### Issues in Economic Systems and Institutions: Part I: Incentives and Motivation

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#### Moral Hazard in Teams (Holmstrom 1982)

- *n* team members: *i* = 1, 2, ..., *n*.
- Each chooses **unobservable** effort  $e_i$  at personal cost  $\frac{1}{2}\lambda_i e_i^2$ .
- Team production function:

$$y = F(\mathbf{e}, \theta) = \theta \sum_{i=1}^{n} e_i$$

- $\theta$  = team productivity,  $\lambda_i$  = individual cost parameter.
- Agent's payoff  $(m_i = \text{monetary earnings})$ :

$$u_i = m_i - \frac{1}{2}\lambda_i e_i^2$$

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#### First Best

The Team Problem

- Suppose *e<sub>i</sub>* is observable, hence contractible.
- Also assume  $\theta$  is known to all agents.
- Any distribution can be achieved by choosing suitable transfers among the agents.
- Pareto efficient arrangements max social surplus:

$$\max_{\mathbf{e}} \theta \sum_{i=1}^{n} e_i - \frac{1}{2} \sum_{i=1}^{n} \lambda_i e_i^2$$

First-best effort level:

$$\widehat{\mathbf{e}}_i = \frac{\theta}{\lambda_i}$$

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#### Teams and Leadership

The Team Problem

#### Partnerships and Second Best

- Now *e<sub>i</sub>* is not observable.
- There is no outside party (budget-breaker) who can take away or contribute funds.
- $(s_i(y), t_i)$  is a **team contract** where
  - $s_i(y) = \text{agent } i$ 's share in output y.
  - $t_i$  = unconditional transfer to agent *i*.
- Budget balancing constraints:

$$\sum_{i=1}^{n} s_i(y) = y$$
$$\sum_{i=1}^{n} t_i = 0$$

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# Inefficiency in Partnerships

# Without loss of generality, assume constant shares: s<sub>i</sub>(y) = s<sub>i</sub>y, where

$$s_i \geq 0$$
 and  $\sum_{i=1}^n s_i = 1$ 

Individual effort choice:

$$e_i(s_i) = rg\max_{e_i} s_i heta \sum_{j=1}^n e_j - rac{1}{2} \lambda_i e_i^2 = rac{s_i heta}{\lambda_i}$$

Note that

 $e_i(s_i) = \widehat{e}_i \;\; ext{if and only if } s_i = 1$ 

There can be no team contract which elicits first best effort levels from all members.

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#### **Optimum Shares**

The surplus maximization problem:

$$\max_{\mathbf{s}} \theta \sum_{i=1}^n e_i(s_i) - \frac{1}{2} \sum_{i=1}^n \lambda_i e_i(s_i)^2 \text{ sub to } \sum_{i=1}^n s_i = 1$$

Form the Lagrangean:

$$\mathcal{L} = \theta \sum_{i=1}^{n} e_i(s_i) - \frac{1}{2} \sum_{i=1}^{n} \lambda_i e_i(s_i)^2 + \mu \left[ 1 - \sum_{i=1}^{n} s_i \right]$$

First order condition:

$$(\theta - \lambda_i e_i(s_i))e'_i(s_i) = \mu$$

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#### **Optimum Shares**

• Using the expression for  $e_i(s_i)$ :

$$(1-s_i)\theta^2 = \mu\lambda_i$$

Summing both sides we get:

$$\mu = \frac{(n-1)\theta^2}{\sum_{i=1}^n \lambda_i}$$

Using this above, we get optimal shares:

$$1-s_i^*=\frac{(n-1)\lambda_i}{\sum_{j=1}^n\lambda_j}$$

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#### Inefficiency

- In the symmetric case (λ<sub>i</sub> = λ<sub>j</sub> = λ), equal sharing is optimal: s<sub>i</sub><sup>\*</sup> = <sup>1</sup>/<sub>n</sub>.
- Reason: due to convex costs, there is diminishing returns to providing incentives to each agent with bonus shares.
- In the symmetric case, equilibrium effort

$$e^* = rac{ heta}{ extsf{n}\lambda} < rac{ heta}{\lambda} = \widehat{e}$$

- Sharing output creates a free rider problem. The inefficiency increases as the size of the team increases.
- In single agent moral hazard theory with a risk-neutral principal, inefficient effort levels arise due to either:
  - ▶ agent risk aversion (need to trade off incentive and insurance).
  - limited liability (cannot punish failure enough).

#### Capitalist Firm

- Ownership and labour are separated.
- Capitalist (principal) can act as a **budget-breaker**: possible to pay the workers less or more than total output produced.
- This restores efficiency by creating credible punishments for underperformance, or credible rewards for meeting targets.
- Under certainty, budget-breaking only happens off the equilibrium path.
- If θ is a random variable, the principal will have to pay or receive money with positive probability.
- Under some assumptions on the distribution of θ, the expected payment to/from the principal can be made vanishingly small.

