Lecture 9: Insurance Contracts

Ram Singh

Department of Economics

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Basics I

Akerlof (1970) and Rothschild and Stiglitz (1976)
Suppose,

- A group of agents/individuals faces risk of accident.
- w is the wealth level possed by each agent
- An accident results in harm/loss L
- Probability of accident is θ; type of agent is denoted by θ; $\theta \in \{\theta_1, \theta_2, ..., \theta_N\}$, where

$$0 < \theta_1 < \theta_2 < ... < \theta_N < 1$$

- ν_i is the proportion (probability) of $\theta = \theta_i$.
- Payoff function of an agent (buyer) is $u(.)$; $u(0) = 0$, $u'(.) > 0$ and $u^{''}(.) < 0$

Basics II

- So, the (reservation) expected utility for agent with type θ_i is

$$\theta_i u(w - L) + (1 - \theta_i)u(w)$$

Since $u^{''}(.) < 0$, the agent with type θ_i is willing to pay more that $\theta_i L$ to get rid of the risk. So,

$$u(w - \theta_i L) > \theta_i u(w - L) + (1 - \theta_i)u(w)$$

- The insurance company is risk-neutral.
- The insurance company can break-even by charging 'actuarially fair' premium.

Provision for Insurance,

- An agent can buy insurance coverage, i.e., if accident happens the insurance company will pay her part of whole of L
- An agent can sign contract with only one insurer

First Best I

Let

- θ be observable
- I_i denote the insurance premium charged by the insurer from agent with type θ_i
- I_i is paid by the agent upfront

Recall, for an agent with type θ_i, the reservation utility (expected utility without insurance contract) is

$$\bar{U}(\theta_i, w, L) = \theta_i u(w - L) + (1 - \theta_i)u(w) = u(\bar{w}_i),$$

where $\bar{w}_i$ the certainty equivalent outside wage for type θ_i. Note, now the reservation utility is type-dependent, since

$$\theta_j > \theta_i \Rightarrow [\bar{U}(\theta_j, w, L) < \bar{U}(\theta_i, w, L)]$$

First Best II

However, if she buys insurance coverage, here expected utility will be

$$U(\theta_i, w, L, I) = \theta_i u(w - I - L + L) + (1 - \theta_i)u(w - I) = u(w - I)$$

So, the agent will buy insurance only if

$$\begin{aligned} U(\theta_i, w, L, I_i) &\geq \bar{U}(\theta_i, w, L), i.e., \\ u(w - I_i) &\geq \theta_i u(w - L) + (1 - \theta_i)u(w). \end{aligned}$$

Let $I_i^* = \theta_i L$. This is ‘actuarially fair’ premium. Moreover,

$$u(w - I_i^*) > \theta_i u(w - L) + (1 - \theta_i)u(w).$$

First Best III

Let $I_i^* = c_I(u, \theta_i)$, where $c_I(u, \theta_i)$ be the insurance premium for type θ_i. Note

$$u(w - c_I) = \theta_i u(w - L) + (1 - \theta_i)u(w).$$

So, each agent will buy full insurance in either case.

Exercise

Show that for any $I_i^* \in [\theta_i L, c_I(u, \theta_i)]$, each agent will be provided full insurance.

Monopoly and Single Contract I

Suppose,

- Insurance company offers full insurance
- Insurance company charges I

Recall,

- the agent with type θ_i is willing to pay up to $c_I(u, \theta_i)$ to get rid of the risk.
- $c_I(u, \theta_i)$ increases with θ
- So, all types such that $I \leq c(u, \theta_i)$ will buy insurance

Monopoly and Single Contract II

The insurance company will solve

$$\max_{I} \left\{ \sum_{i=j}^{N} \nu_i I - \sum_{i=j}^{N} \nu_i \theta_i L \right\}$$

where

$$\begin{aligned} u(w-I) &\geq \theta_i u(w-L) + (1-\theta_i) u(w) \text{ for all } i = j, j+1, ..., N \\ u(w-I) &< \theta_i u(w-L) + (1-\theta_i) u(w) \text{ for all } i = 1, ..., j-1 \end{aligned}$$

In an equilibrium

- only the highest risk type may go for insurance
- however, only some of low-risk types may not buy insurance - the rest may go for it

In any case,

- an equilibrium will be constraint Pareto optimum. Why?

Second Best under Monopoly I

Let
$\alpha \in [0, 1]$ denote the proportion of loss recovered from insurer if accident.

Now, Agent's expected utility can be written as

$$\begin{aligned} U(\theta, \alpha, I) &= \theta u(w - I - L + \alpha L) + (1 - \theta)u(w - I) \\ &= \theta u(w - I - (1 - \alpha)L) + (1 - \theta)u(w - I) \\ &= \theta u(w - L + t^a) + (1 - \theta)u(w - t^n) \\ &= \theta u^a + (1 - \theta)u^n \end{aligned}$$

where $t^a = \alpha L - I$ and $t^n = I$; $u^a = u(w - L + t^a)$.

