Moral Hazard: Applications

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Wholesale contracts I

Examples:

- Principal as a (car) Manufacturer and Agent as a Sale agency
- Principal as a Producer and Agent as a Retailer
- Principal as an Owner of a brand and Agent as a (Franchiser)

Model:

- $c =$ marginal cost of production
- $p =$ is the final MRP of the the good/service
- $D(p, e, \epsilon)$ is the market demand of the the good/service
- $\epsilon =$ a random variable, a noise term;
- $D(p) \in \{D_L(p), D_H(p)\}$ where $D_L(p) < D_H(p)$
- $e =$ effort level opted by the agent; $e \in \{0, 1\}$.

Wholesale contracts II

- $\psi(0) = 0$ and $\psi(1) = \psi$.
- $\pi_1 = Pr(D(p) = D_H(p)|e = 1)$; and $\pi_0 = Pr(D(p) = D_H(p)|e = 0)$; $\pi_1 > \pi_0$.
- $w =$ wage/profit share paid by the principal to the agent; $w(.) = w(D(p))$.
- Let $w(D_L) = w_L$ and $w(D_H) = w_H$.

Payoffs:

- Principal: $V(x) = x,\ V' > 0,\ V'' = 0$;
- Agent: $u(w, e) = u(w) - \psi(e)$, where $u' > 0$, $u'' < 0$.

Wholesale contracts III

Suppose the P wants to induce $e = 1$.

First Best: In the FB, i.e., when e is contractible, risk-neutral P solves

$$\max_{(w_L, p_L),(w_H, p_H)} \{\pi_1[(p_H - c)D_H(p_H) - w_H] + (1 - \pi_1)[(p_L - c)D_L(p_L) - w_L]\}$$

s.t.

$$\pi_1 u(w_H) + (1 - \pi_1)u(w_L) - \psi \geq 0 \qquad (IR)$$

Ex: Show that IR will bind and the FB entails $w_L = w_H = w^*$ s.t.

$$p_1 u(w^*) + (1 - p_1)u(w^*) = u(w^*) = \psi \tag{0}$$

Moreover, the FB p_L^* and p_H^*, respectively, solve

$$p_L + \frac{D_L(p_L)}{D_L'(p_L)} = c \tag{1}$$

$$p_H + \frac{D_L(p_H)}{D_L'(p_H)} = c \tag{2}$$

Wholesale contracts IV

Let $h() = u^{-1}(.)$. The FB cost of inducing effort $e = 1$ is

$$C^{FB} = w^* = h(\psi). \tag{2}$$

Second Best: In SB e is not contractible. Suppose the P wants to induce $e = 1$. Then, risk-neutral P will solve

$$\max_{(w_L, p_L),(w_H, p_H)} \{\pi_1[(p_H - c)D_H(p_H) - w_H] + (1 - \pi_1)[(p_L - c)D_L(p_L) - w_L]\}$$

s.t.

$$\begin{aligned} \pi_1 u(w_H) + (1 - \pi_1)u(w_L) - \psi &\geq 0 \\ \pi_1 u(w_H) + (1 - \pi_1)u(w_L) - \psi &\geq \pi_0 u(w_H) + (1 - \pi_0)u(w_L) \end{aligned}$$

Replace $u(w_H)$ with u_H and $u(w_L)$ with u_L.

Wholesale contracts V

$$\pi_1 u_H + (1-\pi_1)u_L - \psi \geq 0 \quad (1)$$
$$\pi_1 u_H + (1-\pi_1)u_L - \psi \geq \pi_0 u_H + (1-\pi_0)u_L \quad (2)$$

As before it is easy to show that both (IR) and (IC) are binding. IR and IC, together give us

$$u(w_H) = \psi + \frac{\psi(1-\pi_1)}{\Delta\pi}$$
$$u(w_L) = \psi - \frac{\psi\pi_1}{\Delta\pi}$$

or

$$w_H = h(\psi + \frac{\psi(1-\pi_1)}{\Delta\pi})$$
$$w_L = h(\psi - \frac{\psi\pi_1}{\Delta\pi})$$

Wholesale contracts VI

Therefore, $w_L \neq w_H$, i.e., risk is shared with the agent. Also, note that

$$\psi = \pi_1 u(w_H) + (1 - \pi_1)u(w_L) < u(\pi_1 w_H + (1 - \pi_1)w_L)$$

The equality holds since (IR) binds and the inequality follows from the concavity of u. That is,

$$h(\psi) < \pi_1 w_H + (1 - \pi_1)w_L, i.e., \tag{-2}$$

Again, the expected wage payment is higher under SB.