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#### Budget-breaking

- Allow only punishments: the principal can capture some of the output in some scenarios.
- Consider a group punishment rule:

$$s_i(y) = \left\{ egin{array}{cc} heta \widehat{e} & ext{if } y \geq n heta \widehat{e} \ 0 & ext{otherwise}, \end{array} 
ight.$$

- Team members are paid equally if and only if they meet the efficient production target.
- $(\hat{e}, \hat{e}, ..., \hat{e})$  is one NE of this game.
- Deviating upwards not profitable: no extra reward.
- If deviating downwards, best to choose e = 0 and earn 0 payoff. However, θe − ½λ (e)² > 0.

#### What is Leadership?

"The key to successful leadership today is influence, not authority." —Kenneth Blanchard

"A good general not only sees the way to victory; he also knows when victory is impossible."

—Polybius

"Not the cry, but the flight of a wild duck, leads the flock to fly and follow."

-Chinese Proverb

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#### Leadership (Hermalin 1998)

- Productivity is uncertain:  $\theta \sim U[0, 1]$ .
- Symmetric agents:  $\lambda_i = \lambda_j = 1$ .
- Information is valuable: expected surplus is higher when agents are informed than when they are not.
- Intuition: when agents know θ, they can customize their efforts to the true state of productivity.
- Expected surplus under full information:

$$S^{f} = \int_{0}^{1} \left[ \theta^{2} - \frac{1}{2}n\left(\frac{\theta}{n}\right)^{2} \right] d\theta = \frac{2n-1}{6n}$$

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#### Value of Information

If nobody is informed, effort choice under under equal shares:

$$\overline{e} = \arg \max_{e} \int_{0}^{1} \left[ \frac{\theta e}{n} - \frac{1}{2} e^{2} \right] d\theta = \frac{1}{2n}$$

Social surplus is:

$$S^{u} = \int_{0}^{1} \left[ \theta \overline{e} - \frac{1}{2} n \overline{e}^{2} \right] d\theta = \frac{1}{8n}$$

Value of information:

$$I=S^f-S^u=rac{8n-7}{24n}>0 \ \ ext{for} \ n\geq 1$$

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#### Leadership

- Suppose one agent, L, is privately informed about the true value of θ.
- Information (knowledge, wisdom, experience, connections) is the true source of leadership in this model.
- ► Welfare will improve if the leader can convey his private information to others. However, he has incentive to lie.
- To convey the information credibly, he must engage in some costly action (signaling).
- Two forms of signaling:
  - burning money/giving gifts (leading by sacrifice).
  - exerting high effort himself (leading by example).

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#### Main Results

- Leading by sacrifice restores welfare level under complete information.
- Equal shares remain optimal.
- Leading by example leads to even higher welfare than under full information (immiserising information!)
- Under equal sharing rule, leader works harder than others due to the signaling motive.
- Equal shares is no longer optimal.
- In the optimal contract, leader gets smaller share than others in small teams but bigger share than others in large teams. Leader's optimal share bounded away from zero.

#### Leading by Sacrifice

- Efforts must be chosen simultaneously/privately.
- If the leader wants to convey θ, he must sacrifice t(θ) (endogenous!) publicly and before efforts are chosen.
- t(.) must be such that leader wants to report  $\hat{\theta} = \theta$ .
- Direct (truth-telling) mechanism. By Revelation Principle, no other mechanism can do better.
- Can also be thought of as the separating equilibrium of a signaling game.
- Since there is a continuum of types, the slope of the t(.) function is pinned down by the incentive constraint.
- t(.) may be a transfer to other agents, but may also be a pure waste (welfare calculations affected).

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#### The Transfer Function

- Assume equal shares.
- Suppose *L* reports  $\hat{\theta}$ . Since followers believe this, each chooses

$$e\left(\widehat{\theta}
ight) = rg\max_{e}\left[rac{\widehat{ heta}e}{n} - rac{1}{2}e^{2}
ight] = rac{\widehat{ heta}}{n}$$

The leader solves:

$$\max_{\widehat{\theta}} \left[ \theta\left(\frac{n-1}{n}\right) \frac{\widehat{\theta}}{n} - t\left(\widehat{\theta}\right) \right]$$

First order condition:

$$t'\left(\widehat{\theta}\right) = rac{(n-1)\theta}{n^2}$$

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#### The Transfer Function

• For truth-telling, the FOC must be satisfied at  $\hat{\theta} = \theta$ . Hence

$$t(\theta) = \int_0^\theta \frac{(n-1)z}{n^2} dz = \frac{(n-1)\theta^2}{2n^2}$$

• Obviously t(0) = 0, so the constant of integration is 0.

- Since equal shares is optimal under full information and information gets revealed by the incentive-compatible transfer function, equal sharing remains optimal.
- The expected transfer the leader makes to convey information can be collected upfront through unconditional transfers from other agents.