Exercise

Find out if the above FB contract is enforceable.

Note

$$\frac{U_{t^a}}{U_{t^n}} = -\frac{\theta}{1 - \theta} \frac{u'(w - L + t^a)}{u'(w - t^n)}$$

Second Best under Monopoly II

So $\frac{\partial(\frac{U_{t^a}}{U_{t^n}})}{\partial\theta} < 0$.

The insurer's OP is

$$\max_{(t_1^a,t_1^n),(t_2^a,t_2^n)} \{\nu(-\theta_1 t_1^a + (1-\theta_1)t_1^n) + (1-\nu)(-\theta_2 t_2^a + (1-\theta_2)t_2^n)\}$$

s.t.

$$\begin{aligned}
\theta_2 u(w - L + t_2^a) + (1-\theta_2)u(w - t_2^n) &\geq \theta_2 u(w - L + t_1^a) + (1-\theta_2)u(w - t_1^n) \\
\theta_1 u(w - L + t_1^a) + (1-\theta_1)u(w - t_1^n) &\geq \theta_1 u(w - L + t_2^a) + (1-\theta_1)u(w - t_2^n)
\end{aligned}$$

and

$$\begin{aligned}
\theta_2 u(w - L + t_2^a) + (1-\theta_2)u(w - t_2^n) &\geq \bar{U}_2 \\
\theta_1 u(w - L + t_1^a) + (1-\theta_1)u(w - t_1^n) &\geq \bar{U}_1
\end{aligned}$$

Note:

Second Best under Monopoly III

- The context is a common value environment
- The constraints are defined in terms of a non-linear function
- We can express the above constraints in terms of u^a and u^n
- Still, IC for high-type and therefore IR for the low type will be relevant

Let $h(.) = u^{-1}(.)$, $h(0) = 0$, $h'(.) > 0$, and $h''(.) > 0$. We can write P's problem as:

$$\max_{(u_1^a, u_1^n), (u_2^a, u_2^n)} \{\nu(-\theta_1 L + w - \theta_1 h(u_1^a) - (1-\theta_1) h(u_1^n))$$
$$+(1-\nu)(-\theta_2 L + w - \theta_2 h(u_2^a) - (1-\theta_2) h(u_2^n))\}$$

s.t.

$$\begin{aligned} \theta_2 u_2^a + (1-\theta_2) u_2^n &\geq \theta_2 u_1^a + (1-\theta_2) u_1^n \\ \theta_1 u_1^a + (1-\theta_1) u_1^n &\geq \bar{U}_1 \end{aligned}$$

Second Best under Monopoly IV

Using Lagrangian Method, the FOCs for u_1^a and u_1^n are

$$-\nu\theta_1 h'(u_1^a) - \lambda\theta_2 + \mu\theta_1 = 0 \quad (0.1)$$

$$-\nu(1-\theta_1)h'(u_1^n) - \lambda(1-\theta_2) + \mu(1-\theta_1) = 0 \quad (0.2)$$

The FOCs for u_2^a and u_2^n are

$$-(1-\nu)\theta_2 h'(u_2^a) + \lambda\theta_2 = 0 \quad (0.3)$$

$$-(1-\nu)(1-\theta_2)h'(u_2^n) + \lambda(1-\theta_2) = 0 \quad (0.4)$$

From (0.3) and (0.4), we get

$$u_2^a = u_2^n = u_2^{SB}, i.e., \ \alpha_2 = 1.$$

Also, we get

$$\lambda = (1-\nu)h'(u_2^{SB}) > 0$$

$$\mu = (1-\nu)h'(u_2^{SB}) + \nu\theta_1 h'(u_1^a) + (1-\theta_1)h'(u_1^n) > 0$$

Second Best under Monopoly V

So, both constraints bind and give us

$$\begin{aligned} u_2^{SB} &= -\Delta\theta(u_1^n - u_1^a) + \bar{u}_1 = -\Delta\theta\Delta u_1 + u(\bar{w}_1) \\ u_1^a &= u(\bar{w}_1) - (1-\theta_1)(u_1^n - u_1^a) = u(\bar{w}_1) - (1-\theta_1)\Delta u_1 \\ u_1^n &= u(\bar{w}_1) + \theta_1(u_1^n - u_1^a) = u(\bar{w}_1) + \theta_1\Delta u_1 \end{aligned}$$

In P's OP, we can replace u_1^n and u_1^a with Δu_1. Differentiating P's problem w.r.t. Δu_1 gives us:

$$\begin{aligned} \frac{(1-\nu)\Delta\theta}{\nu\theta_1(1-\theta_1)} h'(-\Delta\theta\Delta u_1 + u(\bar{w}_1)) &= h'(\theta_1\Delta u_1 + u(\bar{w}_1)) \\ & \quad - h'(-(1-\theta_1)\Delta u_1 + u(\bar{w}_1)) \end{aligned}$$