Wholesale contracts VII

However, the SB prices, p_L^{SB} and p_H^{SB}, respectively, solve

$$
\begin{aligned}
p_L + \frac{D_L(p_L)}{D_L'(p_L)} &= c \qquad (-1)\\
p_H + \frac{D_L(p_H)}{D_L'(p_H)} &= c \qquad (0)
\end{aligned}
$$

That is,

$$
\begin{aligned}
p_L^{SB} &= p_L^*\\
p_H^{SB} &= p_H^*
\end{aligned}
$$

So, there is no distortion as far as MRPs are concerned.

Insurance Contracts: Monopoly I

Model:
Principal is a risk-neutral Insurance Company and Agent is a risk-averse individual

- Agent has wealth W and faces a risk of accident
- Loss in the event of accident is $d > 0$;
- $e =$ precautionary effort level opted by the agent; $e \in \{0, 1\}$.
- $\psi(e)$ is cost of effort, $\psi(0) = 0$ and $\psi(1) = \psi$.
- π is the probability of accident $\pi = \pi(e)$
- $\pi_1 = \pi(e = 1)$ and $\pi_0 = \pi(e = 0)$, $\pi_1 > \pi_0$

Suppose the P wants to induce $e = 1$.

Insurance Contracts: Monopoly II

Contract: $(I, \delta d)$:

- I is the insurance premium charged Company
- δd is compensation payed by the insurance company if accident, $\delta \in [0, 1]$
- If no accident, then insurance company pays no compensation

Payoffs:

- Company: $V(x) = x$, $V' > 0$, $V'' = 0$;
- Individual: $u(w, e) = u(w) - \psi(e)$, where $u' > 0$, $u'' < 0$.

Assuming that the agent takes care and if no accident occurs the payoffs are

- Company: $V(I) = I$;
- Individual: $u(W - I) - \psi = u(w_H) - \psi$, where $w_h = W - I$.

Insurance Contracts: Monopoly III

If accident, the payoffs are

- Company: $V() = I - \delta d$;
- Individual: $u(W - I - d + \delta d) - \psi = u(w_L) - \psi$, where $w_L = W - I - d + \delta d$.

Note

- $w_H = W - I$, i.e., $I = W - w_H$.
- $w_L = W - I - d + \delta d$, i.e., $I - \delta d = W - d - w_L$.
- $\delta < 1 \Rightarrow w_L < w_H$.

Insurance Contracts: Monopoly IV

In the absence of contract, if the agent takes care his expected utility is

$$\pi_1 u(W) + (1 - \pi_1)u(W - d) - \psi.$$

If he does not take care his expected utility is

$$\pi_0 u(W) + (1 - \pi_0)u(W - d).$$

Assume that

$$\pi_1 u(W) + (1 - \pi_1)u(W - d) - \psi > \pi_0 u(W) + (1 - \pi_0)u(W - d).$$

Therefore, the reservation utility is

$$\bar{u} = \pi_1 u(W) + (1 - \pi_1)u(W - d) - \psi.$$

Insurance Contracts: Monopoly V

First Best: In the FB, i.e., when e is contractible, risk-neutral P solves

$$\max_{I,\delta}\{\pi_1 I + (1-\pi_1)(I-\delta d)\}$$

$$\max_{w_L, w_H}\{\pi_1(W-w_H) + (1-\pi_1)(W-d-w_L)\}$$

s.t.

$$\pi_1 u(w_H) + (1-\pi_1)u(w_L) - \psi \geq \bar{u} \qquad (IR)$$

Ex: Show that IR will bind and the FB entails $\delta = 1$, i.e., $w_L = w_H = w^*$ s.t.