#### Leading by Example

- The leader observes  $\theta$  and chooses his effort.
- Other agents observe leader's effort and choose their own.
- Let  $s_l = s$  be leader's share;  $s_f = \frac{1-s}{n-1}$  each follower's share.
- ► To convince others that productivity is  $\hat{\theta}$ , leader has to choose effort level  $e_l(\hat{\theta})$  (endogenous!)
- Follower's effort choice:

$$e_f\left(\widehat{\theta}\right) = \arg\max_e \left[s_f\widehat{\theta}e - \frac{1}{2}e^2\right] = \frac{(1-s)\widehat{\theta}}{n-1}$$

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#### Leader's Incentive Constraint

The leader solves

$$\max_{\widehat{\theta}} \left[ s\theta \left\{ (n-1) e_f\left(\widehat{\theta}\right) + e_l\left(\widehat{\theta}\right) \right\} - \frac{1}{2} e_l\left(\widehat{\theta}\right)^2 \right]$$

First order condition:

$$s heta\left[1-s+e_{l}^{\prime}\left(\widehat{ heta}
ight)
ight]=e_{l}\left(\widehat{ heta}
ight)e_{l}^{\prime}\left(\widehat{ heta}
ight)$$

• Must hold at 
$$\widehat{\theta} = \theta$$
 (truth-telling):

$$\left[ e_l( heta) - s heta 
ight] e_l'( heta) = s(1-s) heta$$

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#### Equal Shares

The solution to the differential equation:

$$e_l( heta)=k(s) heta$$
 where  $k(s)=rac{1}{2}\left(s+\sqrt{4s-3s^2}
ight)$ 

• If equal shares, 
$$s = \frac{1}{n}$$
 and

$$k\left(rac{1}{n}
ight)=rac{1+\sqrt{4n-3}}{2n}\in\left(rac{1}{n},1
ight)$$
 for  $n\geq 2$ 

- Leader works harder than others.
- ▶ Leader's effort is in between his effort in a full information world and his first best effort  $\hat{e}$ . Social welfare is higher than under full information.

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#### Why is Example Better Than Sacrifice?

- There are two problems in the team:
  - followers do not know productivity.
  - members do not work hard enough.
- > The leader conveys productivity by working harder himself.
- Two wrongs make a right, two vices produce a virtue!
- A more general principle: theorem of the second best (Lipsey and Lancaster).
- In many signaling models, the private signaling activity is socially wasteful. Here it is socially beneficial.

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#### Equilibrium Mimicry

 For some values of the shares, followers literally mimic the leader's effort. This involves

$$s_{l} = \tilde{s} = \frac{1}{1 + n(n-1)}$$
  
$$s_{f} = \frac{1 - \tilde{s}}{n-1} = \frac{n}{1 + n(n-1)}$$

The leader's effort coefficient same as follower's:

$$k(\widetilde{s}) = \frac{n}{1+n(n-1)} = s_f$$

The leader works just as hard as anyone else even though he has a lower stake!

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#### **Optimal Shares**

Optimum s solves:

$$\max_{s} \int_{0}^{1} \theta \left[ e_{l}(\theta) + (n-1)e_{f}(\theta) \right] - \frac{1}{2} \left[ e_{l}(\theta)^{2} + (n-1)e_{f}(\theta)^{2} \right] d\theta$$

Using the expressions derived before:

$$\max_{s} \int_{0}^{1} \left[ \{k(s) + 1 - s\} - \frac{1}{2} \left\{ k(s)^{2} + \frac{(1 - s)^{2}}{n - 1} \right\} \right] \theta^{2} d\theta$$

First order condition:

$$k'(s^*) - 1 = k(s^*)k'(s^*) - rac{1-s^*}{n-1}$$

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#### **Optimal Contract: Properties**

- Leader's share is decreasing in *n*.
- Leader always works harder than his teammates.
- For n ≤ 6, leader's share is less than that of teammates, but greater if n ≥ 7.
- As n→∞, followers' shares converge to 0, but leader's share is bounded below by <u>s</u> = 0.128843.
- In small teams, since leader will work hard to signal, shares are transferred to followers to boost their incentives further.
- In large teams, followers mostly free ride anyway, so transferring shares back to leader is better.

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#### A Different Kind of Leadership

▶ Video clip from Charlie Chaplin's Modern Times.

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Intrinsic vs Extrinsic Motivation

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#### The Conventional View



"A raise might destroy their initiative. The good old carrot and stick bonus keeps them focused."

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Intrinsic vs Extrinsic Motivation

Counter-Productive Incentives

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#### Incentives: Unintended Consequences



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#### Multitasking (Holmstrom and Milgrom 1991)

"If quality were poorly measured, it would be expensive or impossible to maintain good quality while using a piece-rate scheme. Similarly, where individuals spend part of their efforts on individual projects and part on team production, and assuming that individual contributions to the team effort are difficult to assess, it would be dangerous to provide incentives for good performance on the individual projects. The problem, of course, is that individuals may shift their attention from the team activity where their individual contributions are poorly measured to the better measured and well-compensated."

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#### The Forbidden Fruit

And the LORD God commanded the man, "You may freely eat of every tree of the garden; but of the tree of the knowledge of good and evil you shall not eat, for in the day that you eat of it you shall die."

But the serpent said to the woman, "You will not die; for God knows that when you eat of it your eyes will be opened, and you will be like God, knowing good and evil."

Genesis, 2:16-17, 3:4-5.

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#### The Daycare Experiment (Gneezy and Rustichini 2000)

- ▶ 10 day care centres observed over 20 weeks in Haifa, Israel.
- Fines introduced for parents who picked up their kids late.
- Weeks 1-4 no fines.
- ▶ Weeks 5-6:
  - (treatment group: 6 centres) fine of NIS 10 per late arrival.
  - (control group: 4 centres) no fine
- Week 7 onwards: fines withdrawn.
- Turns out fines are counter-productive: parents come late more often!

