# Lecture 2: Hidden Action

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# SB: Risk-averse agent I

**Question**

*What are the implications of risk-aversion of agent*

- *for choice of wage rates by the Principal?*
- *for the choice of minimum acceptable wage?*
- *for cost of inducing high effort?*

As before, the P will solve:

$$\max_{w_i, e}\{p(e)V(1-w_1)+(1-p(e))V(-w_0)\}$$

s.t.

$$p(e)u(w_1)+(1-p(e))u(w_0)-e \geq 0. \qquad \text{(IR)}$$

$$p'(e)[u(w_1)-u(w_0)]=1 \qquad \text{(IC)}$$

# SB: Risk-averse agent II

Form the Lagrangian

$$\begin{aligned}\mathcal{L}() &= p(e)V(1-w_1) + (1-p(e))V(-w_0) \\ &+ \lambda[p(e)u(w_1) + (1-p(e))u(w_0) - e] \\ &+ \mu[p'(e)[u(w_1) - u(w_0)] - 1]\end{aligned} \quad (1)$$

the foc w.r.t. $w_1$ and $w_0$ are, respectively,

$$\frac{V'(1-w_1)}{u'(w_1)} = \lambda + \mu\frac{p'(e)}{p(e)} \quad (2)$$

$$\frac{V'(-w_0)}{u'(w_0)} = \lambda - \mu\frac{p'(e)}{1-p(e)} \quad (3)$$

The FOC for $e$ is:

$$p'(e)[V(1-w_1) - V(-w_0)] + (\lambda p'(e) + \mu p''(e))[u(w_1) - u(w_0)] - \lambda = 0. \quad (4)$$

# SB: Risk-averse agent III

### Exercise

Show that

- $\lambda > 0$ and $\mu > 0$, i.e., both IR and IC bind.
- if $w_1^{SB}$ and $w_0^{SB}$ solve (2) and (3), then $w_0^{SB} < w_1^{SB}$.

Recall, in FB, i.e., if IC was not an issue, the foc w.r.t. $w_1$ and $w_0$ will be, respectively,

$$\frac{V'(1 - w_1)}{u'(w_1)} = \lambda \tag{5}$$

$$\frac{V'(-w_0)}{u'(w_0)} = \lambda \tag{6}$$

When P is risk-neutral, we get $w_1 = w_0$.

# SB: Risk-averse agent IV

Now, when P is risk-neutral, the FOCs can be written as

$$\frac{1}{u'(w_1)} = \lambda + \mu \frac{p'(e)}{p(e)} \tag{7}$$

$$\frac{1}{u'(w_0)} = \lambda - \mu \frac{p'(e)}{1 - p(e)} \tag{8}$$

So $w_0 \neq w_1$, i.e., the agent in not insured even when the P is risk-neutral (Why?). Moreover, from foc for $e$, you can verify that

$$e^{SB} < e^*.$$

# Insurance-Efficiency Trade-off: An illustration I

Model:

- $q =$ output; $q = q(e, \epsilon)$; $q \in \{q_L, q_H\}$, $q_L < q_H$.
- Monetary worth of $q = q$ (assume price is 1)
- $\epsilon =$ a random variable, a noise term;
- $e =$ effort level opted by the agent; $e \in \{0, 1\}$.
- $\psi(0) = 0$ and $\psi(1) = \psi$.
- $p_H = Pr(q = q_H|e = 1)$ is the probability of the realized output being $q_H$; and $p_L = Pr(q = q_H|e = 0)$.
- $w =$ wage paid by the principal to the agent; $w(.) = w(q)$.
- Let $w(q_L) = w_L$ and $w(q_H) = w_H$.

# Insurance-Efficiency Trade-off: An illustration II

Payoffs:

- Principal: $V(x) = x$, $V' > 0$, $V'' = 0$;
- Agent: $u(w, e) = u(w) - \psi(e)$, where $u' > 0$, $u'' < 0$.

Let

$$p_H q_H (1 - p_H) q_L - \psi > p_L q_H (1 - p_L) q_L$$

Suppose the P wants to induce $e = 1$.

**First Best:** In the FB, i.e., when $e$ is contractible, risk-neutral P solves

$$\max_{w_L, w_H} \{p_H(q_H - w_H) + (1 - p_H)(q_L - w_L)\}$$

s.t.

$$p_H u(w_H) + (1 - p_H) u(w_L) - \psi \geq 0 \qquad (IR)$$

# Insurance-Efficiency Trade-off: An illustration III

**Ex:** Show that IR will bind and the FB entails $w_L = w_H = w^*$ s.t.

$$p_H u(w^*) + (1 - p_H)u(w^*) = u(w^*) = \psi \quad (9)$$

Let $h() = u^{-1}(.)$.

