Lecture 8: Insurance Contracts

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Questions

We are ready to address the following questions:

Question

- What are the properties of insurance contract under pure adverse selection?
- What is the meaning of allocative inefficiency in the context of insurance contracts?
- Will market always supply insurance to all types?
- Do the previous results -on rent-extraction, allocative inefficiency, and efficiency-rent trade-off - hold ?
- When is bunching likely to emerge?
- Does equilibrium always exist?

Certainty Equivalent I

Consider a decision maker with u, and the initial wealth level $\bar{x}$. Now this person's utility is given by

- $\int u(\bar{x} + \tilde{z})dF(\tilde{z})$, if s/he gets lottery $F(\tilde{z})$
- $u(\bar{x} + c(F, u, \bar{x}))$, if s/he gets amount $c(F, u, \bar{x})$ with certainty.

Definition

Certainty Equivalent: For a decision maker with u, and the initial wealth level $\bar{x}$,

$c(F, u, \bar{x})$ is the certainty equivalent of the lottery $F(\tilde{z})$ if

$$u(\bar{x} + c(F, u, \bar{x})) = \int u(\bar{x} + \tilde{z})dF(\tilde{z}). \tag{0.1}$$

Certainty Equivalent II

Property

The following statements are equivalent:
u is concave;
u exhibits risk-aversion;
$(\forall F(.) \in \mathcal{L})[c(F, u, \bar{x}) \leq \int \tilde{z} dF(\tilde{z})]$

Certainty Equivalent III

Proof.

$$c(F,u,\bar{x}) \leq \int \tilde{z}dF(\tilde{z}) \Leftrightarrow \bar{x} + c(F,u,\bar{x}) \leq \int (\bar{x}+\tilde{z})dF(\tilde{z})$$

$$\Leftrightarrow u(\bar{x}+c(F,u,\bar{x})) \leq u(\bar{x} + \int (\tilde{z})dF(\tilde{z}))$$

Since $u(\bar{x}+c(F,u,\bar{x})) = \int u(\bar{x}+\tilde{z})dF(\tilde{z})$, we get

$$\int u(\bar{x}+\tilde{z})dF(\tilde{z}) \leq u(\int (\bar{x}+\tilde{z})dF(\tilde{z})$$

i.e., u is concave. □

Note that $\int \tilde{z}dF(\tilde{z}) \leq 0 \Rightarrow c(F,u,\bar{x}) \leq 0$.

Risk Premium I

Consider a decision maker with u, and the initial wealth level $\bar{x}$. Now this person's utility is given by

- $\int u(\bar{x}+\tilde{z})dF(\tilde{z})$, if s/he gets lottery $F(\tilde{z})$
- $u(\bar{x}+\int \tilde{z}dF(\tilde{z}))$, if s/he gets the expected value of the lottery $F(\tilde{z})$ with certainty

Definition

Risk Premium: Consider a decision maker with u at wealth level $\bar{x}$. Now, $\rho(\bar{x},\tilde{z})$ is the risk premium for risk/lottery $\tilde{z}$ with distribution $F(\tilde{z})$ if

$$\int u(\bar{x}+\tilde{z})dF(\tilde{z}) = u(\bar{x}+\int \tilde{z}dF(\tilde{z}) - \rho(\bar{x},\tilde{z})). \tag{0.2}$$

That is, at the wealth level $\bar{x}$, the decision maker is indifferent b/w bearing the risk $\tilde{z}$ and having a sure amount of $\int \tilde{z}dF(z) - \rho(\bar{x},\tilde{z})$.

Risk Premium II

From (0.1) and (0.2),

$$c(F,u,\bar{x}) = \int \tilde{z}dF(\tilde{z}) - \rho(\bar{x},\tilde{z}), i.e., \ \rho(\bar{x},\tilde{z}) = \int \tilde{z}dF(\tilde{z}) - c(F,u,\bar{x}). \quad (0.3)$$

When u exhibits risk-aversion, i.e., $(\forall F(.) \in \mathcal{L})[c(F,u,\bar{x}) \leq \int \tilde{z}dF(\tilde{z})]$,

$$\rho(\bar{x},\tilde{z}) \geq 0.$$

Definition

Insurance Premium: For given wealth level $\bar{x}$, let's add risk $\tilde{z}$ with distribution $F(\tilde{z})$. Insurance Premium $c_I(F,u,\bar{x})$ is given by

$$u(\bar{x} - c_I(F,u,\bar{x})) = \int u(\bar{x}+\tilde{z})dF(\tilde{z}). \quad (0.4)$$

the insurance premium, $c_I(F,u,\bar{x})$ is the amount that makes the decision maker indifferent b/w accepting the risk $\tilde{z}$ and a payment of $c_I(F,u,\bar{x})$.