$$\pi_1 u(w^*) + (1-\pi_1)u(w^*) - \psi = u(w^*) - \psi = \bar{u} \tag{-1}$$

Let $h() = u^{-1}(.)$. The FB 'cost' of inducing effort $e = 1$ is

$$C^{FB} = w^* = h(\psi + \bar{u}). \tag{0}$$

Insurance Contracts: Monopoly VI

Second Best: In SB e is not contractible. Suppose the P wants to induce $e = 1$. Then, risk-neutral P will solve

$$\max_{w_L, w_H} \{\pi_1(W - w_H) + (1 - \pi_1)(W - d - w_L)\}$$

s.t.

$$\begin{aligned} \pi_1 u(w_H) + (1-\pi_1)u(w_L) - \psi &\geq \bar{u} \\ \pi_1 u(w_H) + (1-\pi_1)u(w_L) - \psi &\geq \pi_0 u(w_H) + (1-\pi_0)u(w_L) \end{aligned} \tag{1}$$

Replace $u(w_H)$ with u_H and $u(w_L)$ with u_L.

$$\begin{aligned} \pi_1 u_H + (1-\pi_1)u_L - \psi &\geq 0 \\ \pi_1 u_H + (1-\pi_1)u_L - \psi &\geq \pi_0 u_H + (1-\pi_0)u_L \end{aligned}$$

Now, you can verify that IR and IC will bind, as before we get

Insurance Contracts: Monopoly VII

$$w_H = h(\psi + \bar{u} + \frac{\psi(1-\pi_1)}{\Delta\pi}) \quad (2)$$

$$w_L = h(\psi + \bar{u} - \frac{\psi\pi_1}{\Delta\pi}) \quad (3)$$

That is $w_L \neq w_H$. Indeed, $w_L < w_H$. Therefore, $\delta < 1$. That is, full insurance coverage is not provided. Also, since IR (1) binds

$$\pi_1 u(w_H) + (1-\pi_1)u(w_L) = \bar{u} + \psi, i.e.,$$

$$u(\pi_1 w_H + (1-\pi_1)w_L) > \pi_1 u(w_H) + (1-\pi_1)u(w_L) = \bar{u} + \psi, i.e.,$$

$$\pi_1 w_H + (1-\pi_1)w_L > h(\bar{u} + \psi), i.e.,$$

$$C^{SB} = \pi_1 w_H + (1-\pi_1)w_L > h(\bar{u} + \psi) = C^{FB}, i.e.,$$

Insurance Contracts: Monopoly VIII

Let the Agency Cost, $AC(\bar{u}(W)) = AC(W) = C^{SB} - C^{FB}$, *i.e.*,

$$AC(W) = \pi_1 w_H + (1-\pi_1)w_L - w^*, \textit{i.e.},$$

$$\begin{aligned} AC(W) &= \pi_1 w_H + (1-\pi_1)w_L - w^* \\ &= \pi_1[h(\psi + \bar{u} + \frac{\psi(1-\pi_1)}{\Delta\pi})] + (1-\pi_1)[h(\psi + \bar{u} - \frac{\psi\pi_1}{\Delta\pi})] \\ &- h(\psi + \bar{u}) \end{aligned}$$

In view of $\bar{u}'(W) > 0$, if $h'()$ is convex, we get

$$\begin{aligned} AC'(W) &= \bar{u}'(W)[\pi_1 h'(\psi + \bar{u} + \frac{\psi(1-\pi_1)}{\Delta\pi}) \\ &+ (1-\pi_1)h'(\psi + \bar{u} - \frac{\psi\pi_1}{\Delta\pi}) - h'(\psi + \bar{u})]. \end{aligned}$$

That is $AC'(W) > 0$.

Insurance Contracts: Competition I

Model:
Principal is a risk-neutral Insurance Company and Agent is a risk-averse individual

- Agent has wealth W and faces a risk of accident
- Loss in the event of accident is $d > 0$;
- $e =$ precautionary effort level opted by the agent; $e \in \{0, 1\}$.
- π is the probability of accident $\pi = \pi(e)$
- $\pi_1 = \pi(e = 1)$ and $\pi_0 = \pi(e = 0)$, $\pi_1 > \pi_0$
- P wants to induce $e = 1$.