The FB cost of inducing effort $e = 1$ is given by

$$u(w^*) = \psi, i.e.,$$

$$C^{FB} = w^* = h(\psi). \quad (10)$$

The P can implement $e = 0$, simply by fixing $w_L = w_H = 0$ or by firing the agent. In that case, P's payoff is

$$p_L q_H + (1 - p_L)q_L$$

# Insurance-Efficiency Trade-off: An illustration IV

So, P will induce $e = 1$ iff

$$\begin{aligned} p_H(q_H - w^*) + (1 - p_H)(q_L - w^*) &\geq p_L q_H + (1 - p_L) q_L, i.e., \\ p_H q_H + (1 - p_H) q_L - w^* &\geq p_L q_H + (1 - p_L) q_L, i.e., \\ \Delta p \Delta q &\geq w^*, i.e., \\ \Delta p \Delta q &\geq h(\psi), \end{aligned} \tag{11}$$

where $\Delta p = p_H - p_L$ and $\Delta q = q_H - q_L$. Clearly,

$\Delta p \Delta q$ is the increase in expected profit when effort is increased from $e = 0$ to $e = 1$.

# Insurance-Efficiency Trade-off: An illustration V

**Second Best:** In SB, $e$ is not contractible. Suppose the P wants to induce $e = 1$. Then, risk-neutral P will solve

$$\max_{w_L, w_H} \{p_H(q_H - w_H) + (1 - p_H)(q_L - w_L)\}$$

s.t.

$$\begin{aligned} p_H u(w_H) + (1 - p_H)u(w_L) - \psi &\geq 0 \\ p_H u(w_H) + (1 - p_H)u(w_L) - \psi &\geq p_L u(w_H) + (1 - p_L)u(w_L) \end{aligned}$$

Replace $u(w_H)$ with $u_H$ and $u(w_L)$ with $u_L$.

$$p_H u_H + (1 - p_H)u_L - \psi \geq 0 \quad (12)$$

$$p_H u_H + (1 - p_H)u_L - \psi \geq p_L u_H + (1 - p_L)u_L \quad (13)$$

Now, P will solve

$$\max_{u_L, u_H} \{p_H[q_H - h(u_H)] + (1 - p_H)[q_L - h(u_L)]\}$$

# Insurance-Efficiency Trade-off: An illustration VI

s.t. (12) and (13) hold. Clearly, since $h()$ is strictly convex, and constraints are linear, now we have a concave programme. Letting $\lambda$ and $\mu$ as multiplier for (12) and (13), respectively. The foc w.r.t. $u_H$ and $u_L$ are

$$\begin{aligned} -p_H h'(u_H) + \lambda \Delta p + \mu p_H &= 0 \\ -(1-p_H)h'(u_L) - \lambda \Delta p + \mu(1-p_H) &= 0, i.e., \end{aligned}$$

Next, we want to show that $\lambda > 0$ and $\mu > 0$. For this rewrite the foc to get

$$\begin{aligned} -\frac{p_H}{u'(w_H)} + \lambda \Delta p + \mu p_H &= 0 \\ -\frac{(1-p_H)}{u'(w_L)} - \lambda \Delta p + \mu(1-p_H) &= 0 \end{aligned}$$

$$\frac{1}{u'(w_H)} = \mu + \lambda \frac{\Delta p}{p_H} \quad (14)$$

$$\frac{1}{u'(w_L)} = \mu - \lambda \frac{\Delta p}{1-p_H} \quad (15)$$

# Insurance-Efficiency Trade-off: An illustration VII

$p_H(14) + (1 - p_H)(15) \Rightarrow$

$$\mu = \frac{p_H}{u'(w_H)} + \frac{1 - p_H}{u'(w_L)} > 0 \tag{16}$$

Also, (14) and (15) give us

$$\lambda = \frac{p_H(1 - p_H)}{\Delta p}\left(\frac{1}{u'(w_H)} - \frac{1}{u'(w_L)}\right) > 0 \tag{17}$$

in view of $u^{''} < 0$ and $w_H > w_L$. Note that (13) implies

$$u(w_H) > u(w_L), \ i.e., \ w_H > w_L$$

(Verify ?). Therefore, both (12), i.e., IR and (13), i.e., IC are binding constraints.
Indeed, (12) and (13), together give us

# Insurance-Efficiency Trade-off: An illustration VIII

$$w_H = h(\psi + \frac{\psi(1-p_H)}{\Delta p}) \quad (18)$$

$$w_L = h(\psi - \frac{\psi p_H}{\Delta p}) \quad (19)$$

Therefore, $w_L \neq w_H$. Also, note that

$$\psi = p_H u(w_H) + (1-p_H)u(w_L) < u(p_H w_H + (1-p_H)w_L)$$

The equality holds since (IR) binds and the inequality follows from the concavity of $u$. That is,

$$\psi < u(p_H w_H + (1-p_H)w_L), i.e.,$$

$$h(\psi) < p_H w_H + (1-p_H)w_L, i.e., \quad (20)$$

# Insurance-Efficiency Trade-off: An illustration IX

comparing (14) and (20), the expected wage payment is higher under SB. In other words,

$$C^{FB} = h(\psi) < p_H w_H + (1 - p_H) w_L = C^{SB}, \tag{21}$$

where $w_H$ and $w_L$ are as above.