Risk Premium III

From (0.1) and (0.4),

$$c_l(F, u, \bar{x}) = -c(F, u, \bar{x}) = \rho(\bar{x}, \tilde{z}) - \int \tilde{z} dF(\tilde{z}). \tag{0.5}$$

When the risk is actuarially fair, i.e., $\int \tilde{z} dF(\tilde{z}) = 0$,

$$c_l(F, u, \bar{x}) = -c(F, u, \bar{x}) = \rho(\bar{x}, \tilde{z}).$$

Since, $\rho(\bar{x}, \tilde{z}) \geq 0$ the decision maker will pay a non-negative amount to get rid of the risk.

Exercise: Show that when u is strictly concave and $\int \tilde{z} dF(\tilde{z}) \leq 0$, $c_l(F, u, \bar{x}) > 0$.

Basics I

Akerlof (1970) and Rothschild and Stiglitz (1976)
Suppose,

- A group of agents/individuals faces risk of accident.
- w is the wealth level possed by each agent
- An accident results in harm/loss L
- type of agent is denoted by π; $\pi \in \{\pi_1, \pi_2, ..., \pi_N\}$, where

$$\pi_1 < \pi_2 < ... < \pi_N$$

- ν_i is the probability of $\pi = \pi_i$.
- Payoff function of an agent (buyer) is $u(.)$; $u'(.) > 0$ and $u^{''}(.) < 0$
- So the expected utility for agent with type π_i is

$$\pi_i u(w - L) + (1 - \pi_i) u(w)$$

Basics II

Since $u^{''}(.) < 0$,

$$u(w - \pi_i L) > \pi_i u(w - L) + (1 - \pi_i)u(w)$$

So, the agent with type π_i is willing to pay more that $\pi_i L$ to get rid of the risk.

- The insurance company is risk-neutral and the market is competitive. So.
- The insurance company is willing to charge 'actuarially fair' premium.

Provision for Insurance,

- An agent can buy full insurance coverage, i.e., if accident happens the insurance company will pay her L
- An agent can sign contract with only one insurer

First Best I

Let

- I_i denote the insurance premium charged by the insurer from agent with type π_i
- I_i is paid by the agent upfront

Recall, for an agent with type π_i, the reservation utility (expected utility without insurance contract) is

$$\bar{U}(\pi_i, w, L) = \pi_i u(w - L) + (1 - \pi_i)u(w)$$

Note

$$\pi_j > \pi_i \Rightarrow [\bar{U}(\pi_j, w, L) < \bar{U}(\pi_i, w, L)]$$

However, if she buys insurance coverage, here expected utility will be

$$U(\pi_i, w, L, I) = \pi_i u(w - I - L + L) + (1 - \pi_i)u(w - I) = u(w - I)$$

First Best II

So, the agent will buy insurance only if

$$\begin{aligned} U(\pi_i, w, L, I_i) &\geq \bar{U}(\pi_i, w, L), i.e., \\ u(w - I_i) &\geq \pi_i u(w - L) + (1 - \pi_i) u(w). \end{aligned}$$

Let

$$I_i^* = \pi_i L$$

This is 'actuarially fair' premium. Moreover,

$$u(w - I_i^*) > \pi_i u(w - L) + (1 - \pi_i) u(w).$$

So, each agent will buy full insurance.