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#### The Daycare Experiment



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Intrinsic vs Extrinsic Motivation

## The Daycare Experiment



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#### Intrinsic Motivation (Benabou-Tirole 2003)

- No team; return to single-agent production.
- Agent has intrinsic motivation: gets some utility/satisfaction if he undertakes a project and it is successful.
- Project success also gives utility to a principal/mentor.
- The principal, like Hermalin's leader, has private information about success probability.
- The principal does not internalize the agent's cost, giving rise to credibility issues.
- > The principal can provide **extrinsic** motivation: wage, bonus.
- Compensation (a) provides incentive (b) conveys information.

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#### The Model

- Agent's unobservable effort  $e \in \{0, 1\}$ . Cost = c.
- Probability of success if agent spends effort is θ ∈ {θ<sub>H</sub>, θ<sub>L</sub>}, and 0 otherwise.
- $\alpha = \Pr(\theta = \theta_H)$  is the agent's initial self-confidence.
- Principal learns  $\theta$ , agent gets a noisy signal  $\sigma$ .
- ► Signal distribution:  $g_H(\sigma)$  and  $g_L(\sigma)$ , with  $\frac{g_H}{g_L}$  increasing in  $\sigma$  (monotone likelihood ratio property).
- If the task is succesfully completed, principal gets W and agent gets V (intrinsic motivation).
- ► The principal can also offer a bonus b ≥ 0 for succesful task completion (no flat wage).

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#### Assumptions

Agent has enough intrinsic motivation to work in H state but not in L state:

$$\theta_L V < c < \theta_H V$$

Principal has enough stake in the outcome to offer a bonus in the L state:

$$W > \frac{c}{\theta_L} - V = b^*$$

Agent suffers from a lack of confidence (will not spend effort without additional information/incentive):

$$\alpha < \frac{c - \theta_L V}{\left(\theta_H - \theta_L\right) V}$$

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# Equilibrium

- We will ignore participation constraints.
- Full information world: principal offers a bonus of 0 to H and b\* to L.
- This is not a PBE under incomplete information, because the principal will always want to pose as H.
- One PBE (semi-separating). Principal's strategy:
  - principal offers 0 when H.
  - when L, offers 0 with prob  $x^*$  and  $b^*$  with prob  $1 x^*$ .
- Agent's strategy:
  - if  $b = b^*$ , work hard (e = 1).
  - if b = 0, work hard iff  $\sigma \ge \sigma^*$ .

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# Equilibrium

- Have to find equilibrium values for  $x^*$  and  $\sigma^*$ .
- Exploit principal's and agent's indifference conditions.
- If π is agent's posterior on H, he is indifferent between e = 0 and e = 1 under no bonus iff

$$\left[\pi heta_{H} + (1-\pi) heta_{L}
ight] V = c \Rightarrow \pi = rac{c - heta_{L} V}{\left( heta_{H} - heta_{L}
ight) V}$$

By Bayes' rule:

$$\pi = \frac{\alpha g_{H}(\sigma^{*})}{\alpha g_{H}(\sigma^{*}) + (1 - \alpha) x^{*} g_{L}(\sigma^{*})}$$
$$= \frac{1}{1 + \left(\frac{1 - \alpha}{\alpha}\right) x^{*} R(\sigma^{*})} \text{ where } R(\sigma) = \frac{g_{L}(\sigma)}{g_{H}(\sigma)} \quad (1)$$

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# Equilibrium

 Principal's indifference condition: indifferent between offering bonus and not when L.

$$[1 - G_L(\sigma^*)] W = W - b^*$$
  
or,  $G_L(\sigma^*) = \frac{c - \theta_L V}{\theta_L W}$  (2)

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- (1) and (2) define  $x^*$  and  $\sigma^*$ .
- This is the unique PBE satisfying the NWBR property of Cho and Kreps (slightly stronger version of the Intuitive Criterion).

- Trust effect: when θ = θ<sub>H</sub>, the principal strictly prefers to show confidence in the agent by offering no bonus.
- Very likely the agent will receive good news (high realization of σ) about his productivity and will work hard anyway.
- The bonus is an additional source of information to the agent apart from his signal. A bonus offer is **bad news**.
- Role of assumptions:
  - it is necessary that the agent gets an independent noisy signal of his productivity.
  - it is necessary that the agent has some **intrinsic motivation**.

- Inefficiency: in a first-best world, it is always efficient to spend effort. Under incomplete information, sometimes e = 0 even though θ = θ<sub>H</sub>.
- Agent may work less hard when productivity is high!
- Usual conflict between profit maximization and efficiency (remember monopoly).
- A utilitarian planner would like to commit to the following policy: offer bonus b<sup>\*</sup> iff θ = θ<sub>L</sub>.

