Problem Set 2, MIEG, Winter Term, 2015

Osborne, An introduction to Game Theory
Chapter 5: 173.4, 174.2, 176.2, 177.1, 177.2
Chapter 6: 183.4, 191.1, 196.3, 196.4
Chapter 7: 210.3, 211.1, 214.1, 227.2

1. Consider the following two players game. First player 1 can choose between two actions 'Stop' or 'Continue'. If she chooses 'Stop' then the game ends with payoffs $(1, 1)$. If she chooses 'Continue' then the players simultaneously announce non-negative numbers and each player's payoff is the product of the announced numbers. Formulate this situation as an extensive game and find its SPNE.

2. There are two players, a buyer and a seller. At period 1, the seller chooses his investment level $I \geq 0$ at cost I. At period 2, the seller produces one unit of a good at cost $c(I)$. Higher the investment, lower the cost of production, that is $c' < 0$. Moreover let us assume that $c'' > 0$ and $c(0) < v$ where v is buyer's utility from consuming the good. Buyer observes the investment I and makes a 'take-it-or-leave-it' price offer to the seller.
(a) Model this interaction as a game.
(b) Find SPNE of this game.
(c) What is the socially optimal level of investment?
(d) Is it possible for the buyer and the seller to agree on a contract (before period 1) which delivers the socially optimal investment? Note that contracts can not written on the level of I, because such contracts are not verifiable.

3. A firm's production function is given by

$$Q(L) = L(100 - L) \text{ if } L \leq 50 \text{ and } Q(L) = 2500 \text{ if } L > 50$$

where L is the number of workers. The price of output is 1. A union that represents workers presents a wage demand (a nonnegative real number w), which the firm either accepts or rejects. If the firm accepts the demand, it

chooses L (a nonnegative real number, not necessarily an integer); if it rejects the demand, no production takes place ($L = 0$). The firm's preferences are represented by its profit whereas the union's preferences are represented by the total wage bill, wL.
(a) Find the subgame perfect equilibria of the above game.
(b) Is there a outcome of the game which is Pareto superior to any subgame perfect equilibrium outcome? What is the maximum joint surplus of this game?
(c) Is there a Nash equilibrium of this game, where the firm keeps the entire joint surplus. Explain.

4. Consider two countries, A and B and a single good which is consumed only in country B. The inverse demand function is given by $p = a - (q_A + q_B)$, where q_i is the total output produced by country i. Let c be the marginal cost of production, same for both the countries.
(a) Suppose that there are two producers, one in each country. Market interaction has two periods. In period 1, government of country A chooses a per unit export tax or subsidy for the home firm. In period 2, both firms choose quantities simultaneously. Firms maximize their profit whereas Country A's government maximizes the sum of its own receipts (tax/subsidy) and the profit of its firm. Find SPNE of this game.
(b) What happens if both the firms are located in Country A?

5. [Two stage model of Hotelling's price competition] Consumers are uniformly distributed over a linear city of length 1. All consumers consume one unit of a good which can be produced at 0 cost. There are two firms, each of which chooses a location (simultaneously) in the city in the first period. In the second period each chooses a price simultaneously. Each consumer buys one unit of the good from the firm for which price plus travel cost is lowest. Travel cost is $C(d) = cd^2$ where d is the distance traveled and c is a constant. Find SPNE of this game.

6. We have three players: a Worker, an Employer, and an Arbitrator. They want to set the wage w. If they determine the wage w at date t, the pay-

offs of the Worker, the Employer and the Arbitrator will be $\delta^t w$, $\delta^t(1 - w)$ *and* $w(1-w)$, respectively, where $\delta \in (0,1)$. The time line is as follows:

- At time t = 0,
  - the Worker offers a wage w_0;
  - the Employer accepts or rejects the offer;
  - if she accepts the offer, then the wage is set at w_0 and the game ends; otherwise we proceed to the next date.
- At time t = 1,
  - the Employer offers a wage w_1;
  - the Worker accepts or rejects the offer;
  - if he accepts the offer, then the wage is set at w_1 and the game ends; otherwise we proceed to the next date;
- at time t =2, the Arbitrator sets a wage $w_2 \in [0,1]$ and the game ends.

Compute the SPNE of this game. How would your answer change if at time t=2, the Arbitrator can only set wage $w_2 \in \{w_0, w_1\}$, i.e., the Arbitrator has to choose one of the offers made by the previous two parties.