Contract: $(I, \delta d)$:
If accident, the payoffs are

- Company: $V() = I - \delta d$;

Insurance Contracts: Competition II

- Individual: $u(W - I - d + \delta d) - \psi = u(w_L) - \psi$, where $w_L = W - I - d + \delta d$.

Note

- $w_H = W - I$, i.e., $I = W - w_H$.
- $w_L = W - I - d + \delta d$, i.e., $I - \delta d = W - d - w_L$.
- $\delta < 1 \Rightarrow w_L < w_H$.

Assume that

$$\pi_1 u(W) + (1 - \pi_1)u(W - d) - \psi > \pi_0 u(W) + (1 - \pi_0)u(W - d).$$

Therefore, the reservation utility is

$$\bar{u} = \pi_1 u(W) + (1 - \pi_1)u(W - d) - \psi.$$

Insurance Contracts: Competition III

Ex: Assume that insurer is a monopolist. Show that under FB, the market will not break down as long as

$$h(\psi + \bar{u}) < W - d(1 - \pi_1)$$

Recall, under FB $w_L = w_H = w^*$ s.t.

$$\pi_1 u(w^*) + (1 - \pi_1) u(w^*) - \psi = u(w^*) - \psi = \bar{u} \tag{4}$$

Let $h() = u^{-1}(.)$. The FB 'cost' of inducing effort $e = 1$ is

$$C^{FB} = w^* = h(\psi + \bar{u}). \tag{5}$$

Ex: Assume that insurance market is competitive. Show that under FB, the agent will get utility U^* such that the following condition holds

$$h(\psi + U^*) = W - d(1 - \pi_1)$$

Insurance Contracts: Competition IV

Second Best: In SB e is not contractible. Suppose the P wants to induce $e = 1$. Then, risk-neutral P will solve

$$\max_{u_L, u_H} \{\pi_1 u(w_H) + (1 - \pi_1) u(w_L) - \psi\}, \textit{i.e.},$$

$$\max_{u_L, u_H} \{\pi_1 u_H + (1 - \pi_1) u_L - \psi\},$$

s.t.

$$\begin{aligned} \pi_1 u(w_H) + (1 - \pi_1) u(w_L) - \psi &\geq \pi_0 u(w_H) + (1 - \pi_0) u(w_L) \\ \pi_1 (W - w_H) + (1 - \pi_1)(W - d - w_L) &\geq 0 \end{aligned}$$

Replace $u(w_H)$ with u_H and $u(w_L)$ with u_L.

$$\begin{aligned} u_H - u_L &\geq \frac{\psi}{\Delta \pi} \\ \pi_1 (W - h(u_H)) + (1 - \pi_1)(W - d - h(u_L)) &\geq 0 \end{aligned}$$

Insurance Contracts: Competition V

You can check that both constraints bind. Let

- U^M denote the utility of consumer in side of the contract.

We can write

$$\begin{aligned} U^M &= \pi_1 u_H + (1-\pi_1)u_L - \psi \\ &= u_H + (1-\pi_1)(u_L - u_H) - \psi \\ &= u_H - (1-\pi_1)\frac{\psi}{\Delta\pi} - \psi. \end{aligned}$$

So, IR for insurer can be written as

Insurance Contracts: Competition VI

$$\begin{aligned}
\pi_1 h(u_H) + (1-\pi_1)h(u_L) &= W - d(1-\pi_1), \\
\pi_1 h(U^M + (1-\pi_1)\frac{\psi}{\Delta\pi} + \psi) + (1-\pi_1)h(U^M - \pi_1\frac{\psi}{\Delta\pi} + \psi) &= W - d(1-\pi_1)
\end{aligned}$$

Clearly, we must have $U^M > \bar{u}$, which implies that we must have

$$\pi_1 h(\bar{u} + (1-\pi_1)\frac{\psi}{\Delta\pi} + \psi) + (1-\pi_1)h(\bar{u} - \pi_1\frac{\psi}{\Delta\pi} + \psi) < W - d(1-\pi_1) \quad (6)$$

However, this condition may or may not hold. So market can break down.