- Selection vs incentive effect: promise of a bonus (probabilistically) increases effort, but in equilibrium, bonus may be correlated with failure, because they are offered when chances of success are low.
- Randomized vs non-randomized trials: The predicted link between incentives and performance is subtle and depends on how the data was generated.
- If agents are randomly selected for performance pay, bonus and performance should be positively correlated.
- If agents are strategically selected for performance pay, bonus and performance should be negatively correlated.
- You need theory to interpret evidence.

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- Self confidence and trust: The principal trusts the agent more often the higher is his initial self-confidence.
- From (1),  $\left(\frac{1-\alpha}{\alpha}\right)x^*$  is a constant in equilibrium (given other parameters).
- When  $\alpha \uparrow$ , we must have  $x^* \uparrow$  to preserve the constancy.
- ► The ex ante probability of no-bonus,  $\alpha + (1 \alpha)x^*$ , also  $\uparrow$  as  $\alpha \uparrow$ .
- Short vs long run effects of incentives: while b\* is in effect, the agent works harder. But the bonus undermines his self-confidence and leads to lower effort when it is withdrawn.

#### Intrinsic vs Extrinsic Motivation

# Flat Wages

- Contracts of the form (a, b), where a = (unconditional) wage, b = (conditional) bonus.
- ► The following is a PBE. Principal's strategy:

• 
$$a_L = 0, \ b_L = b^* = \frac{c}{\theta_L} - V.$$

• 
$$a_H = c - \theta_L V$$
,  $b_H = b^* = 0$ .

- Agent's beliefs: for any other contract, believe  $\theta = \theta_L$  with probability 1.
- Accept either contract and choose e = 1.
- This is the only equilibrium that satisfies the refinement, NWBR property.

Intrinsic vs Extrinsic Motivation

# Verifying Equilibrium

• Principal's IC when  $\theta = \theta_H$ :

$$heta_{H}\left(W-b_{H}
ight)-\mathsf{a}_{H}\geq heta_{H}\left(W-b_{L}
ight)-\mathsf{a}_{L}\Rightarrow\mathsf{a}_{H}-\mathsf{a}_{L}\leq heta_{H}\left(b_{L}-b_{H}
ight)$$

• Principal's IC when 
$$\theta = \theta_H$$
:

$$\theta_{L}\left(W-b_{L}\right)-\mathsf{a}_{L}\geq\theta_{L}\left(W-b_{H}\right)-\mathsf{a}_{H}\Rightarrow\mathsf{a}_{H}-\mathsf{a}_{L}\geq\theta_{L}\left(b_{L}-b_{H}\right)$$

Replacing the values and combining inequalities:

$$heta_L\left(rac{c}{ heta_L}-V
ight)\leq c- heta_LV\leq heta_H\left(rac{c}{ heta_L}-V
ight)$$

This is always true. The agent's IC is trivially satisfied.

- Offering a bonus is more costly than offering flat wages when chance of success is high. Therefore, flat wage is a credible signal of productivity.
- Information is fully revealed and efficiency is restored.
- Unlike in standard moral hazard models, unconditional rewards can boost morale and encourage hard work.
- > The bonus has similar properties as before:
  - signals bad news.
  - encourages effort, if information were held constant.
  - negatively correlated with productivity/performance.
  - dampens effort and productivity in the long run.

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## Blood Donation: Voluntary or Paid?

- Richard Titmuss (1971) criticized the American system of blood transfusions.
- Purely voluntary system in Britain; mixture of voluntary, paid and family credit systems in USA.
- Low proportion of voluntary donors in the US.
- ▶ High incidence of serum hepatitis among American recipients.
- Greater wastage of stored blood in the American system.
- Chronic shortage in American hospitals.
- American donors disproportionately poor.

## Blood Donation: Voluntary or Paid?

- Criticism from Solow (1971) and Arrow (1972).
- Questions about reliability of data and definitions.
- Cross-sectional comparison: many other potential factors (e.g., cultural differences).
- Main skepticism: supply curves are not downward sloping; money motivation should complement altruistic motives.
- Defense of Titmuss by Singer (1973): reciprocity, categorical imperative. Payments may "crowd out" voluntary donations.

"To try and repay in some small way some unknown person whose blood helped me recover from two operations and enable me to be with my family." –anonymous donor in Britain.

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# The Swedish Experiment (Mellstrom-Johannesson 2008)

- Sahlgrenska University Hospital, Gothenberg, Sweden.
- 262 subjects (undergraduates) divided into 3 groups and asked if they will donate blood:
  - Treatment 1: no rewards offered.
  - **Treatment 2:** compensation of SEK 50 (US \$7) for donation.
  - **Treatment 3:** SEK 50 to be donated to charity.
- Personal payment of SEK 50 can always be donated to charity!
- Subjects drawn from 3 disciplines: (i) medicine (ii) economics and commercial law (iii) education.
- Those who donated blood in the previous 5 years excluded.

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# The Swedish Experiment:Results



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Counter-Productive Incentives

Pro-social Behaviour

### Is Learning Economics Socially Harmful?



# Generosity and Egotism

"[A] millionaire does not really care whether his money does good or not, provided he finds his conscience eased and his social status improved by giving it away."

-George Bernard Shaw, Socialism for Millionaires.

"I have to admit it humbly, my friend, I was always bursting with vanity... I admitted only superiorities in me and this explained my good will and serenity. When I was concerned with others, I was so out of pure condescension, in utter freedom, and all the credit went to me: my self-esteem would go up a degree."