Second Best: Single Contract I

Suppose,

- Insurance company offers full insurance
- Insurance company charges I

Recall,

- the agent with type π_i is willing to pay more that $\pi_i L$ to get rid of the risk.
- So, all types such that $\pi_i L \geq I$ will buy insurance

In equilibrium, types $i = j, j+1, ..., N$ will buy insurance if the following hold:

$$\begin{aligned} u(w - I) &\geq \pi_i u(w - L) + (1 - \pi_i) u(w) \text{ for all } i = j, j+1, ..., N \\ u(w - I) &< \pi_i u(w - L) + (1 - \pi_i) u(w) \text{ for all } i = 1, ..., j-1 \end{aligned}$$

and

$$\sum_{i=j}^{N} \beta_i I = \sum_{i=j}^{N} \beta_i \pi_i L,$$

Second Best: Single Contract II

β_i is the proportion of type π_i. In an equilibrium

- only the highest risk type may go for insurance
- however, only some of low-risk types may not buy insurance - the rest may go for it

In any case,

- an equilibrium will be constraint Pareto optimum. Why?

So,

- there is case for universal subsidy for insurance
- funded by flat and tax

Multiple Contracts I

Suppose,

- there are only two types of agents; low-risk and high-risk type
- π_1 and $\pi_2 = 1 - \pi_1$ are probability of low-risk and high-risk type, respectively.
- Contract offered to type π_i is (I_i, D_i)

Now, if an agent buys insurance coverage, her expected utility will be

$$U(\pi, w, L, I_i, D_i) = \pi_i u(w - I_i - L + L - D_i) + (1 - \pi_i)u(w - I)$$

So, the agent will buy insurance only if $U(\pi, w, L, I_i, D_i) \geq \bar{U}(\pi_i, w, L)$, *i.e.*,

$$\begin{aligned}
\pi_i u(w - I_i - L + L - D_i) + (1 - \pi_i)u(w - I_i) &\geq \pi_i u(w - L) + (1 - \pi_i)u(w), i.e. \\
\pi_i u(w - I_i - D_i) + (1 - \pi_i)u(w - I_i) &\geq \pi_i u(w - L) + (1 - \pi_i)u(w)
\end{aligned}$$

No Pooling Equilibrium I

Question

Can there be a pooling equilibrium, under competitive supply of insurance?

Suppose, there is a pooling equilibrium. Let the equi. contract be (I, D). Competitive supply means,

$$I = [\pi_1\beta + \pi_2(1-\beta)](L-D)$$

Can there be another contract (I', D') such that:

$$\begin{aligned}
\pi_1 u(w-I'-D') + (1-\pi_1)u(w-I') &\geq \pi_1 u(w-I-D) + (1-\pi_1)u(w-I) \\
\pi_2 u(w-I'-D') + (1-\pi_2)u(w-I') &< \pi_2 u(w-I-D) + (1-\pi_2)u(w-I)
\end{aligned}$$

These inequalities imply:

$$(\pi_2-\pi_1)[u(w-I-D)-u(w-I'-D')] < (\pi_2-\pi_1)[u(w-I)-u(w-I')]$$

No Pooling Equilibrium II

- Since $\pi_2 > \pi_1$, there exists (I', D') such that $D' > D$ and $I' < I$ that satisfies the above inequalities.
- If we choose (I', D') sufficiently close to (I, D), it will
  - Insurer will earn almost same profit from low risk types
  - But, will not incur loss from the high risk types
- So (I', D') is better than (I, D).

Separating Equilibrium I

Question

Which type has incentive to mimic as the other type?

Suppose, contract offered to type π_i is (I_i, D_i). Competitive supply insurance means for (I_2, D_2) we have

$$\begin{aligned} D_2 &= 0 \\ I_2 &= \pi_2 L \end{aligned}$$

However, (I_1, D_1) will be solution of:

$$\max\{\pi_1 u(w - I_1 - D_1) + (1 - \pi_1) u(w - I_1)\}$$

Subject to

$$\begin{aligned} I_1 &\geq \pi_1 (L - D_1) \\ u(w - I_2) &\geq \pi_2 u(w - I_1 - D_1) + (1 - \pi_2) u(w - I_1) \end{aligned}$$

The equi. has the following properties:

Separating Equilibrium II

- Full insurance for high-risk types
- Partial insurance for low risk types
- Constraint Pareto optimality

Also, it can be shown that:

- D_1 does not depend on β_i
- D_1 increases with $\pi_2 - \pi_1$
- That is risk borne by low types increases with $\pi_2 - \pi_1$
- For sufficiently large $\pi_2 - \pi_1$ and hence D_1, low types will be better under pooling equi.
- But, there cannot be a pooling equi.
- So, equi may not exist