Since LHS > 0 and $h'(.) > 0$ we have $\Delta u_1^{SB} > 0$. Hence $(u_1^n)^{SB} > (u_1^a)^{SB}$. That is,

- Only partial insurance for the low risk types $\alpha_1 < 1$

Competitive Insurance: Single Contract I

Suppose,

- Insurance company offers full insurance
- Insurance company charges I

Recall,

- the agent with type θ_i is willing to pay more that $\theta_i L$ to get rid of the risk.
- So, all types such that $\theta_i L \geq c_I(\theta)$ will buy insurance

In equilibrium, types $i = j, j+1, ..., N$ will buy insurance if the following hold:

$$\begin{aligned} u(w - I) &\geq \theta_i u(w - L) + (1 - \theta_i) u(w) \text{ for all } i = j, j+1, ..., N \\ u(w - I) &< \theta_i u(w - L) + (1 - \theta_i) u(w) \text{ for all } i = 1, ..., j - 1 \end{aligned}$$

and

$$\sum_{i=j}^{N} \nu_i I = \sum_{i=j}^{N} \nu_i \theta_i L,$$

Competitive Insurance: Single Contract II

ν_i is the proportion of type θ_i. In an equilibrium

- only the highest risk type may go for insurance
- however, only some of low-risk types may not buy insurance - the rest may go for it

In any case,

- an equilibrium will be constraint Pareto optimum. Why?

So,

- there is case for universal subsidy for insurance
- funded by flat and tax

Multiple Contracts I

Suppose,

- there are only two types of agents; low-risk and high-risk type
- θ_1 and θ_2 are probability of accident for the low-risk and the high-risk type, respectively.
- Contract offered to type θ_i is (I_i, D_i)

Now, if an agent buys insurance coverage, her expected utility will be

$$U(\theta, w, L, I_i, D_i) = \theta_i u(w - I_i - L + L - D_i) + (1 - \theta_i) u(w - I)$$

So, the agent will buy insurance only if $U(\theta, w, L, I_i, D_i) \geq \bar{U}(\theta_i, w, L)$, *i.e.*,

$$\begin{aligned} \theta_i u(w - I_i - L + L - D_i) + (1 - \theta_i) u(w - I_i) &\geq \theta_i u(w - L) + (1 - \theta_i) u(w), i.e., \\ \theta_i u(w - I_i - D_i) + (1 - \theta_i) u(w - I_i) &\geq \theta_i u(w - L) + (1 - \theta_i) u(w) \end{aligned}$$

No Pooling Equilibrium I

Question

Can there be a pooling equilibrium, under competitive supply of insurance?

Suppose, there is a pooling equilibrium. Let the equi. contract be (I, D). Competitive supply means,

$$I = [\theta_1 \nu + \theta_2 (1-\nu)](L - D)$$

Can there be another contract (I', D') such that:

$$\begin{aligned} \theta_1 u(w - I' - D') + (1-\theta_1) u(w - I') &\geq \theta_1 u(w - I - D) + (1-\theta_1) u(w - I) \\ \theta_2 u(w - I' - D') + (1-\theta_2) u(w - I') &< \theta_2 u(w - I - D) + (1-\theta_2) u(w - I) \end{aligned}$$

These inequalities imply:

$$(\theta_2 - \theta_1)[u(w - I - D) - u(w - I' - D')] < (\theta_2 - \theta_1)[u(w - I) - u(w - I')]$$

No Pooling Equilibrium II

- Since $\theta_2 > \theta_1$, there exists (I', D') such that $D' > D$ and $I' < I$ that satisfies the above inequalities.
- If we choose (I', D') sufficiently close to (I, D), it will
  - Insurer will earn almost same profit from low risk types
  - But, will not incur loss from the high risk types
- So (I', D') is better than (I, D).

Separating Equilibrium I

Question

Which type has incentive to mimic as the other type?

Suppose, contract offered to type θ_i is (I_i, D_i). Competitive supply insurance means for (I_2, D_2) we have

$$\begin{aligned} D_2 &= 0 \\ I_2 &= \theta_2 L \end{aligned}$$

However, (I_1, D_1) will be solution of:

$$\max\{\theta_1 u(w - I_1 - D_1) + (1 - \theta_1)u(w - I_1)\}$$

Subject to

$$\begin{aligned} I_1 &\geq \theta_1(L - D_1) \\ u(w - I_2) &\geq \theta_2 u(w - I_1 - D_1) + (1 - \theta_2)u(w - I_1) \end{aligned}$$

The equi. has the following properties:

Separating Equilibrium II

- Full insurance for high-risk types
- Partial insurance for low risk types
- Constraint Pareto optimality

Also, it can be shown that:

- D_1 does not depend on ν_i
- D_1 increases with $\theta_2 - \theta_1$
- That is risk borne by low types increases with $\theta_2 - \theta_1$
- For sufficiently large $\theta_2 - \theta_1$ and hence D_1, low types will be better under pooling equi.
- But, there cannot be a pooling equi.
- So, equi may not exist