-Albert Camus, The Fall.

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# Charitable Donations (Harbaugh 1998)

- Data: alumni donations to a prestigious law school.
- Before: names of all donors published and exact donation amounts reported.
- After: donors were placed in categories, depending on their donated amounts (e.g., \$100-249 or \$500-999, etc.)
- Donations increased at the lowest amounts within a category but decreased elsewhere.
- Clustering hard to explain if donors are not image conscious.

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Pro-social Behaviour

# Charitable Donations (Harbaugh 1998)



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# Incentives & Pro-Social Behaviour (Benabou-Tirole 2006)

- People have 3 reasons to engage in pro-social behaviour:
  - rewards/punishments (extrinsic motivation)
  - caring about others (intrinsic motivation)
  - caring about image (reputational motivation)
- Increasing rewards/punishments has two effects:
  - direct effect: increases extrinsic motivation
  - indirect effect: may reduce reputational motivation because more bad types engage in pro-social behaviour
- Rewards/punishments may "crowd out" pro-social behaviour.
- Multiple equilibria, reflecting the importance of social norms.

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## The Model

- Action choice: a ∈ {0,1}; a = 1 (pro-social action) carries cost c and reward y.
- Agent gets intrinsic payoff v from choosing a = 1, where v ~ F(v) and private information.
- Agent's payoff is 0 if a = 0, and if a = 1:

$$v + y - c + \mu r$$

where r is reputation gain (endogenous) and  $\mu$  reflects concern about reputation.

- Equilibrium: agent chooses a = 1 iff  $v \ge \overline{v}$ .
- Equilibrium reputation:

$$r(\overline{v}) = \mathbf{E}(v|v \ge \overline{v}) - \mathbf{E}(v|v < \overline{v})$$

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## Equilibrium: Uniform Distribution

• Agent must be indifferent between the two actions at  $v = \overline{v}$ :

$$\Psi(\overline{v}) = \overline{v} + \mu r(\overline{v}) + y = c$$

• Uniform distribution: suppose  $v \sim U[0, 1]$ . Then

$$r(\overline{v}) = \int_{\overline{v}}^{1} v\left(\frac{1}{1-\overline{v}}\right) dv - \int_{0}^{\overline{v}} v\left(\frac{1}{\overline{v}}\right) dv = \frac{1}{2}$$

Equilibrium condition:

$$\Psi(\overline{v}) = \overline{v} + rac{\mu}{2} + y = c$$

• Unique equilibrium and  $y \uparrow$  leads to  $\overline{v} \downarrow$ .

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# Equilibrium: Uniform Distribution



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#### Increase in Rewards



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# Equilibrium: Convex Distribution

- Convex distribution: Let  $F(v) = v^2$ , i.e., f(v) = 2v.
- Reputation function:

$$\begin{aligned} r(\overline{v}) &= \int_{\overline{v}}^{1} v\left(\frac{1}{1-\overline{v}^{2}}\right) 2v dv - \int_{0}^{\overline{v}} v\left(\frac{1}{\overline{v}^{2}}\right) 2v dv \\ &= \frac{2}{3(1+\overline{v})} \end{aligned}$$

Equilibrium condition:

$$\begin{split} \Psi(\overline{\nu}) &= \overline{\nu} + \frac{2\mu}{3(1+\overline{\nu})} + y = c \\ \Psi'(\overline{\nu}) &= 1 - \frac{2\mu}{3(1+\overline{\nu})^3} < 0 \quad \text{if } \mu > 6 \end{split}$$

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# Convex Distribution: Multiple Equilibria



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# Stable Equilibria at Corners



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# Multiple Equilibria and Social Norms

- If society regards some behaviours as a sign of being a good person, good (and even not-so-good) people have an incentive to undertake them, making it self-fulfilling.
- Multiplicity in pure gestures: handshake/namaste.
- Compliance says very little, non-compliance says a lot.
- When norms attach on actions that have costs/benefits of their own, reputation must be weighed against them.
- Compliance may be a stronger signal for **costly actions**.
- If the action involves positive externalities (e.g. charity, public service), society is better off.
- If the action involves negative externalities (e.g., wasteful ceremonies, political correctness), the norm is harmful.

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### Beware of Disequilibrium!



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# Crowding Out

- Binary actions:  $a \in \{0, 1\}$ .
- ► Two types: generous (prob α) and greedy (prob 1 − α). Payoff from a = 1:

generous:	$v - c + \mu r$
greedy:	$y - c + \mu r$

- c ∈ {c<sub>L</sub>, c<sub>H</sub>}, with Pr[c = c<sub>L</sub>] = q. Types and costs independently drawn, private information.
- Reputation gain:

$$r = \Pr(\text{generous}|a = 1) - \Pr(\text{generous}|a = 0)$$

# Crowding Out

#### Assumption:

- $\mu < c_L < v < c_H < v + \mu$
- Implications:
  - greedy never contributes without monetary incentives.
  - generous always contributes if cost is low.
  - when cost is high, generous contributes only if there is enough reputational gain.
- High cost greedy never contributes, and low cost generous always does.
- Focus on low cost greedy and high cost generous. Let σ be the probability of contribution from low cost greedy.

