all $k \geq 1$ such that $i - k \geq 1$.

We call (8) as LDIC for θ_i.

Finite Types IX

We can define LUIC for θ_i as

$$\theta_i u(q_i) - T_i \geq \theta_i u(q_{i+1}) - T_{i+1}.$$

It is possible to show that LUICs imply that: for θ_i

$$\theta_i u(q_i) - T_i \geq \theta_i u(q_{i+k}) - T_{i+k}. \tag{12}$$

holds for all $k = 1, 2, ..$ such that $i + k \leq n$.

(11) and (12) imply that for each agent we can replace $n - 1$ ICs with just two constraints; the LDIC and the LUIC.

Exercise: Ignoring LUICs, show that at the optimum all of LDICs will bind. This, in view of the monotonicity of consumption, implies that all LUICs are satisfied.

Finite Types X

In view of SM condition, the principal solves

$$\max_{(q_i, T_i)} \sum \{\nu_i [T_i - cq_i]\}$$

s.t.

$$\begin{aligned} \theta_1 u(q_1) - T_1 &= 0 \\ (\forall i > 1)[\ \theta_i u(q_i) - T_i &= \theta_i u(q_{i-1}) - T_{i-1}] \\ \theta_i > \theta_j &\Rightarrow q_i \geq q_j \end{aligned}$$

We can solve this without considering monotonicity constraints. Form the Lagrangian

Finite Types XI

$$\mathfrak{L} = \sum_{i=1}^{n}\{\nu_i[T_i - cq_i]\} + \sum_{i=2}^{n}\{\lambda_i[\theta_i u(q_i) - \theta_i u(q_{i-1}) - T_i + T_{i-1}]\} + \mu[\theta_1 u(q_1) - T_1$$

For $i = n$

$$\frac{\partial \mathfrak{L}}{\partial q_n} \ : \ \lambda_n \theta_n u'(q_n) = c\nu_n \tag{13}$$

$$\frac{\partial \mathfrak{L}}{\partial T_n} \ : \ \nu_n - \lambda_n = 0, i.e., \ \nu_n = \lambda_n \tag{14}$$

That is,

$$\theta_n u'(q_n) = c, i.e., \ q_n^{SB} = q_n^*.$$

Finite Types XII

For $i = 1$, the foc are:

$$\frac{\partial \mathfrak{L}}{\partial q_1} \quad : \quad [\mu\theta_1 - \lambda_2\theta_2]u'(q_1) = c\nu_1 \tag{15}$$

$$\frac{\partial \mathfrak{L}}{\partial T_1} \quad : \quad \nu_1 + \lambda_2 - \mu = 0, i.e., \ \nu_1 = \mu - \lambda_2 \tag{16}$$

(16), in view of $\theta_2 > \theta_1$ implies

$$\theta_1\nu_1 > \theta_1\mu - \theta_2\lambda_2.$$

Now, in view of this, (15) can be written as

$$[\mu\theta_1 - \lambda_2\theta_2]\theta_1 u'(q_1) = c\nu_1\theta_1, i.e.,$$

$$\theta_1 u'(q_1) = \frac{\nu_1\theta_1}{[\mu\theta_1 - \lambda_2\theta_2]} c > c.$$

Therefore, $q_1^{SB} < q_1^*$.

Finite Types XIII

For $1 < i < n$ foc are

$$\frac{\partial \mathfrak{L}}{\partial q_i} \quad : \quad \lambda_i \theta_i u'(q_i) - \lambda_{i+1} \theta_{i+1} u'(q_i) = c\nu_i \tag{17}$$

$$\frac{\partial \mathfrak{L}}{\partial T_i} \quad : \quad \nu_i - \lambda_i + \lambda_{i+1} = 0 \tag{18}$$

That is,

$$\theta_i u'(q_i) = \frac{c\theta_i \nu_i}{\lambda_i \theta_i - \lambda_{i+1} \theta_{i+1}}$$

(18), in view of $\theta_{i+1} > \theta_i$ implies $\theta_i \nu_i > \lambda_i \theta_i - \lambda_{i+1} \theta_{i+1}$. Therefore,

$$(\forall 1 < i < n)[q_i^{SB} < q_i^*].$$