Insurance: General Case I

Suppose

- q denotes the loss in case of accident
- $q = q(e, \theta) \in (-\infty, 0]$, where
- Θ is the set of states of nature and captures randomness.
- $e \in \mathcal{E} \subseteq \mathcal{R}$ and $\theta \in \Theta$
- $\frac{\partial q(e,\theta)}{\partial e} \geq 0$.

Let

- $F(q|e)$ is a conditional cumulative distribution of q
- $f(q|e)$ is the associated conditional density function
- Note: $\frac{\partial q(e,\theta)}{\partial e} \geq 0 \Rightarrow F_e(q|e) \geq 0$ and $\frac{\partial q(e,\theta)}{\partial e} > 0 \Rightarrow F_e(q|e) > 0$.

Payoff functions:

Insurance: General Case II

- Insurance company is risk neutral. So, let

$$V(x) = x, \quad V' > 0, \quad V'' \leq 0$$

- Insuree is risk-averse. So, let

$$u(w, e) = u(w) - \psi(e), \quad u' > 0, \quad u'' \leq 0,$$

  where $\psi(e)$ is the (money) cost of effort e, $\psi' > 0$ and $\psi'' \geq 0$.

Contract:

- The set of contracts is

$$\mathcal{A} = \{(q, w) : q \in \mathcal{R}_{-}, w(q) \in \mathcal{R}\}.$$

- $\bar{w} =$ Certainty equivalent of the reservation (outside) wage
- $\bar{u} = u(\bar{w})$, the reservation utility

Insurance: General Case III

Therefore, in the above setup, the P will solve:

$$\max_{w(q),e}\{\int V(q-w(q))f(q|e)dq\}$$

s.t. IR

$$\int u(w(q))f(q|e)dq-\psi(e)\geq\bar{u}. \tag{7}$$

and

$$e=\arg\max_{\hat{e}}\{\int u(w(q))f(q|\hat{e})dq-\psi(\hat{e})\} \tag{IC}$$

Insurance: General Case IV

the foc w.r.t. $w(q)$ is

$$(\forall q)[\frac{V'(q - w(q))}{u'(w(q))} = \lambda + \mu \frac{f_e(q|e)}{f(q|e)}] \tag{8}$$

Moreover, since P is risk-neutral, the foc is

$$(\forall q)[\frac{1}{u'(w(q))} = \lambda + \mu \frac{f_e(q|e)}{f(q|e)}] \tag{9}$$

Note that risk-sharing is FB *only if*

$$(\forall q)[\frac{V'(q - w(q))}{u'(w(q))} = \text{constant}] \tag{10}$$

Insurance: General Case V

Let $w_\lambda(q)$ solve

$$\frac{1}{u'(w(q))} = \lambda, \tag{11}$$

where λ is the same as in (9). Recall, $w(q)$ solves (9), i.e.,

$$(\forall q)[\frac{1}{u'(w(q))} = \lambda + \mu\frac{f_e(q|e)}{f(q|e)}]$$

Suppose the accident technology is such that

$$f_e(0,e) > 0 \text{ and } (\forall q < 0) f_e(q,e) < 0. \tag{12}$$

That is effort reduces probability of all accidents.
Now, $\mu > 0$ implies that the SB contract is such that:

$$\begin{cases} w(q) \geq w_\lambda(q), & \text{on } Q_+ = \{q|f_e(q|e) \geq 0\}; \\ w(q) < w_\lambda(q), & \text{on } Q_- = \{q|f_e(q|e) < 0\}. \end{cases} \tag{13}$$

Insurance: General Case VI

So, we get

$$\begin{cases} w(q) > w_\lambda(q), & \text{when } q = 0\}; \\ w(q) < w_\lambda(q), & \text{when } q < 0\}. \end{cases} \tag{14}$$

That is,

- Full insurance is not provided
- In case of no accident, the insuree is rewarded
- $w(q)$ is discontinuous at $q = 0$
- Even small accidents invite penalties called deductibles.