# Equilibrium

- **Region 1:** For  $y \le y_1 = c_L \mu$ , we have  $\sigma = 0$ .
- ► Region 2: For y ∈ (y<sub>1</sub>, y<sub>2</sub>], where y<sub>2</sub> = v − c<sub>H</sub> + c<sub>L</sub>, high cost generous contributes and low cost greedy randomizes: σ ∈ (0, 1).
- Latter's indifference condition:

$$y + \mu r(y) = c_L \Rightarrow r(y) = \frac{c_L - y}{\mu}$$
 (3)

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Define r\* be the reputation level at which high cost generous is indifferent between a = 1 and a = 0:

$$v + \mu r^* = c_H \Rightarrow r^* = \frac{c_H - v}{\mu} \le r(y) \Rightarrow y \le v - c_H + c_L = y_2$$

# Equilibrium

By Bayes' Rule:

$$r(y) = \frac{\alpha}{\alpha + (1 - \alpha)q\sigma(y)}$$
(4)

Image: A math a math

Equating (3) and (4):

$$\sigma(y) = \frac{1}{q} \cdot \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{\mu + y - c_L}{c_L - y}\right)$$

As y ↑, σ ↑. Generous contributes regardless of cost, greedy's contribution probability qσ(y) rises. Monetary incentives have a positive effect.

# Equilibrium

- ▶ **Region 3:** For  $y \in (y_2, c_L)$ , high cost generous chooses a = 0, and low cost greedy randomizes:  $\sigma \in (0, 1)$ .
- The function σ(y) jumps discontinuously downwards at y = y<sub>2</sub>, but rises thereafter.
- For y > y₂, r(y) < r\* so the high cost generous type's strategy is a best response.</p>
- Belief updation formula revised:

$$r(y) = \frac{\alpha q}{\alpha q + (1 - \alpha)q\sigma(y)}$$
(5)

• Combining (3) and (5):

$$\sigma(y) = \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{\mu + y - c_L}{c_L - y}\right)$$

### Equilibrium: Summing Up

Low cost greedy's contribution:

$$\sigma(y) = 0 \text{ if } y \le y_1$$
$$= \frac{1}{q} \cdot \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{\mu+y-c_L}{c_L-y}\right) \text{ if } y_1 < y \le y_2$$

$$= \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{\mu+y-c_L}{c_L-y}\right) \quad \text{if } y > y_2$$

• High cost generous's contribution: 1 if  $y \le y_2$ , 0 if  $y > y_2$ .

• Equilibrium reputational gain:  $r(y) = \frac{c_L - y}{\mu}$  for  $y \in (y_1, c_L)$ .

Counter-Productive Incentives

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Pro-social Behaviour

## Equilibrium in Pictures



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## Choking under Pressure (Arieli et al. 2009)

- ▶ 87 subjects from Madurai did 6 tasks each.
- > The tasks tested creativity, memory and motor skills.
- 3 treatments: high (Rs 400), medium (Rs 40) and low (Rs 4) stakes per task.
- "Very good" performance earns full reward, "good" performance earns half reward, bad performance earns 0.
- ▶ Per capita monthly consumption, rural households: Rs 495.
- Experiment on MIT students: math and physical tasks.
- Another on U. Chicago students: tasks undertaken privately or publicly.

More Experiments

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## The Madurai Results



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## Counter-productive Incentives: Summary

- Domain switch/mental accounting: instead of "doing it for its own sake", people start to think of "doing it for money".
- Negative signal of worth: incentives send the message that the activity is unpleasant or costly, success is unlikely, etc.
- Signaling pro-social motives: money dampens the motive to signal pro-social preferences, since even selfish people will respond to monetary incentives.
- Weakens pro-social motives: since people will be doing it for money anyway, we don't need good Samaritans.
- Choking under pressure: incentives increase effort, but may reduce effectiveness.

## What Shouldn't be for Sale? (Sandel 2012)

- Friendship, love, respect, admiration.
- Traditional problem areas: sweatshop labour, prostitution, kidney sales, child labour.
- Children being paid for each book they read.
- Drug addicted women paid to not have children.
- Customized apologies and wedding toasts bought online.
- Scalping of tickets for Pope's midnight mass.
- Patients paying high subscription fees can see the doctor without waiting in line.
- UK visa application: Premium Service Lounge (cost Rs 2000).

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## Elaine's Birthday Gift from Seinfeld

Video clip from Seinfeld.

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# Sandel's Objections

- Inequality: Price based allocations create unequal access if the income distribution is unequal. Rations or queues give rich and poor an equal shot. We may want that in some domains (e.g., health care).
- Corruption: Paying for some goods and services diminish their value.
  - the immediate supply response (quantity or quality) may be perverse.
  - payment may degrade attitudes and values to have a detrimental effect in the long run.

## Some Responses

- Some markets under criticism are black markets: they are a response to the official suppression of markets.
- Non-market mechanisms can create not only inefficiency but also corruption and injustice (e.g., 2G spectrum allocation, under priced railway tickets, land acquisition).
- Price can signal positive things: appreciation, admiration, etc. (paying a pittance for an artist's work is disrespectful).
- Isn't priority pricing a better response to inequality? Make the rich pay, ration the poor but give it free.
- Do economists not get it? Many of the studies he quotes are from economics journals!

## Allocating Scarce Goods

"Economically efficient allocation... is achieved through the allocation of water to uses that are of high value to society and away from uses with low value. Efficient allocation occurs in a competitive, freely functioning market when supply is in equilibrium with demand."

- Food and Agricultural Organization, UN.

"In a society where everything is for sale, life is harder for those of modest means... If the only advantage of affluence were the ability to afford yachts, sports cars, and fancy vacations, inequalities of income and wealth would matter less than they do today."

- Michael Sandel.

"From each according to his ability, to each according to his need."

– Karl Marx.

Image: A mathematical states and a mathem

Markets and Morality

## High Price, Guaranteed Access

### Alan Gilbert Conducts Scheherazade

Location: Avery Fisher Hall (Directions) Price Range \$30,00 - \$90,00

Fri, Sep, 28, 2012 2:00 PM

Sat, Sep, 29, 2012 8:00 PM

Tue, Oct, 2, 2012 7:30 PM





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Counter-Productive Incentives

Markets and Morality

## Low Price, Rationing

## Student Rush



# An Amazing Orchestra at an Amazing Price. What a Rush!

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Students can purchase \$18:50 rush tickets for select concerts up to 10 days before the concert at nyphil.org/studentrush. Concerts may also be available to purchase at the Avery Fisher Hall Box Office on the day of the performance only. Rush tickets are available on a first come, first serve basis, are subject to availability, and are not available for Orchestra 1, First Tier Center and First Tier Front Boxes.

Students High School and above MUST present a valid ID at the Avery Fisher Hall Box Office to purchase or pick up tickets. Two tickets maximum can be purchased under each valid ID.

\*Senior tickets can only be purchased at the Avery Fisher Hall box office the day of the performance. For senior rush ticket availability please call 212-875-5656.

## A Simple Model

- A continuum of agents of measure 1.
- Each consumes either 0 or 1 unit of a good.
- (Willingness to pay) reservation price  $v \sim U[0, 1]$ .
- (Ability to pay) a fraction have wealth w > 1, fraction have wealth 0.
- Both *v* and *w* are private information.
- Provider has  $x \in (0, 1)$  units available.
- Provider wants to maximize aggregate value (sum of v's).

## Allocation Rules and Welfare

• Market Mechanism: Charge market clearing price  $\hat{p}$ :

$$\lambda (1 - \widehat{p}) = x \Rightarrow \widehat{p} = \frac{\lambda - x}{\lambda}$$

Social welfare (expected):

$$W_m = rac{x}{2} \left(1 + \widehat{p}
ight) = rac{x(2\lambda - x)}{2\lambda}$$

 Lotteries: Give it free but randomly choose recipients. Welfare:

$$W_l = \frac{x}{2}$$

- Assume  $x < \lambda$  (scarcity). Then  $W_m > W_l$ .
- Market mechanism  $\Rightarrow$  higher profits as well as welfare.

## **Price Discrimination**

- Consumers given a choice:
  - subscribe at price p and get guaranteed delivery.
  - subscribe for free and get delivery with prob  $\alpha$ .
- Let v be the cutoff wealthy type who chooses the high price.
   Then

$$\overline{\mathbf{v}} - \mathbf{p} = \alpha \overline{\mathbf{v}} \Rightarrow \mathbf{p} = (1 - \alpha) \overline{\mathbf{v}}$$
 (6)

Rationing variable endogenous:

$$\alpha = \frac{x - \lambda \left(1 - \overline{v}\right)}{1 - \lambda \left(1 - \overline{v}\right)} \tag{7}$$

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## Maximizing Welfare

The measure of welfare:

$$W_{d} = \lambda \left(1 - \overline{\nu}\right) \frac{1}{2} \left(1 + \overline{\nu}\right) + \alpha \lambda \overline{\nu} \cdot \frac{\overline{\nu}}{2} + \alpha (1 - \lambda) \cdot \frac{1}{2}$$

• Using the expression for  $\alpha$ :

$$1 - W_d = \frac{(1 - x)(1 - \lambda + \lambda \overline{v}^2)}{1 - \lambda(1 - \overline{v})}$$

Maximizing W<sub>d</sub> gives the first order condition:

$$\lambda \overline{v}^2 + 2(1-\lambda)\overline{v} - (1-\lambda) = 0$$

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▶ Plug in the solution  $v^*$  in (1) and (2) to get  $\alpha^*$  and  $p^*$ .

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## Features

- For example, suppose  $\lambda = \frac{1}{2}$ .
- ► Then  $v^* = \sqrt{2} 1$ ,  $\alpha^* = 1 \sqrt{2}(1 x)$  and  $p = (2 \sqrt{2})(1 x)$ .
- In this example,  $\alpha^* > 0 \Rightarrow x > 1 \frac{1}{\sqrt{2}}$ .
- Price discrimination does not arise under sufficient scarcity.
- Profit motives lead to all customers treated equally. Social motives create hierarchies!